NOME: Sabrina Araújo da Silva   NºUSP: 12566182

P3 - MAT2453

questão 1.

a) $\int x \arcsen(x^2)dx$

usando integração por partes, temos

$$u = \arcsen(x^2) \qquad du \overset{RC}{=} \frac{1}{\sqrt{1-(x^2)^2}} \cdot x^{2\prime}$$

$$= \frac{1}{\sqrt{1-x^4}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}dx$$

$$dv = x\,dx \qquad v = \frac{x^2}{2}$$

$$\int x \arcsen(x^2)dx = \arcsen(x^2) \cdot \frac{x^2}{2} - \int \frac{2x}{\sqrt{1-x^4}} \cdot \frac{x^2}{2} dx$$

$$= \frac{1}{2} \cdot x^2 \cdot \arcsen(x^2) - \int \frac{x^3}{\sqrt{1-x^4}} dx$$

usando substituição

$$u = 1 - x^4 \qquad du = -4x^3 dx$$

$$\Rightarrow -\frac{du}{4} = x^3 dx$$

$$\int \frac{-1}{4\sqrt{u}} du = -\frac{1}{4}\int \frac{1}{\sqrt{u}} du$$

$$= -\frac{1}{4}\int u^{-\frac{1}{2}} du = -\frac{1}{4}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)$$

$$= -\frac{1}{4} \cdot 2 \cdot u^{\frac{1}{2}} = -\frac{1}{2} \cdot u^{\frac{1}{2}}$$

$$= -\frac{1}{2}(1-x^4)^{\frac{1}{2}} = -\frac{1}{2}\sqrt{1-x^4}$$

$$= \frac{1}{2} x^2 \cdot \text{arcsen}(x^2) - \left(-\frac{1}{2}\sqrt{1-x^4}\right)$$

$$\boxed{= \frac{1}{2} x^2 \cdot \text{arcsen}(x^2) + \frac{1}{2}\sqrt{1-x^4} + C}$$

b) $\int \frac{e^x}{e^x(e^x-3)} dx$

$$\int \frac{\cancel{e^x}}{\cancel{e^x}(e^x-3)} dx = \int \frac{1}{(e^x-3)} dx$$

usando substituição

$$u = e^x \quad ; \quad du = e^x dx \Rightarrow \frac{1}{e^x} du = dx$$

$$\int \frac{1}{u-3} \cdot \frac{1}{e^x} du \Rightarrow \int \frac{1}{u-3} \cdot \frac{1}{u} du$$

$$= \int \frac{1}{u \cdot (u-3)} du =$$

usando fração parcial

$$= \int \frac{1}{u(u-3)} du =$$

$$= \frac{1}{u(u-3)} = \frac{A}{u} + \frac{B}{u-3}$$

$$= \frac{1}{u(\cancel{u-3})} = \frac{A(u-3) + Bu}{u(\cancel{u-3})}$$

$$= 1 = A(u-3) + Bu$$

$$u = 3$$

$$= 1 = A \cdot 0 + 3B$$

$$B = \frac{1}{3}$$

$$u = 0$$

$$= 1 = -3A + B \cdot 0$$

$$A = -\frac{1}{3}$$

$$= \frac{-\frac{1}{3}}{u} + \frac{\frac{1}{3}}{u-3} = -\frac{1}{3} \cdot \frac{1}{u} + \frac{1}{3} \cdot \frac{1}{(u-3)}$$

$$= -\frac{1}{3u} + \frac{1}{3(u-3)}$$

$$= \int -\frac{1}{3u} + \frac{1}{3(u-3)}\, du = \int -\frac{1}{3u}\, du + \int \frac{1}{3(u-3)}\, du$$

$$\int -\frac{1}{3u}\, du = -\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} (\ln|u|) + C$$

$$\int \frac{1}{3(u-3)}\, du = \frac{1}{3} \int \frac{1}{u-3}\, du$$

por substituição

$$(t = u-3 \qquad dt = 1\, du)$$

$$= \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} (\ln|t|) = \frac{1}{3} (\ln|u-3|) + C$$

