ARIMA | Model ID

Model ID for ARIMA(p, d, q) Models

$$(B)(1 \quad B)^{d}(X_t \qquad) = \quad (B)a_t$$

where

- $\varphi(B)$ and $\theta(B)$ are pth and qth order operators, respectively
- All roots of $\varphi(z) = 0$ and $\theta(z) = 0$ are outside the unit circle

Properties of ARIMA Models

Recall:

- The $(1-B)^d$ factor dominates the stationary components
  - In the realizations
  - Autocorrelations
  - Spectral densities (not shown) all have peaks at $f = 0$
- For $d > 1$ this domination is even more striking
- The true autocorrelations are equal to one for all lags
- The sample autocorrelations will always damp (in part because of the way they are calculated)
- **Slowly damping sample autocorrelations is an indication of an ARIMA data**

Recall the Issues with Parameter Estimation in Stationary Models with Roots *Close* to the Unit Circle

$$(1 \quad 2.195B + 1.994B^2 \quad .796B^3)X_t = a_t$$

Real root very close to unit circle

Factor	**Abs recip**	**System freq**
$1\text{-}0.995B$	0.9950	0.0
$1\text{-}1.2B+0.8B^2$	0.8944	0.133

	φ_1	φ_2	φ_3
True	2.195	–1.994	.796
YW	1.34	–0.57	.11
Burg	2.12	–1.90	.75
MLE	2.14	–1.92	.76

- Burg and ML estimates are better than YW in near "nonstationary" case

Model ID in Nonstationary Case: General Comments

- YW and Burg estimates always produce stationary AR models
  - Even if data are from an ARIMA model
- ML estimates may produce nonstationary models
  - However, the nonstationarity is often due to roots inside the unit circle rather than on the unit circle like *(1-B)*.
    - i.e. unacceptable models

Key Point

- A decision whether to fit a stationary or nonstationary model must be made by the time series analyst
  - Simply noting that a fitted model has roots outside the unit circle does not in itself indicate stationarity
- The decision whether to fit a nonstationary model may depend on the physical problem being considered
  - For example, data from stationary models have an attraction to a mean value.
    - Is this a reasonable assumption for your data?

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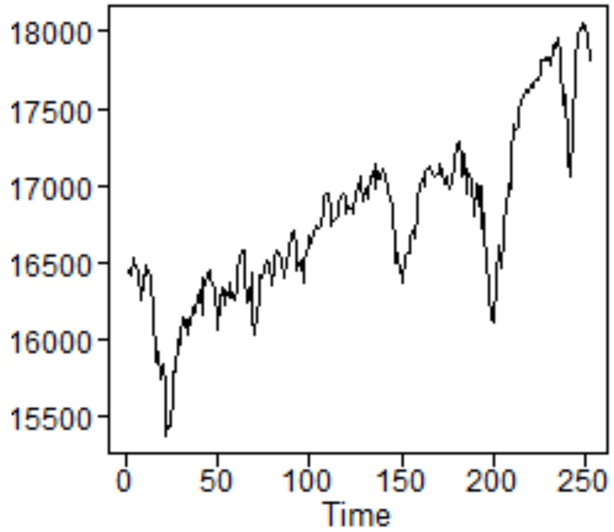
ARIMA | Dow Jones Example Part 1

Dow Jones Data

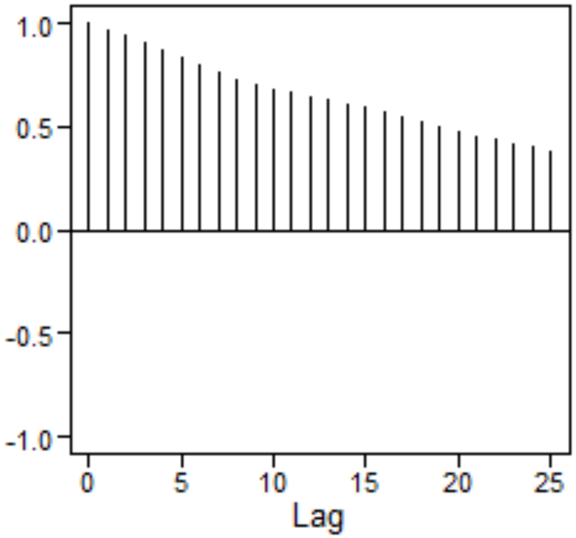
The graphs below are

- A plot of daily closing values of the DOW Jones for 2014
- The associated (damping) sample autocorrelations

DOW Jones 2014



Sample autocorrelations



Dow Jones Data

The tswge commands:

```
data(dowjones2014)
aic.wge(dowjones2014,p=0:5,q=0:2)
```

select the AR(1) model:

$$(1 - .9816B)(X_t - 16778) = a_t$$

Question: Should we use this (stationary) model or a model that includes a $(1-B)$ factor?

- $\hat{\varphi}_1$ is very close to 1
- Sample autocorrelations damp, but they do so slowly (the autocorrelation at lag 10 is nearly .7)
- Do you expect the DOW Jones average to eventually pull back to a mean value like 16,778?
  - Historically the answer to this last question is ***No.***

A nonstationary model is suggested

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ARIMA Box-Jenkins Model ID Procedure

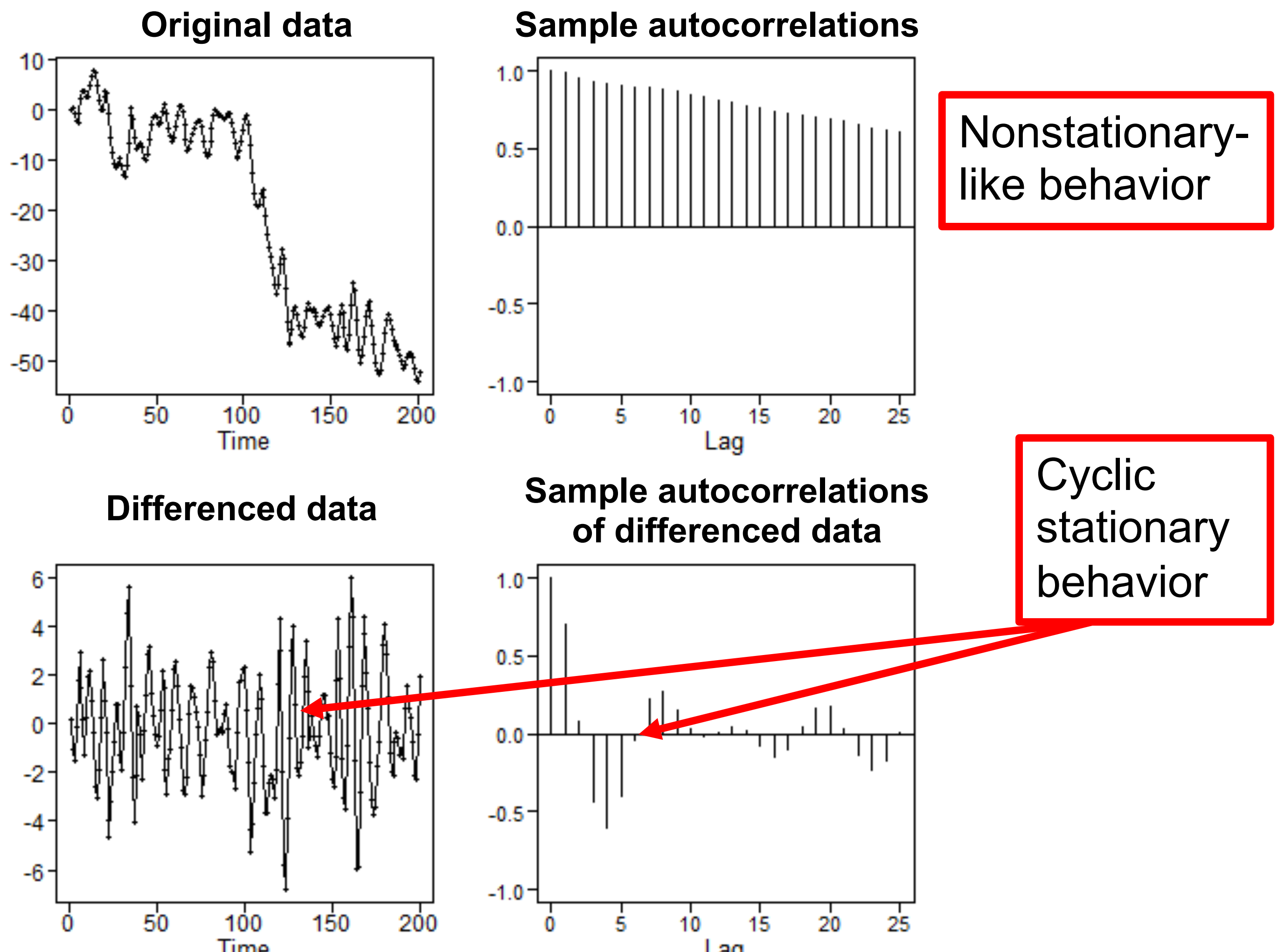
Classical Box-Jenkins Procedure* for Including a Unit Root in the Model

- If data are wandering and sample autocorrelations damp slowly
  - Difference the data
- If differenced data still show evidence of a unit root
  - Difference the data again, etc.

* Reference to "Box-Jenkins" procedures apply to techniques proposed in the Classic Box-Jenkins (1970) text on time series analysis. These include
  - The forecasting procedure we used
  - The stationary model ID procedure we do not recommend
  - The nonstationary differencing techniques listed above

A Model with a Unit Root

$$(1 - B)(1 - 1.2B + .8B^2)X_t = a_t$$



Steps for Obtaining an ARIMA(p,d,q) Model

- Difference the data (possibly multiple times) until the data appear stationary
- Estimate the parameters of the "stationarized data"

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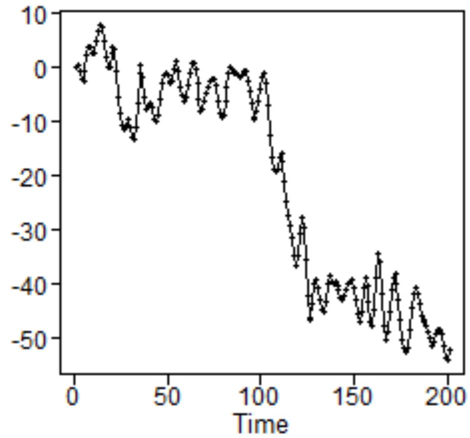
Example 1

1 Unit Root

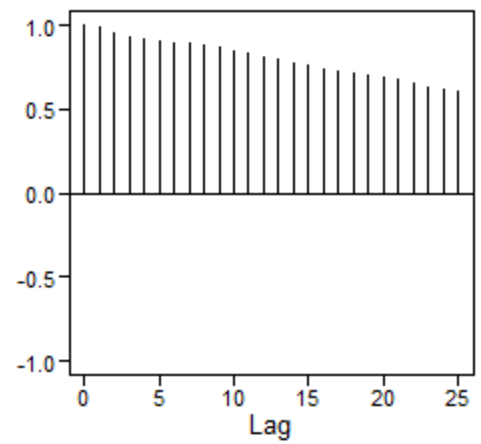
Model with a Unit Root

$$(1 \quad B)(1 \quad 1.2B + .8B^2)X_t = a_t$$

Original data



Sample autocorrelations



Notes:

