Artificial Intelligence

Assignment 2

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GROUP HOLLERITH - SOLUTION

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1 Propositional Logic (60 Points)

The semantics of propositional logic was defined in the lecture for propositional formulae. This definition can be extended to sets of formulae. A interpretation I satisfies a set of propositional formulae Δ, written $I \models \Delta$ iff $\forall \psi \in \Delta : I \models \psi$. $\mathsf{Mod}(\Delta)$ and $\Delta \vdash \phi$ can be defined analogous.

Explain why the following statements are correct—if possible. Otherwise, argue why they are incorrect (Δ and Γ represent sets of propositional formulae).

a) $\{p \Rightarrow r, q \Rightarrow r, p \vee q, \neg r\}$ is satisfiable.

b) $\{p \Rightarrow (q \vee r)\} \vdash (p \Rightarrow r)$.

c) $\{a \vee b, p, q, \neg p\} \vdash a$.

d) If $\Gamma \vdash \phi$ and $\Delta \nvdash \phi$ then $\Gamma \cup \Delta \vdash \phi$.

e) If $\Gamma \vdash \phi$ and $\Delta \vdash \phi$ then $\Gamma \cap \Delta \vdash \phi$.

f) If $\Gamma \nvdash \phi$ then $\Gamma \vdash \neg\phi$.

SOLUTION:

Truth table:

p	q	r	$p \Rightarrow r$	$q \Rightarrow r$	$p \vee q$	$q \vee r$	$\neg r$	$p \Rightarrow (q \vee r)$
T	T	T	T	T	T	T	F	T
T	F	T	T	T	T	T	F	T
F	T	T	T	T	T	T	F	T
F	F	T	T	T	F	T	F	T
T	T	F	F	F	T	T	T	T
T	F	F	F	T	T	F	T	F
F	T	F	T	F	T	T	T	T
F	F	F	T	T	F	F	T	T

a) As per definition, a formula is satisfiable if ther[illegible]ny interpretation that satisfies it. With the truth table, there are more than one possible assignment to be it satisfiable.

b) As per definition, a formula is satisfiable if there is any interpretation that satisfies it. With the truth table, there are more than one possible assignment to be it satisfiable.

Here, $\{p \Rightarrow (q \vee r)\}$ satisfies $(p \Rightarrow r)$. According to the truth table, both sides of this statement are satisfiable. Therefore, the statement is correct.

c) $\{a \vee b, p, q, \neg p\} \vdash a$. This statement is correct.

d) As an interpretation *I* satisfies a set of propositional formulae Δ, written $I \models \Delta$ iff $\forall \psi \in \Delta : I \models \psi$.

Mod(Δ) and $\Delta \vdash \phi$ can be defined analogous. So, the statement is correct.

$\Gamma \cup \Delta \vdash \phi$ is correct as Γ satisfies ϕ but Δ doesn't satisfy ϕ. But either of these satisfies ϕ. So, the statement is true.

e) As an interpretation *I* satisfies a set of propositional formulae Δ, written $I \models \Delta$ iff $\forall \psi \in \Delta : I \models \psi$.

Mod(Δ) and $\Delta \vdash \phi$ can be defined analogous. So, the statement is correct.

$\Gamma \cap \Delta \vdash \phi$ is correct as Γ satisfies ϕ and Δ satisfies ϕ. Both of these satisfies ϕ. Thus, the statement is true.

f) As an interpretation *I* satisfies a set of propositional formulae Δ, written $I \models \Delta$ iff $\forall \psi \in \Delta : I \models \psi$.

Mod(Δ) and $\Delta \vdash \phi$ can be defined analogous. So, the statement is not correct.

If $\Gamma \vdash \phi$ then $\Gamma \vdash \neg\phi$ where Γ doesn't satisfies ϕ, thus, Γ doesn't guarantee to satisfy negation of ϕ.

2 Penguins and Polar Bears (40 Points)

Formalize the statements below in first-order logic. Use only the following predicates and equality for this:

- *polarBear(X)*: X is a polar bear
- *penguin(X)*: X is a penguin
- *human(X)*: X is a human
- *mammal(X)*: X is a mammal
- *fish(X)*: X is a fish
- *raisedByHand(X)*: X was raised by hand
- *likes(X, Y)*: X likes Y
- *eats(X, Y)*: X eats Y
- *feeds(X, Y)*: X feeds Y

and the constants *tom* and *knut*.

With equality (inequality), you can state that terms are equal (not equal). With that, you could state for example that each student has a unique enrollment number

$\forall X((student(X) \wedge enrollmentNr(X, N_1) \wedge enrollmentNr(X, N_2)) \Rightarrow N_1 = N_2))$.

Formalize:

(a) Tom is a human.

(b) Knut likes Tom.

(c) Knut is a polar bear and was raised by hand.

(d) All polar bears are mammals.

(e) Penguins only eat fish.

(f) Polar bears that have been hand-raised like at least one human.

(g) Polar bears that have been hand-raised do not like other polar bears.

(h) If two polar bears like each other, neither of them was hand-raised.

(i) Penguins raised by hand like all humans.

(j) Penguins like any person who feeds them.

SOLUTION:

a) human (Tom)

b) likes (Knut, Tom)

c) polarBear(Knut) ∧ raisedByHand(Knut)

d) $\forall_x$ (polarbear(x) ⇒ mammal(x))

e) $\exists_x$ (fish(x) ⇒ eat(penguin,x))

f) raisedByHand(polarbears) ⇒ likes(polarbears, $\exists_y$(human(y)))

g) $\forall_x$(polarbear(x) ∧ raisedByHand(x)) ⇒ ¬like((polarbear(x) ∧ raisedByHand(x)), ($\forall_y$ (pb(y) ∧ ¬ (raisedByHand(y))))

h) (polarbear(x) ∧ polarbear (y) ∧ like(x,y)) ⇒ ¬(raisedByHand(x) ∧ raisedByHand(y))

i) $\forall_y$(polarbear(x) ∧ raisedByHand(x)) ⇒ like((polarbear(x) ∧ raisedByHand(x)), human(y))

j) $\forall_x$ ((person(x) ∧ feed(x, penguin)) ⇒ (like(penguin,x)))