$$= -\frac{1}{3} (\ln|u|) + \frac{1}{3} (\ln|u-3|) + C \qquad (u = e^x)$$

$$= -\frac{1}{3} (\ln(e^x)) + \frac{1}{3} (\ln|e^x - 3|) + C$$

$$\boxed{= -\frac{1}{3} x + \frac{1}{3} (\ln(e^x - 3)) + C}$$

questão 2

Calcule $\int_1^{\sqrt{3}} \frac{1}{x^2\sqrt{x^2+1}}\,dx.$

$$\int_1^{\sqrt{3}} \frac{1}{x^2\sqrt{x^2+1}}$$

usando substituição trigonométrica

$x = tg\,\theta$

$dx = sec^2\theta\;d\theta$

$\sqrt{x^2+1} = sec\,\theta$

$$\int_1^{\sqrt{3}} \frac{1}{tg^2\theta\sqrt{tg^2\theta+1}} \cdot sec^2\theta\;d\theta = \int_1^{\sqrt{3}} \frac{sec^2\theta}{tg^2\theta\sqrt{tg^2\theta+1}}\,d\theta$$

$$= \int_1^{\sqrt{3}} \frac{sec^2\theta}{tg^2\theta\sqrt{sec^2\theta}}\,d\theta = \int_1^{\sqrt{3}} \frac{sec^2\theta}{tg^2\theta\; sec\,\theta}\,d\theta$$

$$= \int_1^{\sqrt{3}} \frac{sec\,\theta}{tg^2\theta}\,d\theta = \int_1^{\sqrt{3}} sec\,\theta \cdot \frac{1}{tg\,\theta} \cdot \frac{1}{tg\,\theta}\,d\theta$$

$$= \int_1^{\sqrt{3}} sec\,\theta \cdot cotg^2\theta\;d\theta$$

$x = tg\,\theta$

$\theta = 1 \Rightarrow x = tg(1) = x = \frac{\pi}{4}$

$\theta = \sqrt{3} \Rightarrow x = tg(\sqrt{3}) = x = \frac{\pi}{3}$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec\theta \cdot \text{cotg}^2\theta \, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec\theta \cdot \text{cotg}\theta \cdot \text{cotg}\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \text{cotg}\,\theta \cdot \text{cossec}\,\theta \quad d\theta$$

$$\left(u = \text{cossec}\,\theta \qquad du = -\text{cossec}\,\theta \cdot \text{cotg}\,\theta \, d\theta \qquad d\theta = -\frac{du}{\text{cossec}\theta \cdot \text{cotg}\theta} \right)$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \text{cotg}\theta \cdot u \cdot \left(-\frac{du}{\text{cossec}\theta \cdot \text{cotg}\theta}\right) = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \text{cotg}\theta \, u \cdot -\left(\frac{du}{\text{cotg}\theta \cdot u}\right)$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\frac{\text{cotg}\theta \, u}{\text{cotg}\theta \, u} \, du = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -1 \, du$$

$$\left\{ \begin{array}{l} u = \text{cossec}\,\theta \\ \theta = \frac{\pi}{4} \qquad u = \text{cossec}\left(\frac{\pi}{4}\right) = \frac{1}{\text{sen}\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} \\ \qquad = \frac{2\sqrt{2}}{2} = \sqrt{2} \\ \theta = \frac{\pi}{3} \qquad u = \text{cossec}\left(\frac{\pi}{3}\right) = \frac{1}{\text{sen}\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \\ \qquad = \frac{2\sqrt{3}}{3} \end{array} \right.$$

$$= \int_{\sqrt{2}}^{\frac{2\sqrt{3}}{3}} -1 \, du = -u \Big|_{\sqrt{2}}^{\frac{2\sqrt{3}}{3}}$$

$$= -\left(\frac{2\sqrt{3}}{3}\right) - \left(-\sqrt{2}\right) = -\frac{2\sqrt{3}}{3} + \sqrt{2}$$

$$\boxed{\int_{1}^{\sqrt{3}} \frac{1}{x^2\sqrt{x^2+1}} = -\frac{2\sqrt{3}}{3} + \sqrt{2}}$$