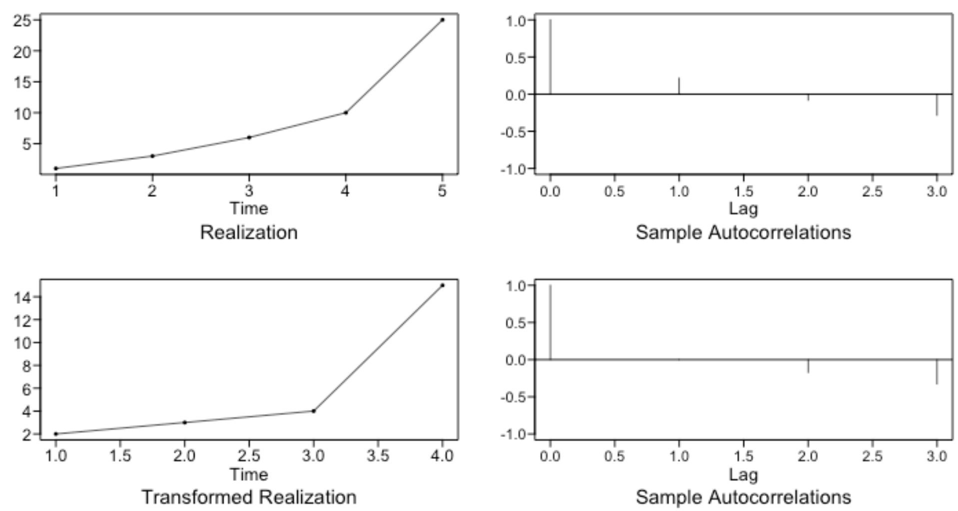
- We decided to difference the data because
  - Dramatic wandering
  - Slowly damping autocorrelations
- Differencing the data adds a factor of $(1–B)$ to the final model of the original data (which will be ARIMA)
- A feasible decision might also have been to go ahead and model the data using a stationary model
  - i.e. fit a model (using AIC) to the original data

Review: “Difference” the Data using tswge

tswge transformation function

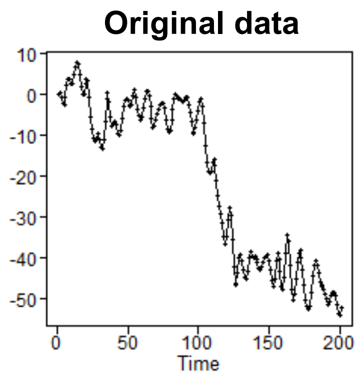
```
# the following command differences the data in x
y=artrans.wge(x,phi.tr=1)
# This simply means that y(i) = x(i) - x(i-1)
# y has length n-1 because x(1) has no x(0) before it.
# Example
x = c(1,3,6,10,25)
y = artrans.wge(x,phi.tr = 1)
y # shows the 4 differences
```

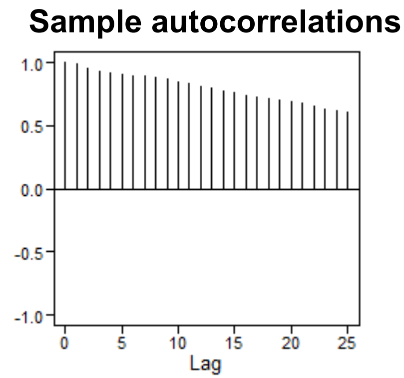
```
> y # shows the 4 differences
[1]  2  3  4 15
```



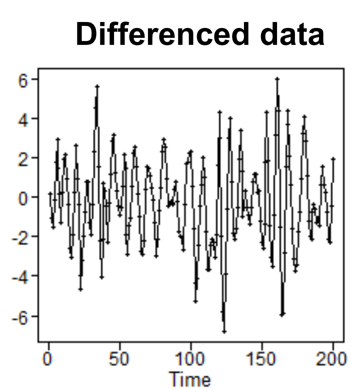
Model with a Unit Root

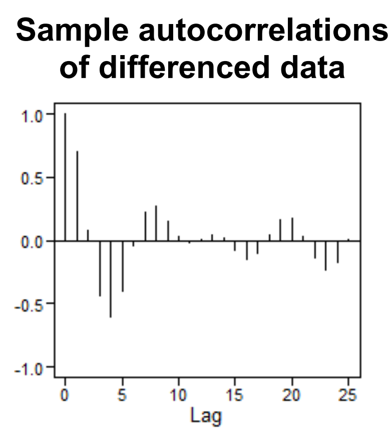
$$(1\quad B)(1\quad 1.2B+.8B^2)X_t = a_t$$






```
# generate ARIMA(2,1,0) data
xd1=gen.arima.wge(n=200,phi=c(1.2,-.8),d=1,sn=56)
# difference the data
xd1.dif=artrans.wge(xd1,phi.tr=1)
# xd1.dif is the differenced data
```

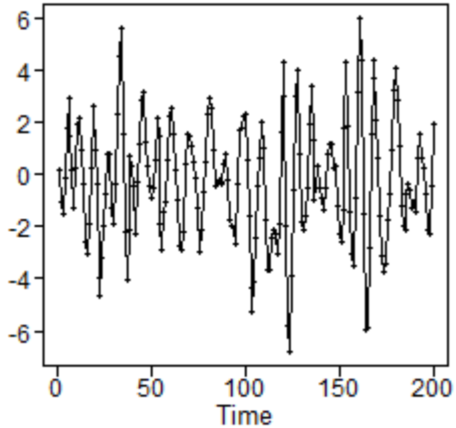




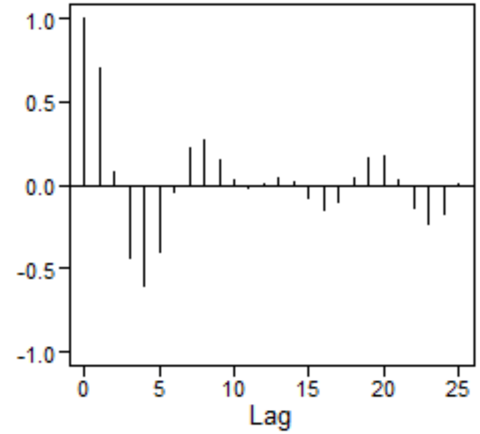
Model with a Unit Root

$$(1 \quad B)(1 \quad 1.2B + .8B^2)X_t = a_t \qquad {}_a^2 = 1$$

Differenced data



Sample autocorrelations of differenced data



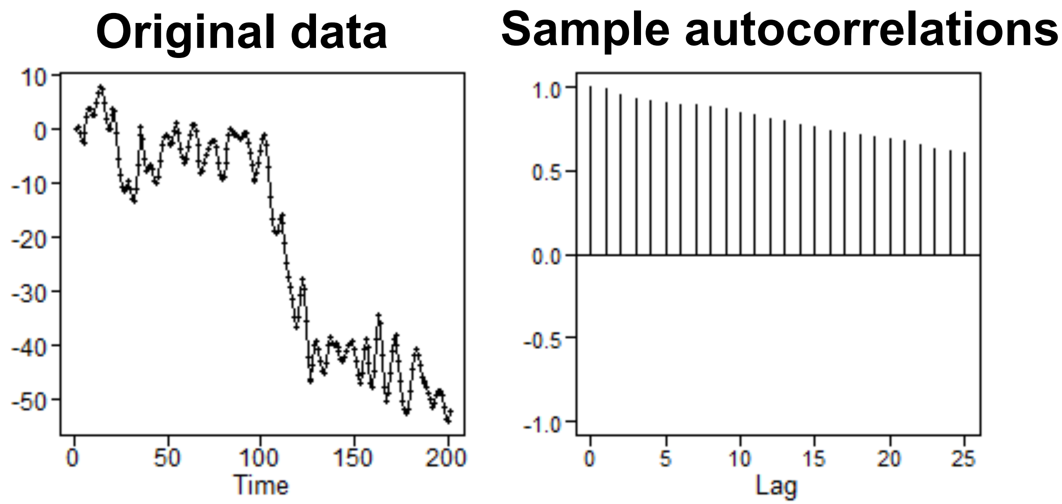
Differenced data (xd1.dif) appear stationary—use AIC

```
aic5.wge(xd1.dif,p=0:5,q=0:2)
#  aic picks AR(2)
est.ar.wge(xd1.dif,p=2)  # $phi [1] 1.265005 -0.802240
#  $avar [1] 0.9293244
mean(xd1)  # -21.22
```

Final model

$$(1-B)(1-1.27B+.80B^2)(X_t+21.22) = a_t \quad \hat{\sigma}_a^2 = .93$$

Or … We Could Have Modelled it with a Stationary Model




```
# generate ARIMA(2,1,0) data
x=gen.arima.wge(n=200,phi=c(1.2,-.8),d=1,sn=56)
# Assume it will eventually make it back to a fixed
# process mean. That is, it is attracted to a constant mean.
aic5.wge(x) # selects a p = 4, q = 0
est.arma.wge(x, p = 4, q = 0)
#phis:  2.3546329 -2.2920782  1.0456011 -0.1101713
#avar: 0.9295115
mean(x) #-21.21951
```

$$(1 - 2.35B + 2.29B^2 - 1.05B^3 + 0.11B^4)(X_t + 21.22) = a_t$$

$$\sigma_a^2 = .9295$$

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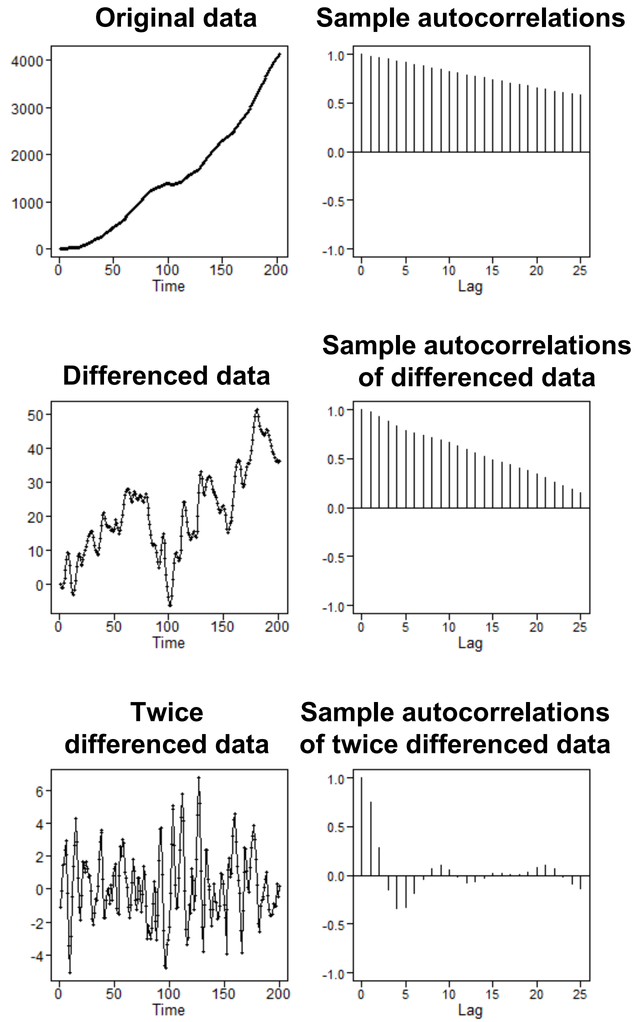
Example 2

2 Unit Roots

A Model with 2 Unit Roots

$$(1-B)^2(1-1.2B+.6B^2)X_t = a_t$$



Nonstationary: no stationary AR(2) behavior visible

Nonstationary: stationary AR(2) behavior not obvious

Stationary AR(2) behavior can now be seen

tswge demo

```
# generate data from the ARIMA(2,2,0) model
x=gen.arima.wge(n=200,d=2,phi=c(1.2,-.6),sn=132,vara=1)
# difference the data
x.d1=artrans.wge(x,phi.tr=1)
# difference the data again since the differenced data
#    still has nonstationary ARIMA characteristics
#     (see previous slide)
 x.d2=artrans.wge(x.d1,phi.tr=1)
#  x.d2 appears to be stationary (see previous slide)
aic5.wge(x.d2,p=0:5,q=0:2)
# AIC picks an AR(2)  # which seems reasonable from
est.ar.wge(x.d2,p=2)
#  $phi[1]  1.2724446 -0.6827008
#   $avar [1] 1.026015
mean(x)  # 1512
```

Final model

$$(1-B)^2(1-1.27B+.68B^2)(X_t-1512)=a_t \quad \hat{\sigma}_a^2=1.03$$

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Comments about ARIMA Models

Comments about ARIMA Models

$$(1 \quad B)(1 \quad 1.2B + .8B^2)X_t = a_t$$

$$(1 - B)(1 - 1.27B + .80B^2)(X_t + 21.22) = a_t \quad \hat{\sigma}_a^2 = .93$$

$$(1 - B)^2(1 - 1.2B + .6B^2)X_t = a_t$$

$$(1 - B)^2(1 - 1.27B + .68B^2)(X_t - 1512) = a_t \quad \hat{\sigma}_a^2 = 1.03$$

- Notice that both of the previous ARIMA models had $\mu = 0$.
- However, the two sample means were $\bar{X} = -21.22$ and 1512, respectively.
- This is because there is no attraction to a mean in ARIMA models.
- For ARIMA models, you can always write the model in zero mean form.

