Министерство образования и науки РФ

Федеральное государственное автономное

образовательное учреждение высшего образования

«Санкт-Петербургский национальный исследовательский университет

информационных технологий, механики и оптики»

**факультет программной инженерии и компьютерной техники**

**КУРСОВАЯ РАБОТА**

по дисциплине

‘Дискретная Математика’

Вариант №20

*Выполнил:*

Студент группы P3109

Маллаев Сабур Н.

*Преподаватель:*

Поляков Владимир

Иванович



Санкт-Петербург, 2022

Функция

Функция ƒ(X1, X2, X3, X4, X5) принимает значение 1 при 5 ≤ (X1X2X3 + X4X5) < 9 и неопределенное значение при (X3X4X5) = 7

Таблица истинности

Таблица 1

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| № | X1 | X2 | X3 | X4 | X5 | X1X2X3 | X4X5 | X3X4X5 | ƒ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 0 |
| 3 | 0 | 0 | 0 | 1 | 1 | 0 | 3 | 3 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 4 | 0 |
| 5 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 5 | 0 |
| 6 | 0 | 0 | 1 | 1 | 0 | 1 | 2 | 6 | 0 |
| 7 | 0 | 0 | 1 | 1 | 1 | 1 | 3 | 7 | d |
| 8 | 0 | 1 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |
| 9 | 0 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 |
| 10 | 0 | 1 | 0 | 1 | 0 | 2 | 2 | 2 | 0 |
| 11 | 0 | 1 | 0 | 1 | 1 | 2 | 3 | 3 | 1 |
| 12 | 0 | 1 | 1 | 0 | 0 | 3 | 0 | 4 | 0 |
| 13 | 0 | 1 | 1 | 0 | 1 | 3 | 1 | 5 | 0 |
| 14 | 0 | 1 | 1 | 1 | 0 | 3 | 2 | 6 | 1 |
| 15 | 0 | 1 | 1 | 1 | 1 | 3 | 3 | 7 | d |
| 16 | 1 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 |
| 17 | 1 | 0 | 0 | 0 | 1 | 4 | 1 | 1 | 1 |
| 18 | 1 | 0 | 0 | 1 | 0 | 4 | 2 | 2 | 1 |
| 19 | 1 | 0 | 0 | 1 | 1 | 4 | 3 | 3 | 1 |
| 20 | 1 | 0 | 1 | 0 | 0 | 5 | 0 | 4 | 1 |
| 21 | 1 | 0 | 1 | 0 | 1 | 5 | 1 | 5 | 1 |
| 22 | 1 | 0 | 1 | 1 | 0 | 5 | 2 | 6 | 1 |
| 23 | 1 | 0 | 1 | 1 | 1 | 5 | 3 | 7 | d |
| 24 | 1 | 1 | 0 | 0 | 0 | 6 | 0 | 0 | 1 |
| 25 | 1 | 1 | 0 | 0 | 1 | 6 | 1 | 1 | 1 |
| 26 | 1 | 1 | 0 | 1 | 0 | 6 | 2 | 2 | 1 |
| 27 | 1 | 1 | 0 | 1 | 1 | 6 | 3 | 3 | 0 |
| 28 | 1 | 1 | 1 | 0 | 0 | 7 | 0 | 4 | 1 |
| 29 | 1 | 1 | 1 | 0 | 1 | 7 | 1 | 5 | 1 |
| 30 | 1 | 1 | 1 | 1 | 0 | 7 | 2 | 6 | 0 |
| 31 | 1 | 1 | 1 | 1 | 1 | 7 | 3 | 7 | d |

Представление булевой функции в аналитическом виде

КДНФ: ƒ =

ККНФ: ƒ =

Минимизация булевой функции методом Квайна–Мак-Класки

Нахождение простых импликант (максимальных кубов)

Получение кубов различной размерности кубического комплекса K(f) и выделение из них простых импликант приведено в таблице 2.