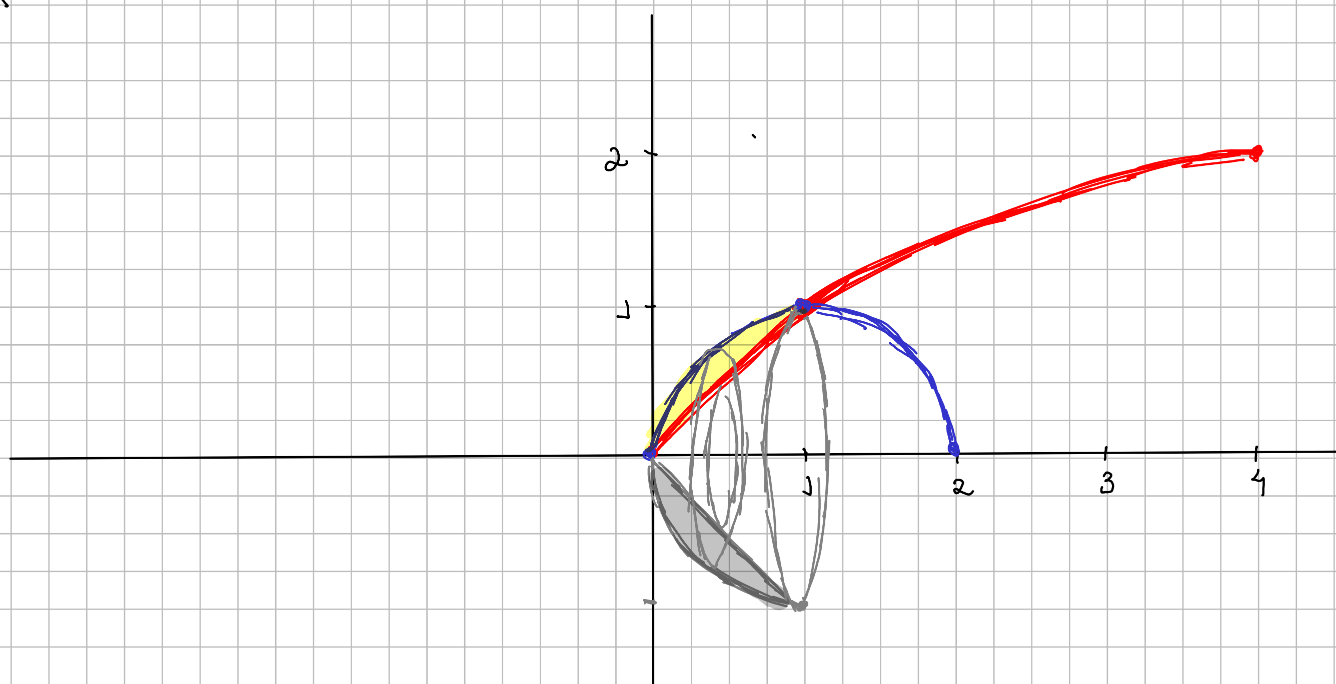
questão 3

Determine o volume do sólido obtido pela rotação da região

$$A = \{(x, y) \in \mathbb{R}^2 : y \geq \sqrt{x} \text{ e } (x-1)^2 + y^2 \leq 1\}$$

ao redor do eixo Ox.

$$\begin{cases} y \geq \sqrt{x} \\ y^2 + (x-1)^2 \leq 1 \quad \Rightarrow \quad y \leq \sqrt{2x - x^2} \end{cases}$$