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Review of Forecasts for ARIMA Models

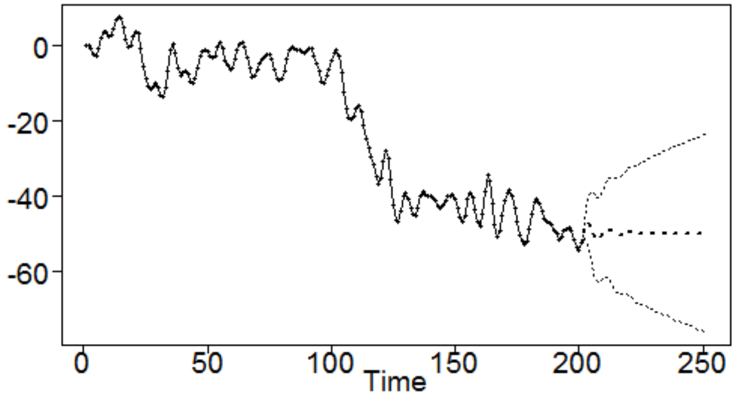
Forecasting with the ARIMA(2,1,0) Model

Fitted model

$$(1-B)(1-1.27B+.80B^2)(X_t+21.22)=a_t \quad \hat{\sigma}_a^2=.93$$

tswge forecasting code

```
# let xd1 denote the realization that produced
#   this model fit
# code for forecasting the next 50 values
fore.aruma.wge(xd1,d=1,phi=c(1.27,-.8),n.ahead=50)
```



Forecasts essentially forecast the last data value (because of a singe $(1-B)$ factor). The AR(2) factor adds a little cyclic behavior for early steps ahead.

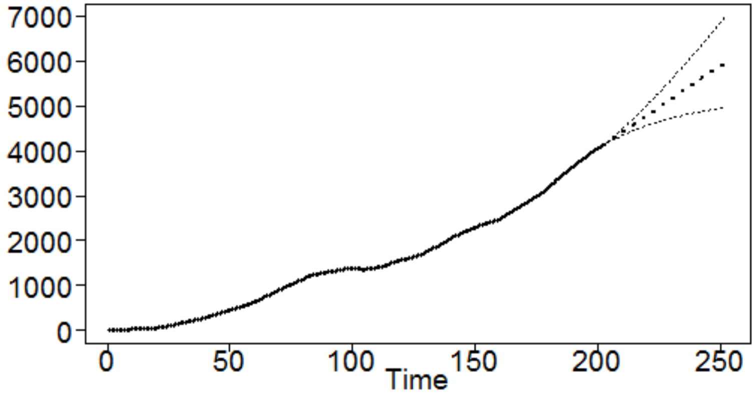
Forecasting with the ARIMA(2,2,0) Model

Fitted model

$$(1-B)^2(1-1.27B+.68B^2)(X_t-1512)=a_t \quad \hat{\sigma}_a^2=1.03$$

tswge forecasting code

```
# let x denote the realization that produced
#   this model fit
# code for forecasting the next 50 values
fore.aruma.wge(x,d=2,phi=c(1.27,-.68),n.ahead=50)
```



Forecasts essentially forecast a linear trend determined by the last two data values. The effect of the AR(2) is not noticeable.

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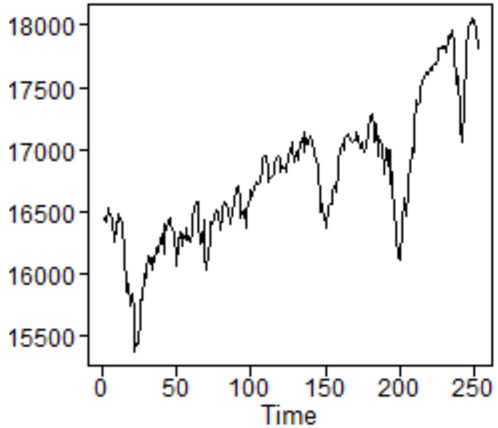
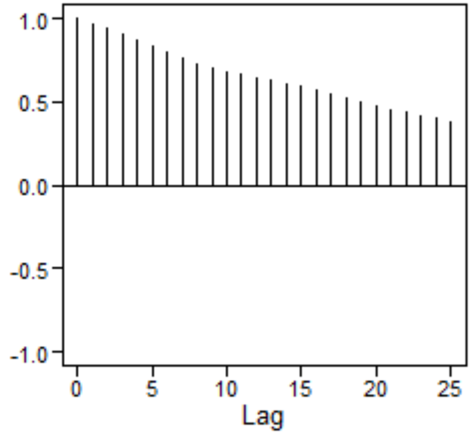
ARIMA | Dow Jones Example Part 2

2014 DOW Jones Data

DOW Jones 2014

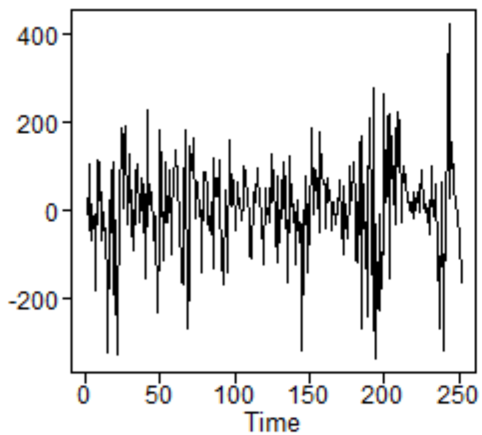


Sample autocorrelations

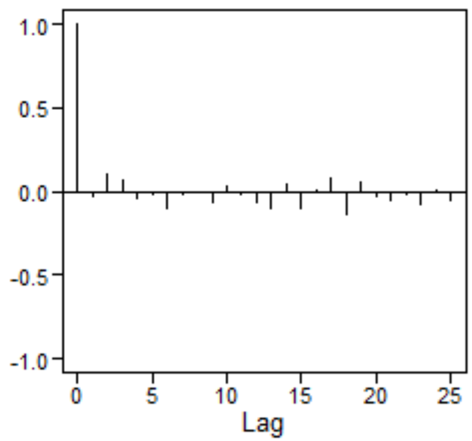


Nonstationary-like behavior

Differenced DOW data



Sample autocorrelations of differenced data



Looks stationary

2014 DOW Jones Data

tswge code for differencing and analyzing the differenced data

```
data(dowjones2014)
# redefine for simplicity
dow=dowjones2014
# difference the data
dow.1=artrans.wge(dow,phi.tr=1)
aic5.wge(dow.1,p=0:5,q=0:2)
#  aic picks ARMA(4,1)
est.arma.wge(dow.1,p=4,q=1)
# $phi [1]  0.9265955  0.1355600 -0.0240474 -
0.1242563
# $theta [1] 0.9999996
#  $avar [1] 12112
mean(dow)  # 16778
```

2014 DOW Jones Data

Let's look at the estimated ARMA(4,1) model

$$(1 \quad .93B \quad .14B^2 + .02B^3 + .12B^4)(X_t \quad 16778) = (1 \quad B)a_t \qquad \hat{\ }_a^2 = 12112$$

which factors into the following factors found using

```
factor.wge(phi=c(.93,.14,-.02,-.12))
```

$$(1 \quad .85B)(1 \quad .68B)(1 + .61B + .21B^2)(X_t \quad 16778) = (1 \quad B)a_t$$

Notes:

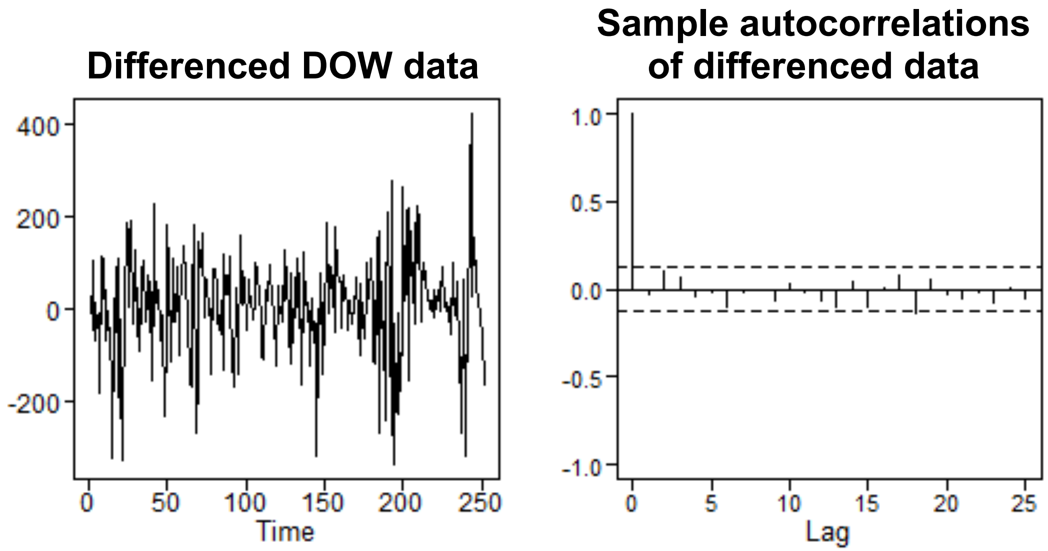
- The above is a noninvertible model, but actually $\hat{\theta}_1$=.9999996 which is "barely" invertible.
- The two factors $(1-.85B)$ and $(1-.68B)$ work to cancel-out the $(1-.9999996B)$ MA factor.
- It appears that a smaller order model is called for.

2014 DOW Jones Data

We try using BIC

```
aic5.wge(dow.1,p=0:5,q=0:2,type='bic')
# BIC selects p=0,q=0, i.e. white noise
```

Looking again at the differenced data we see that white noise is plausible.



We should have modeled it as white noise before applying AIC!

2014 DOW Jones Data

Final model

$$(1 \quad B)(X_t \quad 16778) = a_t$$

or

$$(1 \quad B)X_t = (1 \quad B)16778 + a_t$$

But, $(1 \quad B)16778 = 16778 \quad 16778 = 0$

So, we can write the model ***without the mean***.

Note:

This is true for all ARIMA models with $d \geq 1$

We could have written the final models for the ARIMA(2,1,0) and ARIMA(2,2,0) examples ***without a mean.***

2014 DOW Jones Data

DOW Jones model

$$(1 \quad B)X_t = a_t$$

Note that the differenced data $Y_t = (1 - B)X_t$ is white noise,

- So we can estimate the white noise variance as the sample variance of the differenced DOW data
- We found $\hat{\sigma}_a^2$ using the R command `sd`

```
sd(dow.1)
```

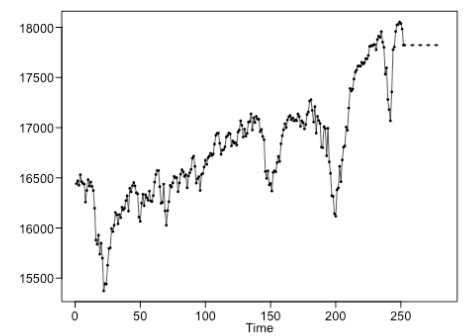
and squaring the result.

Final model $(1 \quad B)X_t = a_t \qquad \hat{\ }_a^2 = 13036$

Note:

- Forecasts from this model will simply repeat the last data value for all steps ahead
- It's difficult to predict the DOW!