Таблица 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **K1(ƒ)** | | | | | | | | |
| 1. | | 100X1 | | | v | 1-8 | | |
| 2. | | 10X01 | | | v | 1-9 | | |
| 3. | | 1X001 | | | v | 1-11 | | |
| 4. | | 1001X | | | v | 2-8 | | |
| 5. | | 10X10 | | | v | 2-10 | | |
| 6. | | **1X010** | | |  | 2-12 | | |
| 7. | | 1010X | | | v | 3-9 | | |
| 8. | | 101X0 | | | v | 3-10 | | |
| 9. | | 1X100 | | | v | 3-13 | | |
| 10. | | 1100X | | | v | 4-11 | | |
| 11. | | **110X0** | | |  | 4-12 | | |
| 12. | | 11X00 | | | v | 4-13 | | |
| 13. | | 0X111 | | | v | 5-14 | | |
| 14. | | X0111 | | | v | 5-15 | | |
| 15. | | **01X11** | | |  | 6-14 | | |
| 16. | | **0111X** | | |  | 7-14 | | |
| 17. | | 10X11 | | | v | 8-15 | | |
| 18. | | 101X1 | | | v | 9-15 | | |
| 19. | | 1X101 | | | v | 9-16 | | |
| 20. | | 1011X | | | v | 10-15 | | |
| 21. | | 11X01 | | | v | 11-16 | | |
| 22. | | 1110X | | | v | 13-16 | | |
| 23. | | X1111 | | | v | 14-17 | | |
| 24. | | 1X111 | | | v | 15-17 | | |
| 25. | | 111X1 | | | v | 16-17 | | |
| **K0(ƒ) ∪ N(ƒ)** | | | | | | | |
| 1. | 10001 | | | v | | | |
| 2. | 10010 | | | v | | | |
| 3. | 10100 | | | v | | | |
| 4. | 11000 | | | v | | | |
| 5. | 00111 | | | v | | | |
| 6. | 01011 | | | v | | | |
| 7. | 01110 | | | v | | | |
| 8. | 10011 | | | v | | | |
| 9. | 10101 | | | v | | | |
| 10. | 10110 | | | v | | | |
| 11. | 11001 | | | v | | | |
| 12. | 11010 | | | v | | | |
| 13. | 11100 | | | v | | | |
| 14. | 01111 | | | v | | | |
| 15. | 10111 | | | v | | | |
| 16. | 11101 | | | v | | | |
| 17. | 11111 | | | v | | | |
| **K2(ƒ)** | | | | | | | | | |
| **10XX1** | | | 1-18 | | | | 2-17 | | |
| **1XX01** | | | 2-21 | | | | 3-19 | | |
| **10X1X** | | | 4-20 | | | | 5-17 | | |
| **101XX** | | | 7-20 | | | | 8-18 | | |
| **1X10X** | | | 7-22 | | | | 9-19 | | |
| **11X0X** | | | 10-22 | | | | 12-21 | | |
| **XX111** | | | 13-24 | | | | 14-23 | | |
| **1X1X1** | | | 18-25 | | | | 19-24 | | |
| **Z(ƒ)** | | |
| 1X010 | | |
| 110X0 | | |
| 01X11 | | |
| 0111X | | |
| 10XX1 | | |
| 1XX01 | | |
| 10X1X | | |
| 101XX | | |
| 1X10X | | |
| 11X0X | | |
| XX111 | | |
| 1X1X1 | | |

Составление импликационной таблицы

Таблица 3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Простые импликанты | | 0-кубы | | | | | | | | | | | | |
| 0 1 0 1 1 | 0 1 1 1 0 | 1 0 0 0 1 | 1 0 0 1 0 | 1 0 0 1 1 | 1 0 1 0 0 | 1 0 1 0 1 | 1 0 1 1 0 | 1 1 0 0 0 | 1 0 0 0 1 | 1 1 0 1 0 | 1 1 1 0 0 | 1 1 1 0 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 1 | 1X010 |  |  |  | X |  |  |  |  |  |  | X |  |  |
| 2 | 110X0 |  |  |  |  |  |  |  |  | X |  | X |  |  |
| 3 | 01X11 | X |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 0111X |  | X |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 10XX1 |  |  | X |  | X |  | X |  |  | X |  |  |  |
| 6 | 1XX01 |  |  | X |  |  |  | X |  |  | X |  |  | X |
| 7 | 10X1X |  |  |  | X | X |  |  | X |  |  |  |  |  |
| 8 | 101XX |  |  |  |  |  | X | X | X |  |  |  |  |  |
| 9 | 1X10X |  |  |  |  |  | X | X |  |  |  |  | X | X |
| 10 | 11X0X |  |  |  |  |  |  |  |  | X |  |  | X | X |
| 11 | XX111 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 | 1X1X1 |  |  |  |  |  |  | X |  |  |  |  |  | X |

Ядро покрытия:

01X11

0111X

XX111

T =

Получим следующую упрощенную импликационную таблицу (Таблица 4):

