Week 5 math436

lemma 3.4 if $\lambda_1, \dots, \lambda_k$ are distinct eigenvalues of a linear map $\alpha: V \to V$ and $v_1, \dots, v_k$ are corresponding eigenvectors, then the set $\{v_1, \dots, v_k\}$ is linearly independent

corollary 3.5 if V is a finite dimensional, and $\alpha: V \to V$ is a linear operator, it has at most $\dim(V)$ different eigenvalues

thm 3.8 fundamental thm of algebra

for a polynomial $p(x)$ with coefficients in $\mathbb{C}$ of degree n, $p(x)$ can be factored into factors

$$p(x) = c(x - z_1) \dots (x - z_n)$$

where $c, z_1, \dots, z_n$ are complex numbers

for a polynomial $q(x)$ of degree n with coefficients in $\mathbb{R}$, $q(x)$ has at most n zeros, not necessarily in $\mathbb{R}$. If z is a root of $q(x)$, then so is the complex conjugate $\bar{z}$.

thm 3.9 let V be a finite-dimensional complex vector space and let $\alpha: V \to V$ be a linear map. Then α has an eigenvalue

Lemma 3.11 For a finite dimensional vector space V with basis $v_1, \dots, v_n$ and a linear map $\alpha: V \to V$ the following are equivalent

(a) the matrix is upper triangular

(b) for each j, $\alpha(v_j) \in \operatorname{span}_{\mathbb{F}}(v_1, \dots, v_j)$

(c) for each j, the space $\operatorname{span}_{\mathbb{F}}(v_1, \dots, v_j)$ is invariant under α

lemma 3.12 let V be a finite dimensional complex vector space and let $\alpha: V \to V$ be a linear map. Then, there is a basis $v_1, \dots, v_n$ of V with respect to which the matrix of α is upper triangular

Lemma 3.13 if $\alpha: V \to V$ is a linear map with an upper-triangular matrix with respect to a basis $v_1, \ldots, v_n$, then α is invertible if & only if all the diagonal entries