DOW Jones



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A More General Approach to ARIMA Model ID

A More General Approach for Detecting Nonstationarities

To this point, our approach (Box-Jenkins) for detecting and removing stationarities is as follows:

- If data are wandering and sample autocorrelations damp slowly
  - *Difference the data*
- If differenced data still show evidence of a unit root
  - *Difference the data again, etc.*

A More General Approach for Detecting Nonstationarities

The ARIMA is an extension of the ARMA model to allow one or more factors of $(1-B)$ in the model

- That is, ARIMA models include roots **on** the unit circle
  - For ARIMA models, these roots are +1

ARIMA models are not the only extension of ARMA models that have roots **on** the unit circle

- These roots may be complex. These models include
  - Seasonal models
  - Cyclic ARUMA models (see Woodward, et al., 2017; but not covered here)

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Tiao/Tsay and Overfit Tables

1 Unit Root

Result (Tiao/Tsay) (loosely stated):

If a high order AR(p) model is fit to a realization from a nonstationary model with roots on the unit circle, then factors associated with these roots will "show up" in the factor table.

- The nonstationary behavior will "show up" in factor tables for a wide range of orders p.
- Fitting several high order models to a set of data to identify nonstationarities is called ***overfitting.***

Overfitting using the `tswge` package:

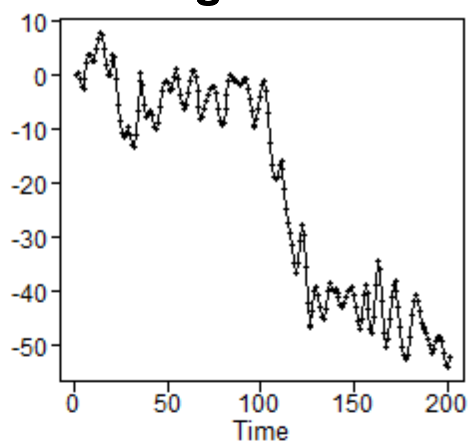
`est.ar.wge` function

- We use Burg estimates (always stationary)
- MLE would be ok

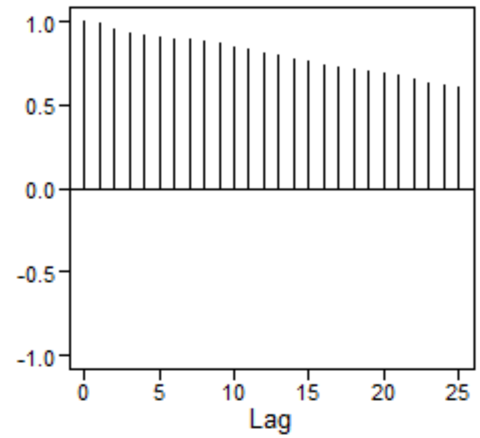
Recall: ARIMA(2,1,0) Model

$$(1 - B)(1 - 1.2B + .8B^2)X_t = a_t$$

Original data



Sample autocorrelations




```
# generate ARIMA(2,1,0) data as before
xd1=gen.arima.wge(n=200,phi=c(1.2,-.8),d=1,sn=56)
# fit an AR(6) and AR(8) to this realization
est.ar.wge(xd1,p=6,type='burg')
est.ar.wge(xd1,p=8,type='burg')
```

Overfit Tables for ARIMA(2,1,0) Data

AR(6) fit

AR factor	Absolute reciprocal of root	System frequency (f_0)
$1-0.9944B$	0.9944	0.0000
$1-1.1978B+0.7584B^2$	0.8708	0.1293
$1-0.4472B+0.0881B^2$	0.2967	0.1142
$1+0.2879B$	0.2879	0.5000

AR(8) fit

AR factor	Absolute reciprocal of root	System frequency (f_0)
$1-0.9951B$	0.9951	0.0000
$1-1.1324B+0.7314B^2$	0.8552	0.1348
$1-1.0807B+0.4241B^2$	0.6513	0.0943
$1+0.4968B$	0.4968	0.5000
$1+0.3615B+0.2464B^2$	0.4964	0.3093

Key point:

Even though the data are not from an AR(6) or AR(8), the factor close to 1-*B* shows up in both factor tables

Recall for ARIMA(2,1,0) Data

We previously analyzed these data and we decided to difference the data because:

- The data are wandering and sample autocorrelations damp slowly.
- We only differenced once because the differenced data appeared to be stationary.

Using overfitting:

- We decide to difference the data because the near 1-*B* factor continued to show up in high order Burg overfits.
- The differenced data will likely appear to be stationary after differencing because only one factor near 1-*B* appears in the overfit tables.

Modeling the ARIMA(2,1,0) Data using Overfitting

Since the factor tables suggest a factor of $(1-B)$, we difference the data and come up with the same model as before:

$$(1-B)(1-1.27B+.80B^2)(X_t+21.22)=a_t \quad \hat{\sigma}_a^2=.93$$

Note:

As mentioned with the DOW data, we can write the model without a mean, i.e.

$$(1-B)(1-1.27B+.80B^2)X_t=a_t \quad \hat{\sigma}_a^2=.93$$

Question:
How do you use overfitting to detect ***more than one unit root?***

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Overfit Factor Tables

2 Unit Roots

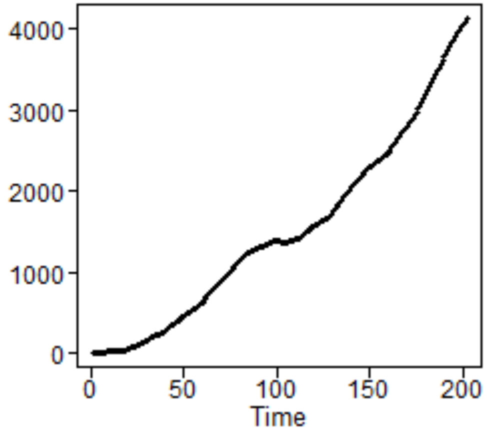
Question:

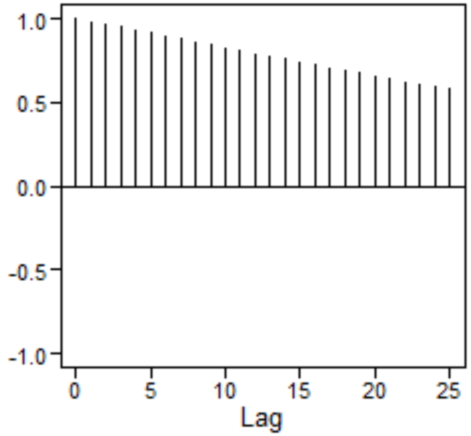
How do you use overfitting to detect ***more than one unit root?***

A Model with 2 Unit Roots

$$(1-B)^2(1-1.2B+.6B^2)X_t = a_t$$

Original data **Sample autocorrelations**






```
# generate ARIMA(2,2,0) data as before
x=gen.arima.wge(n=200,d=2,phi=c(1.2,-.6),sn=132)
# fit an AR(8) and AR(10) to this realization
est.ar.wge(x,p=8,type='burg')
est.ar.wge(x,p=10,type='burg')
```

Overfit Tables for ARIMA(2,2,0) Data

AR(8) fit

Factor	Abs recip	System freq
$1-1.9919B+0.9924B^2$	0.9962	0.0036
$1-1.2575B+0.6987B^2$	0.8359	0.1145
$1-0.8229B$	0.8229	0.0000
$1+0.1891B+0.2798B^2$	0.5290	0.2786
$1+0.4792B$	0.4792	0.5000

AR(10) fit

$1-1.9903B+0.9909B^2$	0.9954	0.0036
$1-0.8684B$	0.8684	0.0000
$1-1.3414B+0.7528B^2$	0.8676	0.1094
$1-0.1407B+0.4330B^2$	0.6581	0.2330
$1+1.0699B+0.4040B^2$	0.6356	0.4092
$1-0.1267B$	0.1267	0.0000

No Factors near 1-*B* ?

Let's examine AR(8) fit further

Factor	Abs recip	System freq
$1-1.9919B+0.9924B^2$	0.9962	0.0036

This factor

- Has roots very close to the unit circle
- Has System Frequency $f_0 = .0036$
  - Associated with period $1/.0036 = 278$
  - This period is longer than the data length (n = 200)
  - Suggests aperiodic data or data with a very long period
- Is very close to $1-2B+B^2 = (1-B)^2$

Suggests two factors of $(1-B)$

Note: AR(10) fit has a similar factor

Recall for ARIMA(2,2,0) Data

We previously analyzed these data and we decided to difference the data twice.

Using overfitting:

- We decide to difference the data twice because of the comments on the previous slide.
- We come up with the same final model as in our previous analysis of this realization.

Note: We did this analysis by looking at factor tables.

- It is ***always*** a good idea to look at the data
  - Before and after differencing, etc.

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ARIMA |
Comments about Unit Roots

Comments about Unit Roots (ARIMA Models)

- We mentioned that the decision whether to include a unit root or other nonstationary component in your model is a difficult one.
- We argued that it is a conscious decision that must be made by you.
- We next discuss the “Dicky-Fuller tests” which are popular in Economics.
- These tests are appealing because they seem to “make this decision for you.”

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Dicky-Fuller Test

Testing for Unit Roots

- Popular in Economics
- These are formal tests designed to help you decide whether to include one or more factors of $(1 - B)$ in your model, i.e. a root of +1
- They involve a test of the hypothesis

  H_0: *model has a root of* $+1$

  H_A: *the model does not have a root of* $+1$
- These are often called Dickey-Fuller tests

A Problem with the Dicky-Fuller Tests

- A problem with these tests is that a decision to include a unit root in the model is based on a decision to **not reject** the null hypothesis!
    - As data scientists you know that failure to reject a null hypothesis does not mean that it is true!
- The following mini-simulation will illustrate this point
- Using function **`adf.test`** in CRAN package **`tseries`**

Mini-Simulation

- 10 realizations were generated from the ***stationary model*** $(1-\varphi_1 B)X_t = a_t$
  - For $\varphi_1 = .9$, $.95$, and $.975$
  - Realization lengths n=100 and 200
- In each case we record the number of times out of 10 that the Dicky-Fuller test ***failed to reject*** the null hypothesis
  - i.e., ***incorrectly decided there was a unit root***

	$\varphi_1 = .9$	$\varphi_1 = .95$	$\varphi_1 = .975$
n=100	9/10	9/10	10/10
n=200	5/10	6/10	8/10

Comments on Simulation Results

The simulations show that unit root tests do not give you a definitive decision concerning whether a root is:

- On the unit circle (nonstationary)

or

- Close to the unit circle (stationary)

Bottom line:

The decision to include a nonstationary factor in your model must be made by ***you***

- And will often involve physical considerations/domain knowledge

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Model ID for Strictly Seasonal Models

Model Identification for Seasonal Models

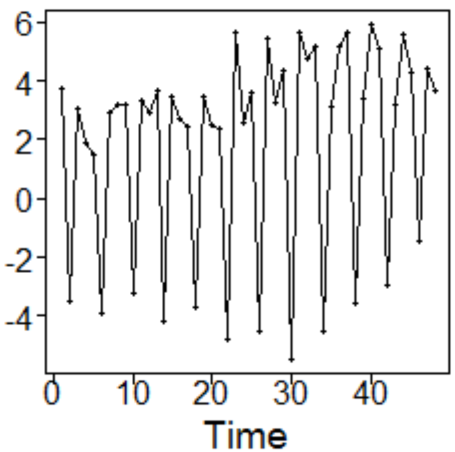
- In the previous slides we have discussed the issues related to model identification of ***ARIMA* models**
- Another type of nonstationary model is the ***seasonal model***, i.e. models that include a factor such as $(1 - B^s)$ for some integer s
  - Fitting seasonal models is the topic of the following material

Recall: Seasonal Models

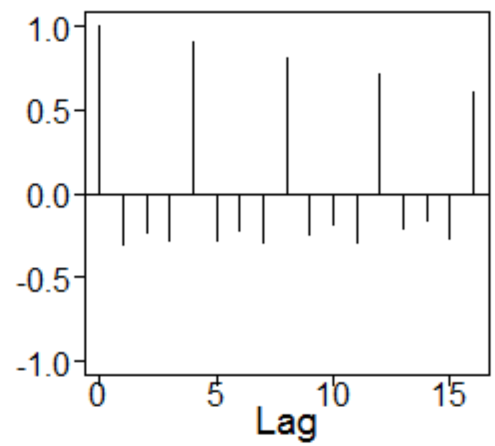
Pure seasonal models $\boxed{(1 - B^s)X_t = a_t}$

Example: $(1 \quad B^4)X_t = a_t$

Realization



Sample autocorrelations



The data go through 20 “irregular periods of 4” in 80 time points. The pattern seems to be 3 values “high” followed by a very low value.

