

**INFO 6205**  
**Program Structures & Algorithms**  
**Fall 2018**  
**Assignment 5-Report**

In this assignment, I ran an experiment for to test that the depth/height of a Binary Search Tree after  $M$  (Hibbard) deletions and insertions.

**1. Conclusion:**

- While doing random insertion and deletion for  $x$  times, the height of the tree is coming to be proportional to square root of  $n$ .  
 $h = O(n^{1/2})$   
Where  $h$  = height of the tree  
 $n$  = No. of nodes.  
 $O()$  = big - Oh
- Initially, the height of tree is equivalent to  $n^{1/2}$  but as the number of nodes increase the height of tree tends towards  $\log(n)$ .  
 $h = \log(n)$  For higher values of  $n$ .

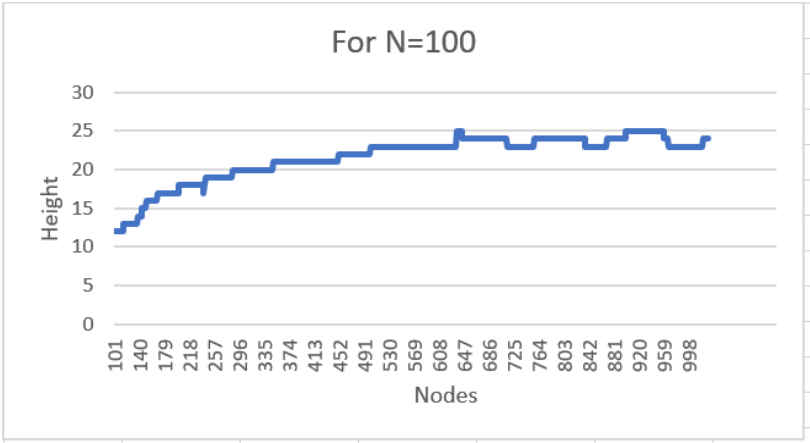
**2. Graphical Representation –**

I made three different graphs to analyze the behavior.

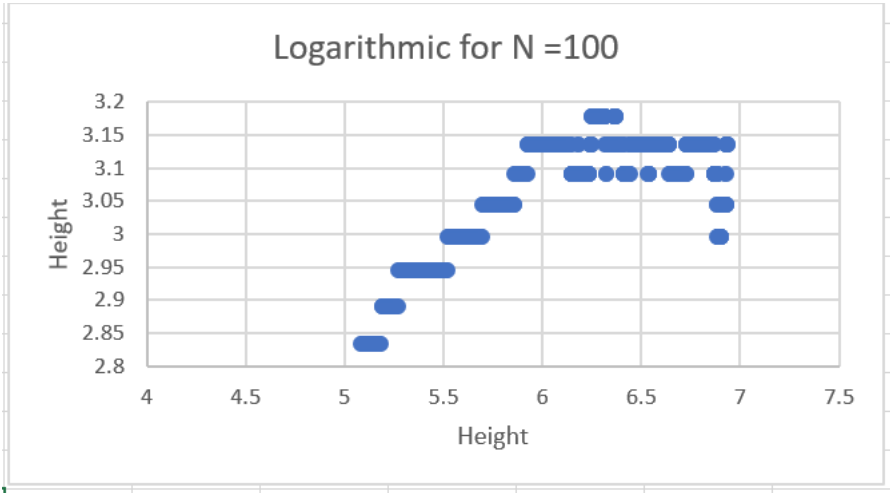
- For node values from  $N = 100$ , in this I also generated an initial binary tree with 100 values than performed the operations.
- Second Graph is between  $\log(n)$  and  $\log(\text{height})$  values.
- Thirdly, I plotted a graph for values from  $N = 1$ .

In the below graphical representations  $(N)$  = No of Nodes.  
The graph is plotted between Nodes and Height. The Y-axis represent the height, whereas the X-axis represent the Nodes.

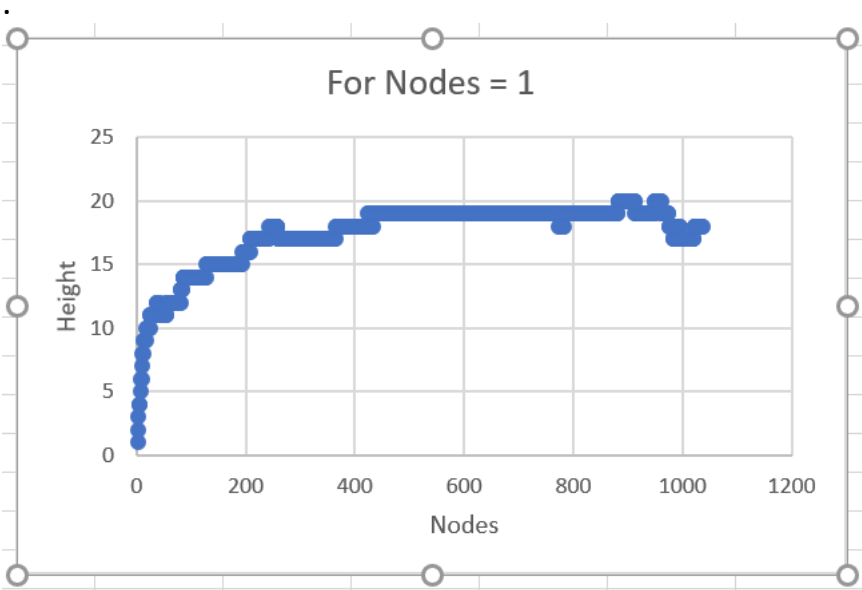
Graph – 1



Graph-2



Graph-3



### 3. Proof:

- To prove the above relationship, I have generated number of nodes in an random order.
- I took a value m, for which I performed insertion and deletion operations. I am performing the insertion and deletion too by generating a random Boolean value.
- Lastly, calculated the height of the tree and analyzed the height with respect to the number of nodes or the size of the tree.
- Analyzing the above graphs, we can get the following result-

From Graph-2, we have plotted a log-log curve, and it appears to be a straight line (although it is not visible properly, because I have plotted the graph of a lot of values and the difference between the values are very less).

So, we can consider the equation of graph as straight line.

That is,  $\text{Log}(d) = m * \text{Log}(n) + c$

For the values taken from the graph-2

$\text{Log}(d) = 3.05$

$\text{Log}(n) = 5.9$

Putting the above values in the equation:-

Lets, calculate the slope of the line,

$$m = 3.05/5.9$$

$$m = \frac{1}{2} \text{ (approx.)}$$

Now, putting the value back to our equation –

$$\text{Log}(d) = m * \text{Log}(n) + c$$

$$\text{Log}(d) = \frac{1}{2} * \text{Log}(n) + c$$

Considering c to be Log of some value So,

$$\text{Log}(d) = \text{Log}(n^{1/2}) + \text{Log}(c)$$

$$\text{Log}(d) = \text{Log}(n^{1/2} * c)$$

Taking anti- log on both the sides,

$$d = n^{1/2} * c$$

Since c is a constant so,

d is proportional to  $n^{1/2}$ .

Hence Proved Height is proportional to  $n^{1/2}$ .

## Hence Proved