Таблица 4

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Простые импликанты | | 0-кубы | | | | | | | | | | |
| 1 0 0 0 1 | 1 0 0 1 0 | 1 0 0 1 1 | 1 0 1 0 0 | 1 0 1 0 1 | 1 0 1 1 0 | 1 1 0 0 0 | 1 0 0 0 1 | 1 1 0 1 0 | 1 1 1 0 0 | 1 1 1 0 1 |
| a | b | c | d | e | f | g | h | i | j | k |
| A | 1X010 |  | X |  |  |  |  |  |  | X |  |  |
| B | 110X0 |  |  |  |  |  |  | X |  | X |  |  |
| C | 0111X |  |  |  |  |  |  |  |  |  |  |  |
| D | 10XX1 | X |  | X |  | X |  |  | X |  |  |  |
| E | 1XX01 | X |  |  |  | X |  |  | X |  |  | X |
| F | 10X1X |  | X | X |  |  | X |  |  |  |  |  |
| G | 101XX |  |  |  | X | X | X |  |  |  |  |  |
| H | 1X10X |  |  |  | X | X |  |  |  |  | X | X |
| I | 11X0X |  |  |  |  |  |  | X |  |  | X | X |
| J | 1X1X1 |  |  |  |  | X |  |  |  |  |  | X |

Определение минимального покрытия

Метод Петрика:

Применим закон поглощения к дизъюнктивным термам, в результате чего в выражении остаются только двухбуквенные термы

Приведем выражение к ДНФ:

Возможны следующие варианты покрытия:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| C1 = |  | T  A  D  G  I |  | C2 = |  | T  B  D  F  H |  | C3 = |  | T  B  E  F  H |  | C4 = |  | T  A  B  D  G  H |  | C5 = |  | T  A  D  F  H  I |  |
| S1a = 24 | | | | S2a = 24 | | | | S3a = 24 | | | | S4a = 28 | | | | S5a = 27 | | | |
| S1b = 29 | | | | S2b = 29 | | | | S3b = 29 | | | | S4b = 34 | | | | S5b = 33 | | | |
| C6 = |  | T  A  E  F  G  I |  | C7 = |  | T  A  E  F  H  I |  | C8 = |  | T  B  D  F  G  I |  | C9 = |  | T  B  E  F  G  I |  |
| S6a = 27 | | | | S7a = 27 | | | | S8a = 27 | | | | S9a = 27 | | | |
| S6b = 33 | | | | S7b = 33 | | | | S8b = 33 | | | | S9b = 33 | | | |

Минимальные покрытия функции:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cmin(f) = |  | 01X11  0111X  XX111  110X0  10X1X  1XX01  1X10X |  | Sa = 24, Sb = 31 |

МДНФ:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cmin(f) = |  | 01X11  0111X  XX111  1X010  11X0X  10XX1  101XX |  | Sa = 24, Sb = 31 |

МДНФ:

Минимизация булевой функции на картах Карно

Определение МДНФ

Используем две четырехмерные карты Карно, различающиеся по переменной X1

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | X4X5 | | | |  |  |  | X4X5 | | | |
|  |  | 00 | 01 | 11 | 10 |  |  |  | 00 | 01 | 11 | 10 |
| X2X3 | 00 |  |  |  |  |  | X2X3 | 00 |  | 1 | 1 | 1 |
| 01 |  |  | d |  |  | 01 | 1 | 1 | d | 1 |
| 11 |  |  | d | 1 |  | 11 | 1 | 1 | d |  |
| 10 |  |  | 1 |  |  | 10 | 1 | 1 |  | 1 |
|  |  | X1 = 0 | | | |  |  |  | X1 = 1 | | | |

Получаем

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cmin(f) = |  | 0111X  01X11  110X0  1XX01  10X1X  11X0X  101XX |  | Sa = 24, Sb = 29 |

МДНФ имеет следующий вид:

Определение МКНФ

Используем две четырехмерные карты Карно, различающиеся по переменной X1

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | X4X5 | | | |  |  |  | X4X5 | | | |
|  |  | 00 | 01 | 11 | 10 |  |  |  | 00 | 01 | 11 | 10 |
| X2X3 | 00 | 0 | 0 | 0 | 0 |  | X2X3 | 00 | 0 |  |  |  |
| 01 | 0 | 0 | d | 0 |  | 01 |  |  | d |  |
| 11 | 0 | 0 | d |  |  | 11 |  |  | d | 0 |
| 10 | 0 | 0 |  | 0 |  | 10 |  |  | 0 |  |
|  |  | X1 = 0 | | | |  |  |  | X1 = 1 | | | |

Получаем

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cmin(f) = |  | 0XX0X  00XXX  X0000  1111X  0X0X0  11X11 |  | Sa = 19, Sb = 25 |

МКНФ имеет следующий вид:

Преобразование минимальных форм булевой функции

Факторизация и декомпозиция МДНФ

|  |  |
| --- | --- |
|  | SQ=29 |
|  | SQ=26 |