Sample autocorrelations at lags 4, 8, … are ”large” which is consistent with the model.

- that is, X_t, X_{t+4}, X_{t+8}, … would be expected to be “similar”

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Model Identification for Seasonal Models

Procedure for including $1 - B^s$ in the model

As in the case of ARIMA models, ***the decision to include a seasonal factor*** in the model ***is one you will have to make*** after considering the evidence.

1. Look at the data
   - Does the behavior in realization tend to repeat every s time periods?
   - Are the sample autocorrelations large at s, 2s, 3s, etc.?
   - Does a seasonal model with season s make sense?

We recommend using factor tables

2. Compare factors in *overfit factor table* with those for $1 - B^s$ under consideration.
3. If "nonstationary" factors match with factors of $1 - B^s$ fairly well, transform the data by $1 - B^s$.

Recall: Seasonal Factor Table, s = 4

Factor	Abs recip	System freq
1−*B*	1	0.0
1+*B*	1	0.5
1+*B*^2	1	0.25

Factor table for seasonal data on previous slide

```
x=gen.aruma.wge(n=48,s=4,sn=23)
est.ar.wge(x,p=8,type='burg')
```

Factor	Abs recip	System freq
1+0.0148*B*+0.9826*B*^2	0.9912	0.2512
1+0.9905*B*	0.9905	0.5000
1−0.9556*B*	0.9556	0.0000
1−0.5126*B*+0.2985*B*^2	0.5464	0.1723
1+0.4753*B*	0.4753	0.5000
1+0.0593*B*	0.0593	0.5000

Factor tables with *p*=10 and *p*=12 are similar

Based on the Factor Tables

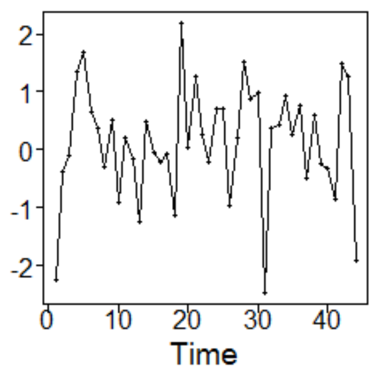
- It appears that the model contains a seasonal factor of the type $(1 - B^4)$.
- The next step is to transform the data to obtain

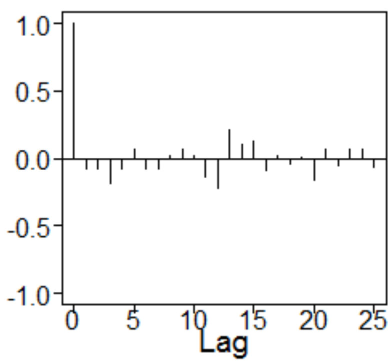
$$Y_t = (1 - B^4)X_t = X_t - X_{t-4}$$

- We can use the tswge command

```
y=artrans.wge(x,phi.tr=c(0,0,0,1))
```

- The transformed data and sample autocorrelations are shown below

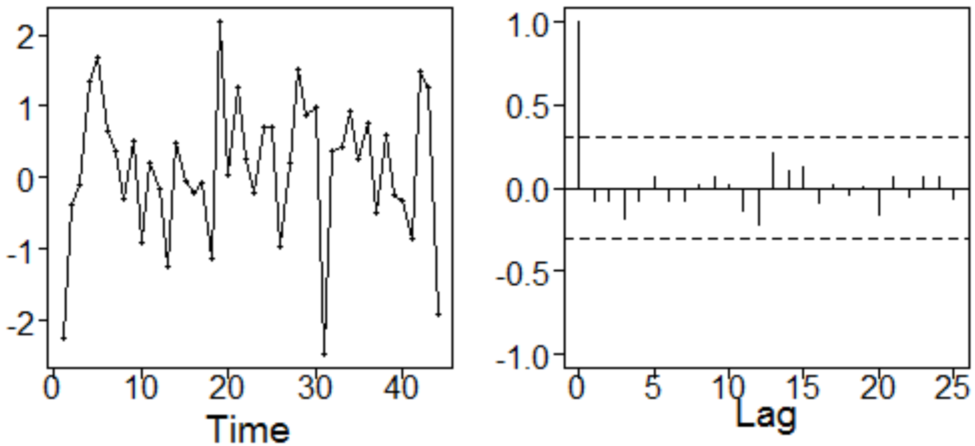




- The next step is to model the transformed data

Finding a Final Model

- Transformed data $Y_t = (1 - B^4)X_t = X_t - X_{t-4}$



- The transformed data appear white
  - Which is consistent with the true model
  - So the **`y`** variable obtained using **`artrans.wge`** is white noise. To find the white noise variance we use

    `wnv=sd(y)^2`

Final model $(1 - B^4)X_t = a_t \qquad \hat{\sigma}_a^2 = 1.006$

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Model ID for More General Seasonal Models

Fitting More General Seasonal Models

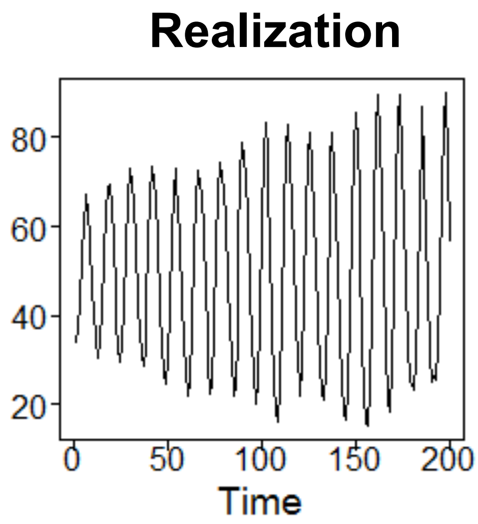
$$(1-B^s)\varphi(B)X_t = \theta(B)a_t$$

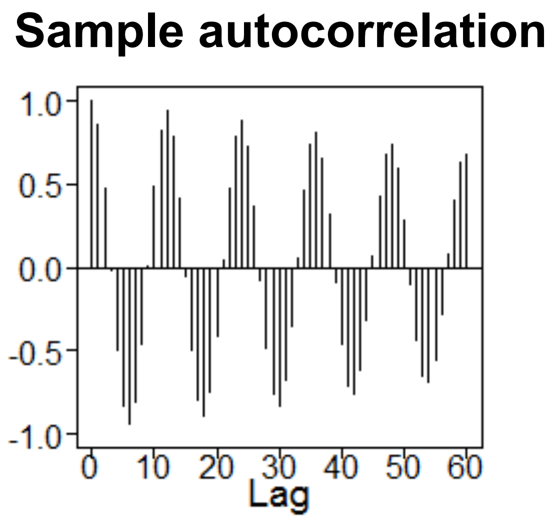
Seasonal factors $(1 - B^s)$ are a type of nonstationary factor. As such they tend to be dominant in factor tables of high order AR models fit to the data (overfitting).

- We have seen this in the previous "purely seasonal" case with *s*=4
- This dominance also tends to occur when stationary components are also in the data
  - As in the case of ARIMA models

Simulated Seasonal Data

$$(1 - B^{12})(1 - 1.5B + .8B^2)(X_t - 50) = a_t$$





Use tswge to simulate and plot the data above

```
x=gen.aruma.wge(n=200,s=12,phi=c(1.5,-.8),sn=87)
x=x+50
plotts.sample.wge(x,lag.max=60)
d15=est.ar.wge(x,p=15, type='burg')
```

Factor table for $(1 - B^{12})$

Factor	Abs recip	System freq
1+*B*+*B*^2	1	0.3333
1+1.732*B*+*B*^2	1	0.4167
1-*B*+*B*^2	1	0.1667
1+*B*	1	0.5000
1-1.732*B*+*B*^2	1	0.0833
1-*B*	1	0.0000
1+*B*^2	1	0.2500

Factor table for AR(15) fit to data on previous slide

```
d15=est.ar.wge(x,p=15, type='burg')
```

Factor	Abs recip	System freq
1-1.7269*B*+0.9978*B*^2	0.9989	0.0839
1+1.0027*B*+0.9948*B*^2	0.9974	0.3338
1-1.0036*B*+0.9944*B*^2	0.9972	0.1661
1-0.0142*B*+0.9907*B*^2	0.9953	0.2489
1+1.7320*B*+0.9904*B*^2	0.9952	0.4180
1+0.9788*B*	0.9788	0.5000
1-0.9522*B*	0.9522	0.0000
1-1.5043*B*+0.7732*B*^2	0.8793	0.0867
1+0.1523*B*	0.1523	0.5000

Factor Table for AR(17) Fit to Data on Previous Slide

```
d15=est.ar.wge(x,p=17, type='burg')
```

Factor	Abs recip	System freq
1-1.7276B+0.9985B^2	0.9992	0.0838
1-1.0028B+0.9955B^2	0.9977	0.1662
1+1.0010B+0.9936B^2	0.9968	0.3337
1-0.0157B+0.9930B^2	0.9965	0.2487
1+1.7329B+0.9899B^2	0.9949	0.4182
1+0.9836B	0.9836	0.5000
1-0.9136B	0.9136	0.0000
1-1.4286B+0.7747B^2	0.8802	0.0993
1-0.7137B	0.7137	0.0000
1+0.7516B+0.3366B^2	0.5801	0.3621

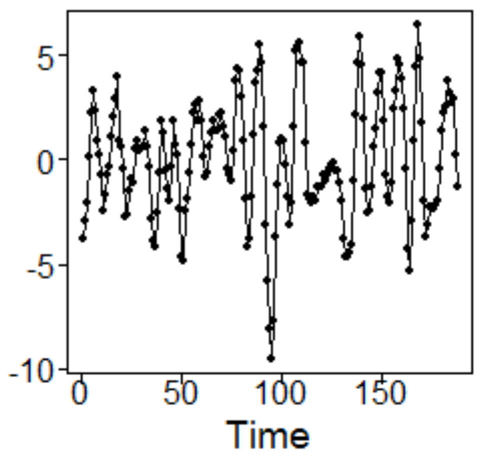
Note: The factor $1 - 0.9136B$ is not very close to the unit circle, but since all other factors are consistent with $(1 - B^{12})$ we conclude that this overfit table suggests $(1 - B^{12})$ also.

Based on Overfit Tables

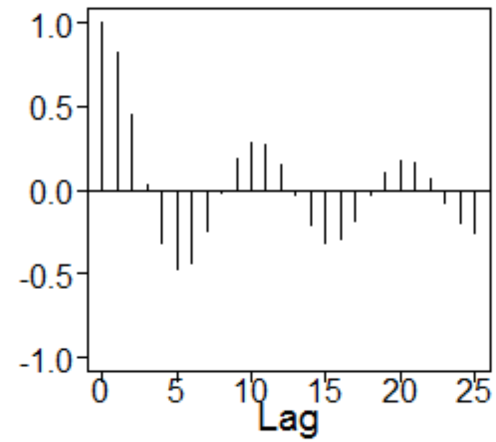
Transform data to create $Y_t = (1 \quad B^{12}) X_t$

```
y=artrans.wge(x,phi.tr=c(0,0,0,0,0,0,0,0,0,0,0,1))
```

Transformed data



Sample autocorrelations



The transformed data appear stationary, so we use AIC to identify a model.