área da secção transversal de 0 a 1

$$A = \pi\left(\sqrt{2x - x^2}\right)^2 - \pi\left(\sqrt{x}\right)^2$$

$$A = \pi\,(2x - x^2) - \pi\,(x)$$

calcular o volume

$$V = \int_0^1 \pi\,(2x - x^2)\,dx - \int_0^1 \pi\,(x)\,dx$$

$$\pi\int_0^1 2x - x^2\,dx = \pi\left(\frac{2x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1$$

$$= \pi\left(x^2 - \frac{x^3}{3}\right) = \pi\left(1 - \frac{1}{3}\right) - \pi\left(0 - \frac{0}{3}\right)$$

$$= \pi\left(\frac{2}{3}\right)$$

$$\int_0^1 \pi(x)\,dx = \pi\left(\frac{x^2}{2}\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{2}\right) - \pi\left(\frac{0}{2}\right) = \pi\left(\frac{1}{2}\right)$$

$$V = \pi\left(\frac{2}{3}\right) - \pi\left(\frac{1}{2}\right) = \frac{\pi 4 - \pi 3}{6} = \boxed{\frac{\pi}{6}}$$

questão 4

Calcule a derivada de $F(x) = \int_0^{\sqrt{x}} (x + t^2)e^{t^2} dt$, para $x > 0$.

$f(x) = (x + t^2)\, e^{t^2}$

seja F uma primitiva

$F'(x) = f(x) = (x + t^2)\, e^{t^2}$

então $g(x) = \int_0^{\sqrt{x}} (x + t^2)\, e^{t^2} dt = F(t) \Big|_0^{\sqrt{x}} = F(\sqrt{x}) - F(0)$

$g'(x) = \left(F(\sqrt{x}) - F(0) \right)' \underset{R.C}{=} F'(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} - F'(0) \cdot 0$

$= (\sqrt{x} + t^2)\, e^{t^2} \cdot \frac{1}{2\sqrt{x}} = \left(e^{t^2}\sqrt{x} + e^{t^2} t^2 \right) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{e^{t^2}\sqrt{x}}{2\sqrt{x}} + \frac{e^{t^2} t^2}{2\sqrt{x}} = e^{t^2} \left(\frac{\sqrt{x} + t^2}{2\sqrt{x}} \right) = \boxed{e^{t^2} \left(\frac{1}{2} + \frac{t^2}{2\sqrt{x}} \right)}$

questão 5

Determine o comprimento do gráfico de $y = \ln(\sec x)$, para $0 \leq x \leq \frac{\pi}{4}$.

$$\begin{cases} y = \ln(\sec x) \\ x \geq 0 \\ x \leq \frac{\pi}{4} \end{cases}$$

temos que,

$$\boxed{\text{comprimento de curvas} \quad L = \int_a^b \sqrt{1 + f'(x)^2}\, dx}$$

$$f(x) = \ln(\sec x)$$

$$f'(x) \overset{R.C}{=} \frac{1}{\sec(x)} \cdot \sec' = \frac{1}{\cancel{\sec(x)}} \cdot \cancel{\sec x} \cdot \operatorname{tg} x$$

$$f'(x) = \operatorname{tg} x$$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \operatorname{tg}^2 x}\, dx \qquad (\operatorname{tg}^2(x) + 1 = \sec^2 x)$$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x}\, dx$$

$$L = \int_0^{\frac{\pi}{4}} \sec x\, dx = \ln|\sec x + \operatorname{tg} x| \Big|_0^{\frac{\pi}{4}}$$

$$= \ln\left|\sec\left(\frac{\pi}{4}\right) + \operatorname{tg}\left(\frac{\pi}{4}\right)\right| - \ln|\sec(0) + \operatorname{tg}(0)|$$

$$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$tg\left(\frac{\pi}{4}\right) = 1$$

$$\ln|\sqrt{2}+1|$$

$$sec(0) = \frac{1}{\cos(0)} = \frac{1}{1} = 1$$

$$tg(0) = 0$$

$$\ln|1+0| = \ln|1| = 0$$

$$L = \ln|\sqrt{2}+1| - 0$$

$$\boxed{L = \ln|\sqrt{2}+1|}$$