Hasanain Alsabonchi

Lab3

1. Please fill in each box with T/F

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| F(n) | G(n) | O | o | Ω | ω | Ɵ |
|  |  | T | T | F | F | F |
|  |  | F | F | T | T | F |
|  |  | T | T | F | F | F |
|  |  | T | T | F | F | F |

**a.     f(n) = Ω(g(n)) implies g(n) = Ω (f (n)) False**

**we can disprove it by a the counterexample, assume its true**

If f(n) = Ω(g(n)) then      0 ≤ c.g(n) ≤ f(n) ∀ n≥ n0

If g(n) = Ω (f (n)) then    0 ≤ c.f(n) ≤ g(n) ∀ n≥ n0

Assume f(n)=n2+2n       g(n)=n2

∃ c=1,f(n) and g(n) no negative function

 0 ≤ c(n2)≤( n2+2n)       True

0 ≤c( n2+2n)  ≤ n2False         **∴ False Does Not apply**

**b.     f(n) =O(g(n)) implies g(n) = O (f (n))  ---> False**

**we can disprove it by a the counterexample, assume its true**

0 ≤ f (n) ≤c g(n)   , ∃ c1=1, ∀n>n0,f(n) and g(n) no negative function

If f(n) = O(g(n))then        0 ≤ f(n) ≤ c.g(n)     ∀ n≥ n0

If g(n) = O (f (n))then      0 ≤ g(n) ≤ c.f(n)      ∀ n≥ n0

Assume f(n)=n2+2n    g(n)=n3

0 ≤  n2+2n  ≤c( n)3   True  ,   0 ≤ n3 ≤c( n2+2n)   False

**c.     f(n) = Ω(g(n)) implies g(n) = O (f (n)) === true**

By definition

0 ≤ cg (n) ≤f (n)

 ∃ c1=1, ∀n>n0

If     f(n) = Ω(g(n))    then **0 ≤ c.g (n) ≤f (n)** ,

If g(n) = O (f (n))  then **0 ≤f (n) ≤c.g (n)**  Divided by 1/c

0 ≤ g (n) ≤1/c(f (n))    True, 0 ≤1/c(f (n)) ≤g (n)  False

d.   **f(n) + g(n) = Ɵ (max f (n), g(n)) ===> true**

0 ≤ c1.g(n) ≤ f (n) ≤ c2.g(n)    by Direct proof

∃ c1=1,c2=10, ∀n>n0,

**Case 1:**If g(n) < f(n)  ,g(n) is smaller than f(n)   True

  0 ≤ c₂.g(n) ≤ f (n)+g(n) ≤ c₁.g(n)    ∀ 0<c1<c2

**Case 2**: If f(n) <g(n)   True

 0 ≤ c₁.f(n) ≤ f (n)+g(n) ≤ c₂.g(n)

∴ satisfy   which is true  ∴ implies **for f(n) >g(n) or g(n) >f(n)**

**e.     f(n) + g(n) = Ɵ (min f (n), g(n))  ---> False**

Disprove by the counterexample

assume it’s  true , 0 ≤ c₁.g(n) ≤ f (n) ≤ c₂.g(n).

f(n) and g(n ) not negative function

∃ c1=1,c2=10, ∀n>n0,

**Case1** : if g(n) <f(n)

                  assume f(n)= n3  ,g(n)= n

0 ≤ c₁.g(n) ≤ **f (n)+g(n)** ≤ c₂.g(n)

0 ≤ n≤ **n3+n≤ 10n**    **False** which is contradict with our assumption.

**Case2** : if f(n) <g(n)

0 ≤c₁.f(n) ≤ g(n)+f(n) ≤ c1.f(n)

0 ≤ n3≤ **n3+n**≤ 10 n3

           alse which is contradict with our assumption.

**∴ False**

**f.     f(n) + g(n) = Ɵ (sum f (n), g(n)) --->True**

0≤c₁ **(**f(n) + g(n)**)**≤**f (n)+ g(n)** ≤c₂.**(**f(n) + g(n)**)**   ∀n>n0,

∴ **it is True  for ∀ 0<c1<c2**

g. **f(n) = Ɵ (f(n/2))   False**

f(n) = Ɵ (f(n/2)) therefore c₁f(n/2) ≤ f(n) ≤ c₂f(n/2)

counterexample:

                                 if f(n) = 2n

Then f(n/2) =

                                       0≤c₁ 2n/2 ≤2n≤c₂ 2n/2

we can not find any c2

that make this equation true. therefore it is false.

**h.     f(n) = Ɵ ((f(n))2) ----> False**

By counter example, assume it’s true

  0 ≤c₁(f(n) )2≤ f(n)≤ c₂(f(n))2

assume it’s

∃ c1=1,c2=10, ∀n>n0, assume f(n)= n2

0 ≤(n)4≤ n2≤ 10(n))4

           Here it's not satisfy  ∴ implies

**i.     f(n) = Ω(f(n))--->true**

 0 ≤  c.g(n) ≤  f(n) ∀n>n0,∃ c1=1

0 ≤  1.f(n) ≤  f(n) True   ∀c=1

**j.f(n) = O(f(n))  ---> True**

0 ≤ f (n) ≤c g(n)   ∀n>n0, ∃ c1=1,

0 ≤ f (n) ≤c f(n)   True ∀c<1

**K.    f(n) + Ω(f(n)) = Ɵ(f(n))====> True**

**0≤g(n)≤ f(n)  by definition of**Ω

0≤c₁f(n)≤ **f(n) + Ω(f(n))**≤c₂ .f(n)  , ∀n>n0,c2=10,

     c₁.f(n)≤f(n)===> (c+1) f(n)

 0≤c₁.f(n)≤(c+1) f(n)≤c₂.f(n)

∴ True

**l.     f(n) + o(f(n)) = Ɵ(f(n))----->False**

**By counter example,**

assume **f(n)= n2 ,o(f(n)) =n3** , non negative functions,

0≤c₁.f(n)≤ **f(n) + o(f(n))**≤c₂.f(n)       ∄ c2

0 ≤c₁n2 ≤ **n2  +**  **n3**≤c₂ n2

**there is no c2 that make this statement true.**