Using tswge to Model the Transformed Data

We use `aic5.wge` to model the transformed data (y) on the previous slide

- When using AIC to model data that has been "stationarized" using the seasonal transform $(1 - B^s)$, it is good practice to allow range of *p* values to include *s*
  - To uncover any seasonal stationary information that might be in the data
  - In R code to follow, we consider the range *p*=0:13 and *q*=0:3

Using tswge to Model the Transformed Data

```
# y is the transformed data
aic5.wge(y,p=0:13,q=0:3)
```

AIC selects an ARMA(4,2) model. Factoring the ARMA(4,2) model we obtain:

AR factors

Factor	Abs recip	System freq
1-0.4098*B*+0.8135*B*^2	0.9019	0.2135
1-1.4733*B*+0.7771*B*^2	0.8815	0.0925

MA factor

Factor	Abs recip	System freq
1-0.4957*B*+0.9262*B*^2	0.9624	0.2085

Note: The dominant behavior of the transformed data is a pseudo cyclic behavior of length about 10 (f_0=.10). The factors associated with frequency about f_0=.2 essentially cancel.

Using tswge to Model the Transformed Data

Based on the comments on the previous slide, we look for a simpler model using BIC.

```
aic5.wge(y,p=0:13,q=0:3,type='bic')
```

BIC picks a simpler AR(2) model. This is consistent with

- The pseudo cyclic data
- Damping cyclical sample autocorrelations

Decision:

We choose to model the transformed data as an AR(2).

Final Model

Based on the decision to fit an AR(2) model, we use the **est.ar.wge** command to obtain ML estimates.

```
est.ar.wge(y,p=2)
#  $phi:  [1] 1.4652837 -0.7596714
#  $avar: [1] 1.036377
mean(x)   #[1] 49.77867
```

So, the final model is

$$(1 - B^{12})(1 - 1.47B + .76B^2)(X_t - 49.78) = a_t \qquad \hat{\sigma}_a^2 = 1.04$$

which is very similar to the model from which the data were simulated:

$$(1 - B^{12})(1 - 1.5B + .8B^2)(X_t - 50) = a_t \qquad \sigma_a^2 = 1$$

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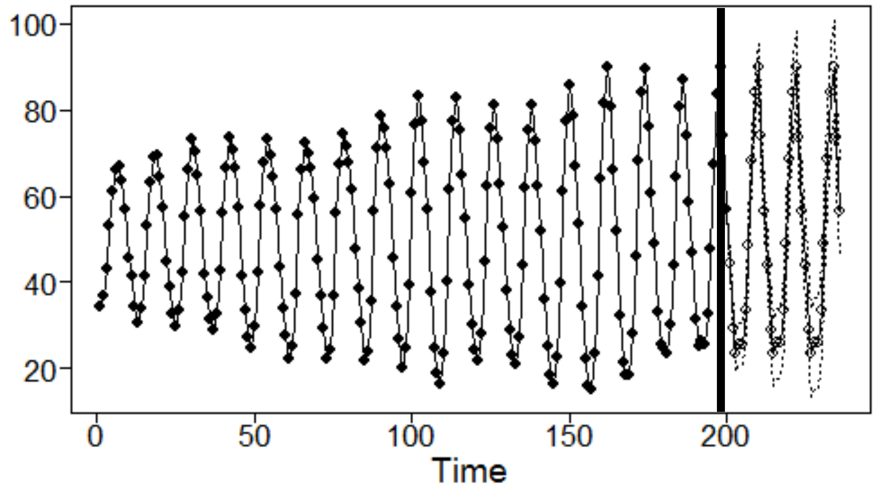
Review of Forecasting with Seasonal Models

Forecasting with the Seasonal Model

$$(1 - B^{12})(1 - 1.47B + .76B^2)(X_t - 49.78) = a_t \qquad \hat{\sigma}_a^2 = 1.04$$

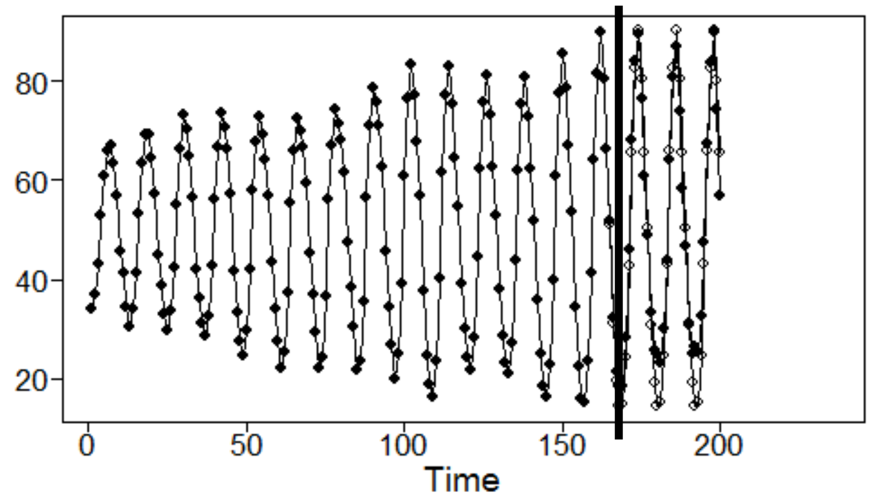
Forecasts: 36 steps ahead

```
x=gen.aruma.wge(n=200,s=12,phi=c(1.5,-.8),sn=87)
x=x+50
fore.aruma.wge(x,s=12,phi=c(1.47,-.76),n.ahead=36,
lastn=FALSE)
```



Forecasts: Last 36 Values

```
fore.aruma.wge(x,s=12,phi=c(1.47,-.76),n.ahead=36,
lastn=TRUE,limits=FALSE)
```



ASE = 26.27

Summary:

- The forecasts (open circles) are very close to the true values.
- We were able to fit a model very close to the true model.
- Forecasts (ahead and last 36) are quite good.

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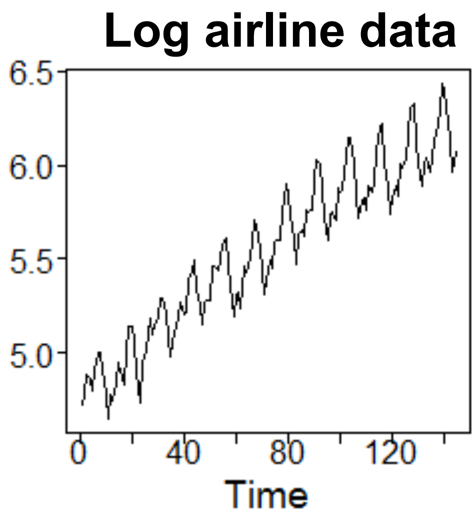
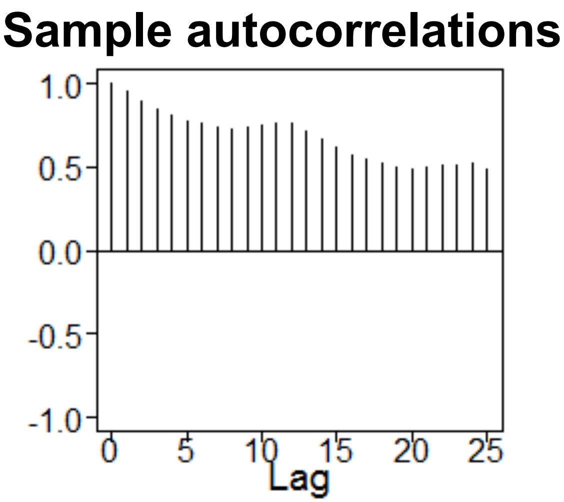
Seasonal Models | Example

Airline Data

Example: Airline Data





Again

- There is evidence of nonstationarity
- We apply the overfit procedure using **`est.ar.wge`**

Factor table for $(1 - B^{12})$

Factor	Abs recip	System freq
1+B+B^2	1	0.3333
1+1.732B+B^2	1	0.4167
1-B+B^2	1	0.1667
1+B	1	0.5000
1-1.732B+B^2	1	0.0833
1-B	1	0.0000 ?
1+B^2	1	0.2500

Factor table for AR(15) fit to log airline data

```
est.ar.wge(lair,p=15,type='burg')
```

Factor	Abs recip	System freq
1-1.7253B+0.9953B^2	0.9976	0.0838
1-0.9861B+0.9950B^2	0.9975	0.1677
1+0.9869B+0.9850B^2	0.9925	0.3328
1-0.0375B+0.9788B^2	0.9893	0.2470
1-1.9697B+0.9704B^2	0.9851	0.0034 ?
1+1.6891B+0.9504B^2	0.9749	0.4168
1+0.8540B	0.8540	0.5000
1+0.6000B	0.6000	0.5000
1-0.1110B	0.1110	0.0000

Notes

- Factor tables for other high orders are similar
- The factor $1-1.9697B+0.9704B^2$ is associated with a frequency of $f_0 = 0.0034$ or a period of $1/.0034 = 294$ (longer than the data record). This suggests aperiodic data or a very long period.
- Also this factor is very close to $1-2B+B^2 = (1-B)^2$ which is associated with frequency $f_0 = 0$
    - We encountered a similar situation with an ARIMA model with $d=2$.
    - $(1-B)^2$ provides the last "piece" needed for a factor of $(1-B^{12})$ plus an extra $(1-B)$ factor
    - That is, the factor table suggests the presence of nonstationary factors $(1-B)(1-B^{12})$
- Although $(1+.85B)$ is not as close to the unit circle "as we would expect for $s=12$ data, the "total picture" suggests a factor of $(1-B^{12})$

In the log airline factor table

$(1-B)^2$ provides the last "piece" needed for a factor of $(1 - B^{12})$ plus an extra $(1-B)$ factor

Recall:

We earlier discussed the following model that is useful for modeling ***seasonal data with a trend***

$$(1 \quad B)(1 \quad B^{s}) \quad (B)(X_t \qquad) = \quad (B)a_t$$

We referred to it as the ***airline model***

- For obvious reasons!

Note:

The airline model allows for ***stationary components***

- To find these, we transform to find

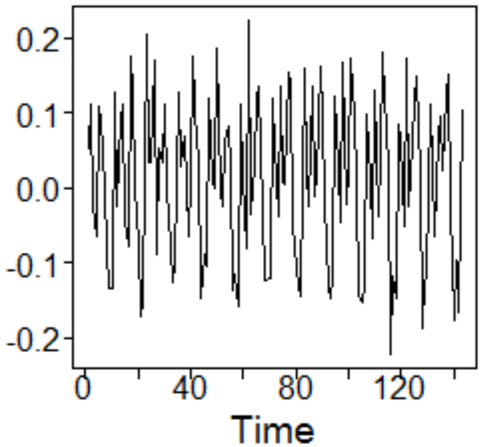
$$Y_t = (1 - B)(1 - B^{12})X_t$$

Transform to Stationarity

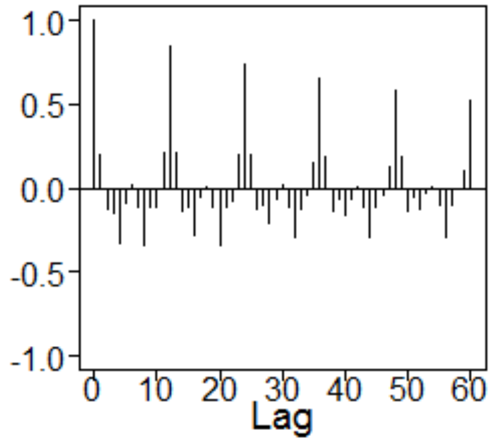
Using **tswge** to transform log airline data (lair) to remove seasonal and ARIMA components

```
#Difference the data
d1=artrans.wge(lair,phi.tr=1)
```

Differenced data (d1)



Sample autocorrelations

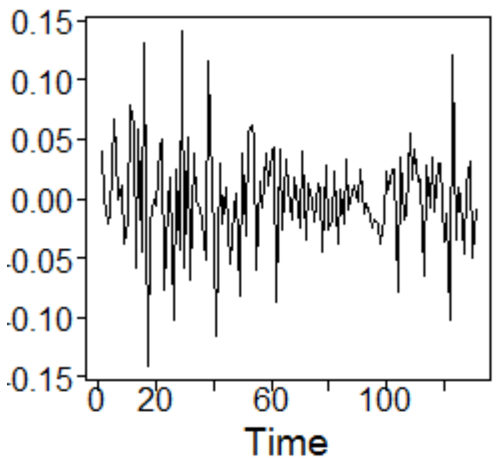


- Trend has been removed
- 12 month seasonal behavior remains

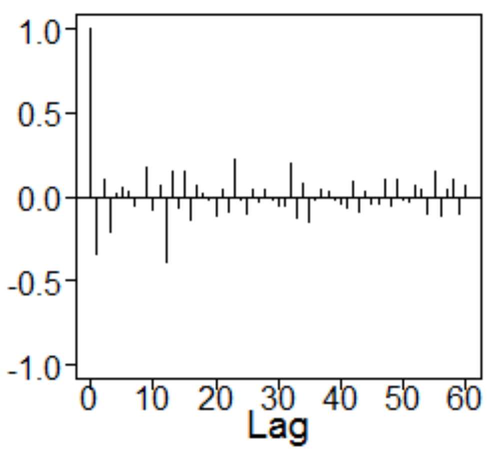
Differenced Data Transformed by (1-B^{12})

```
# Transform differenced data with "seasonal difference"
d1.12=artrans.wge(d1,phi.tr=c(0,0,0,0,0,0,0,0,0,0,0,1))
```

(1-*B*)(1-*B*^{12})X_t (d1.12)



Sample autocorrelations



- Appears to be stationary
- No trending or seasonality present
- We will find a stationary model for this realization

Finding a Final Model for the (log) Airline Data

Use `tswge` to model the transformed data, `d1.12`

```
aic5.wge(d1.12,p=0:13,q=0:3)
```

AIC picks an ***ARMA(12,1).***

- In order to see if a lower order model could satisfactorily model the data, we use BIC

```
aic5.wge(d1.12,p=0:13,q=0:3,type='bic')
```

BIC picks an ***MA(1)*** as the first choice and an ***AR(1)*** as second choice.

- Examining the data and sample autocorrelations, neither model seems appropriate
- We decide to use the ARMA(12,1) model chosen by AIC

Final Model for (log) Airline Data

We estimate the parameters of the ARMA(12,1) model using est.arma.wge

```
est.arma.wge(d1.12,p=12,q=1)
# $phi  [1]  0.00846435  0.07967002 -0.10710183 -0.02069279
0.08039524  0.04050674 -0.05516681  0.03629598  0.13277817
[10] -0.05297698 -0.12340975 -0.40343184
# $theta  [1] 0.4536856
# $avar  [1] 0.001310444
mean(lair)   # [1] 5.542176
```

$(1-B)(1-B^{12})\varphi_{12}(B)(X_t - 5.54) = (1-.45B)a_t \;\; \hat{\sigma}_a^2 = .0013$

$\varphi_{12}(B)$=$1 - .008B - .080B^2 + .107B^3 + .021B^4 - .080B^5 - .041B^6$
$+.055B^7 - .036B^8 - .133B^9 + .053B^{10} + .0123B^{11} + .403B^{12}$

Comments

- Recall from an earlier unit that forecasts using the airline model are quite good
  - We previously used the AR(13) (Woodward / Gray Model) instead of the ARMA(12,1) (Box Model) but forecasts are very similar
- Recall also that $(1-B)(1-B^{12})$ contains two factors of $(1-B)$ which accounts for the fact that the trend in the airline data is predicted to continue
- It is clear that seasonal models are useful for data that occur monthly or quarterly or some other sampling interval where similar patterns are likely to be repeated

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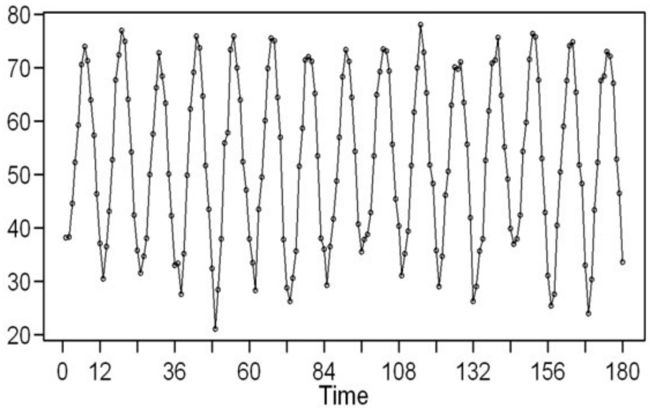
Seasonal Models | Example

Pennsylvania Temperature Data

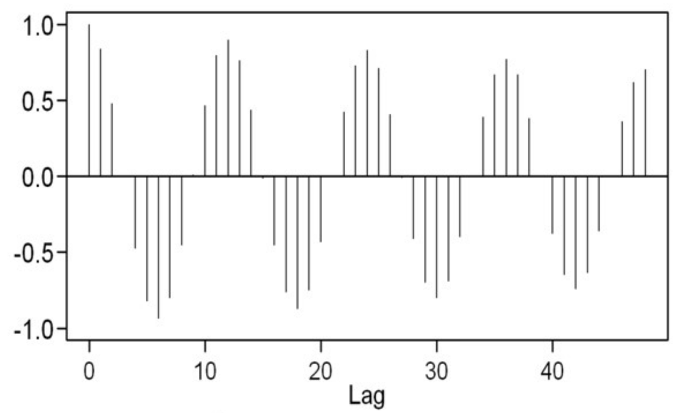
Pennsylvania Monthly Temperature Data

We conclude this section by analyzing another monthly time series

- Monthly average temperatures for Pennsylvania over a 15 year period



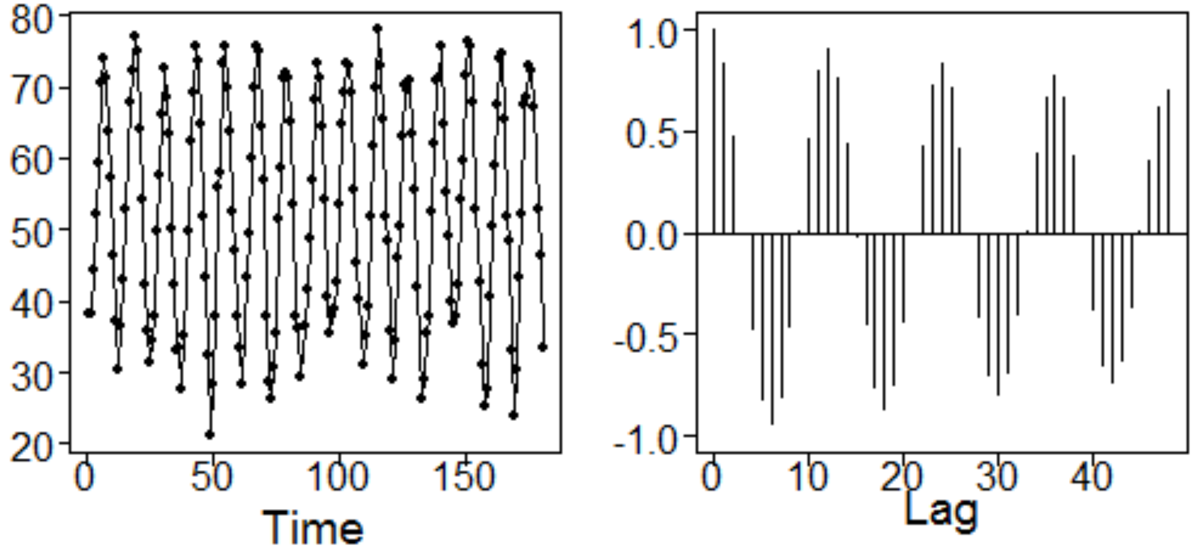
(a) Monthly Pennsylvania temperature data



(b) Sample autocorrelations for (a)

Again! Use overfit procedure

Pennsylvania Monthly Temperature Data



Note:

- There certainly seems to be a “seasonal” pattern
- Autocorrelations damp slowly
- ***Again! Use overfit procedure***

Factor Tables for p=14 and 15
Overfit to Pennsylvania Temperature Data (patemp)

```
data(patemp)  # the data set is available in tswge
est.ar.wge(patemp,p=14,type='burg')
```

Factor	Abs recip	System freq
1-1.7267B+0.9971B^2	0.9985	0.0838
1-0.9007B	0.9007	0.0000
1+0.1854B+0.8005B^2	0.8947	0.2665
1-0.8397B+0.7441B^2	0.8626	0.1691
1+0.9827B+0.6948B^2	0.8335	0.3503
1+1.5999B+0.6922B^2	0.8320	0.4557
1+0.4007B	0.4007	0.5000
1-0.3283B	0.3283	0.0000
1+0.1672B	0.1672	0.5000

```
est.ar.wge(patemp,p=15,type='burg')
```

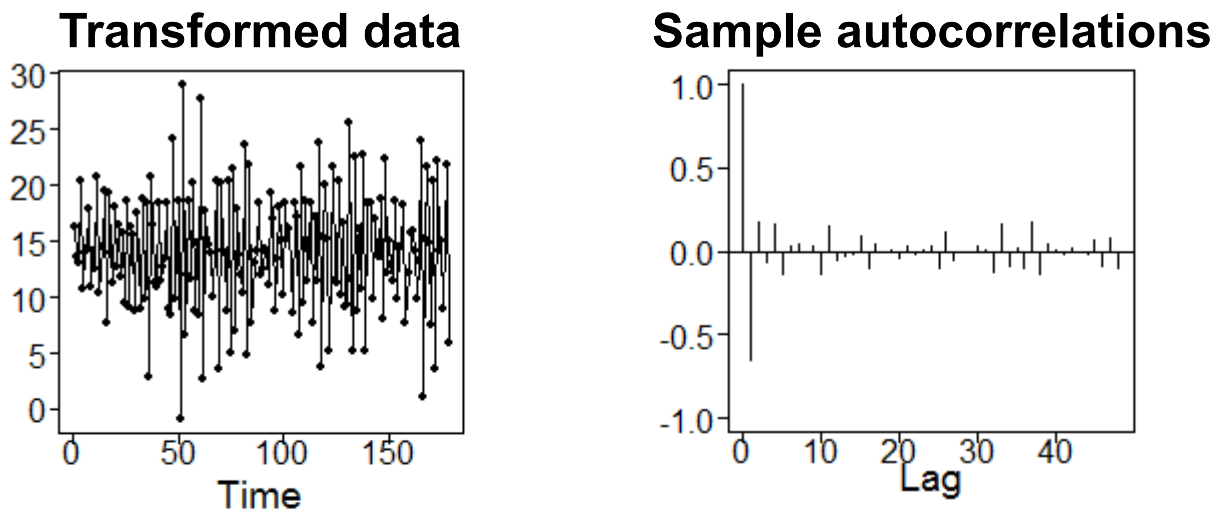
Factor	Abs recip	System freq
1-1.7268B+0.9973B^2	0.9987	0.0838
1-0.8743B	0.8743	0.0000
1-0.8987B+0.7623B^2	0.8731	0.1640
1+0.2200B+0.7559B^2	0.8694	0.2702
1+1.0663B+0.7499B^2	0.8660	0.3556
1+1.6060B+0.7278B^2	0.8531	0.4452
1+0.7830B	0.7830	0.5000
1+0.0141B+0.4368B^2	0.6609	0.2517
1-0.6487B	0.6487	0.0000

Factor Tables for Pennsylvania Temperature Data

- These factor tables are ***not what we expected***
  - We expected to see the factors of $(1 - B^{12})$ approximated by factors with roots very close to the unit circle
  - This is what we saw in the previous two seasonal examples
- The only factor of $(1 - B^{12})$ that has the behavior we expected is $(1 - 1.732B + B^2)$
  - Each factor table had a factor very close to this one with roots close to the unit circle
- This suggests that the stationarizing transformation is $Y_t = (1 - 1.732B + B^2)X_t$

We Transform the Data by $Y_t = (1 - 1.732B + B^2)X_t$

```
y.tr=artrans.wge(patemp,phi.tr=c(1.732,-1))
```



- The transformed data appear stationary
- Using aic5.wge with p=0:13, q=0:3, an AR(9) is selected
- Using aic5.wge with the same range for p and q and using BIC, an AR(3) is selected
- Let's go with the simpler model

Final Model

Estimating the parameters of the AR(3) model we get

```
est.ar.wge(y.tr,p=3)
#  $phi  [1] -1.1425939 -0.8431575 -0.4144801
# $avar  [1] 10.76795
mean(patemp)   #  [1] 52.62667
```

$$(1 - 1.732B + B^2)(1 + 1.14B + .84B^2 + .41B^3)(X_t - 52.63) = a_t,$$
$$\hat{\sigma}_a^2 = 10.77$$

Note:

The above model is an example of what Woodward, et al., 2017, refer to as an ARUMA model

- It is a more general nonstationary model that can contain roots on the unit circle that are not necessarily +1
- Seasonal models are ARUMA models

Notes

- The issue here is that the seasonal behavior is very nearly sinusoidal
  - It can be explained with a second order factor (with roots on the unit circle and a cycle length of 12)
  - The other seasonal data sets, for example the airline data, had a non-sinusoidal pattern within the 12 month year
    - For the airline data November and December are high and January is low, etc.
    - A pattern, but not sinusoidal
- A signal-plus-noise with a sine or cosine signal might also be a good model for these data

Key Point

Do not include $1-B^{12}$, for example, in the model simply because there is a 12^{th} order nonstationarity or because the data show a period of 12

- Or just because you have monthly data

Check the factor tables!!

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Signal + Noise Models | Testing for Trend

OLS Method

Fitting Signal-Plus-Noise Models

In our discussion of fitting nonstationary models to data we have considered

- ARIMA models
- Seasonal models

We next address the topic of fitting deterministic ***signal-plus-noise models*** to data

Deterministic Signal-Plus-Noise Models

$$\boxed{X_t = s_t + Z_t}$$

s_t is a deterministic signal

Z_t is a zero-mean, stationary process

Example signals: $s_t = a + bt$

$s_t = a + bt + ct^2$

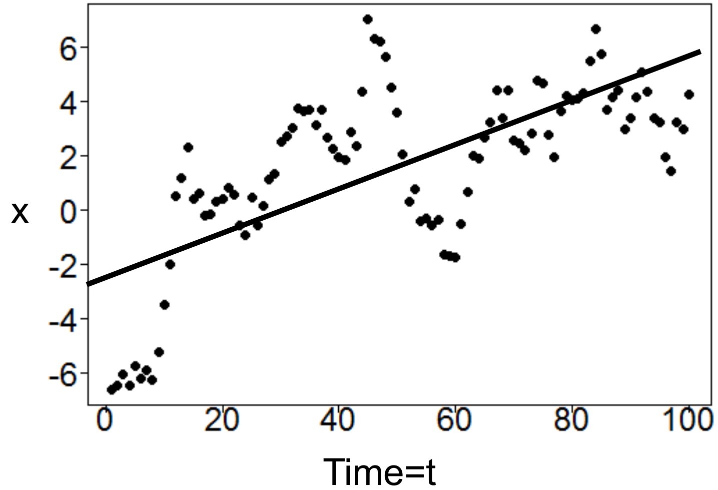
$s_t = A\cos(2\pi ft + C)$, C constant

Testing for Linear Trend

Recall: It is difficult to distinguish between random trends and deterministic trends.

- We will look at this issue in more detail in this section

Question: Is there a deterministic linear trend component in the data below?



Note:

It looks like a linear regression problem.

Linear Regression Strategy to Test for Trend

- Let *t* = Time be the independent variable
- Let *x* = X_t be the dependent variable
- Consider the standard regression model
  $X_t = a + bt + Z_t$
- Conclude there is a trend if $H_0: b = 0$ is rejected.

Results:

Base R program **`lm`** can be used for regression. If

- *x* is the vector of length 100 plotted on vertical axis
- t=1:100

The R code:

```
re=lm(formula=x~t)
summary(re)
```

produces (among other things) the regression model

$$X_t = -2.082 + .073t + Z_t$$

and $H_0: b = 0$ is rejected with $p < .001$.

Conclusion

There is clearly a significant (upward) trend

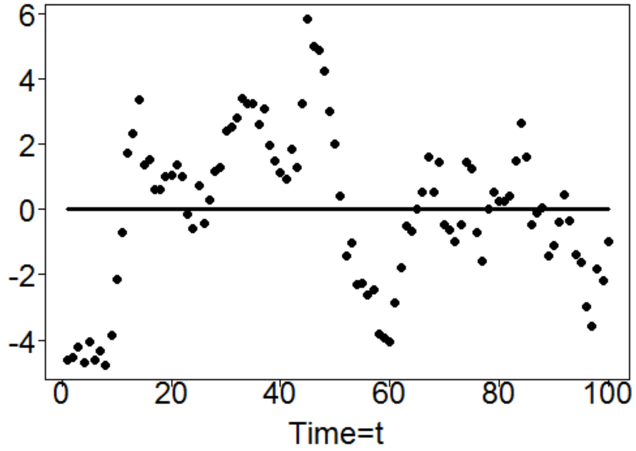
Check residuals

As good data scientists, we know to check the residuals to see if they are ***uncorrelated.***

Using the R code

```
z=x -(2.082+.073*t)
```

we obtain the residual plot below:



Oops!

Obviously the Residuals Are Correlated

Question:

Does that really make a difference? (Or is that just something statisticians worry about?)

Note:

The data set in the previous analysis satisfies the model $X_t = 0 + 0 * t + Z_t$ where Z_t is a realization of length n=100 from the AR(1) model $Z_t - .95Z_{t-1} = a_t$

That is:

$H_0: b = 0$ is ***True*** (and we rejected it dramatically!)

Was This Just a Fluke?

- We expect to reject $H_0: b = 0$ about 5% of the time when $b = 0$.
- Maybe this is just one of those 5%
- In order to check this we have run a Mini-Simulation
  - We simulate several realizations from models $X_t = 0 + 0 * t + Z_t$ where Z_t is from AR(1) models like $Z_t - .95Z_{t-1} = a_t$
    - How unusual is it to reject $H_0: b = 0$ (which is true in these simulated realizations)?

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Mini-Simulation

Type 1 Error Rate

Mini-Simulation: Using Regression

- We consider the model
  $X_t = a + bt + Z_t$ where $(1 - \varphi_1 B)Z_t = a_t$ ***where b=0***
  - We test at the .05 level of significance
- 10 realizations were generated from the model above
  - For $\varphi_1 = .9,\ .95.\ .975$
  - Realization lengths n=100 and 200
- In each case we record the number of times out of 10 we ***(incorrectly)*** rejected $H_0{:}\, b = 0$

	$\varphi_1 = .9$	$\varphi_1 = .95$	$\varphi_1 = .975$
n=100	8/10	8/10	9/10
n=200	5/10	8/10	7/10

Realizations from $X_t = 0 + 0 * t + Z_t$ where Z_t satisfies $Z_t - .95Z_{t-1} = a_t$

Obviously, $H_0: b = 0$ is true in the above model.

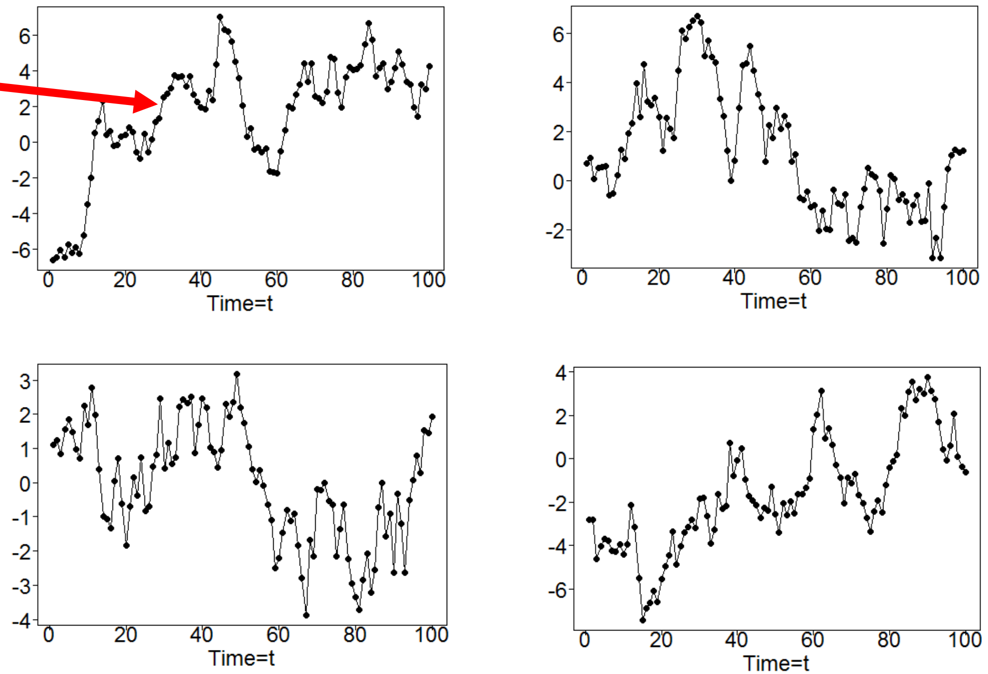
- These realizations show "random trends" that are difficult to distinguish from "deterministic trends"
- And cause $H_0: b = 0$ to be incorrectly rejected

Realization we previously analyzed and rejected

$H_0: b = 0$

with $p < .001$.

We'll come back to this



Comments on Simulation Results

The simulations show that using standard regression techniques to test $H_0: b = 0$ with AR(1) residuals

- Where φ_1 is close to +1
- Ignoring the fact that the residuals are correlated

Results in the null hypothesis are being rejected ***much more than 5% of the time.***

Bottom line

This is a really bad way to test for trend in time series data

- You will detect a (deterministic) trend in many cases in which no such trend exists
- The test is picking up on the random trends in AR(1) data

Caution:

When we have tests with inflated “observed significance rates”

- Then if you find a significant deterministic trend, you don’t have “confidence” that the trend is actually deterministic
- The next example illustrates the problems this can cause

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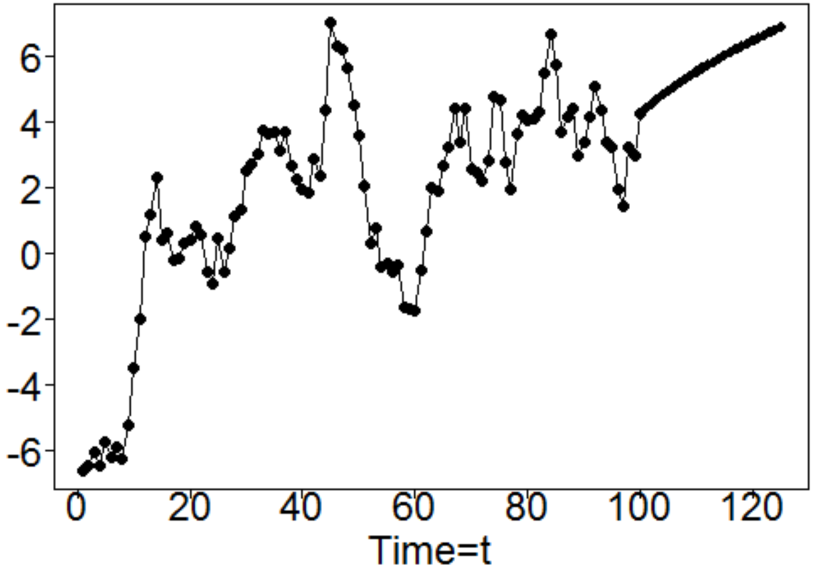
Forecasting and False Trend

True model: $X_t = 0 + 0 * t + Z_t$ **where** Z_t **satisfies** $Z_t - .95Z_{t-1} = a_t$

We fit the model: $X_t = -2.082 + .073t + Z_t$

($H_0: b = 0$ **is rejected with** $p < .001$.)

Forecasts vs. actual next 25 data values from "random trend" model



Note: The trend didn't continue as predicted by deterministic trend model!

oops again!!

Clearly We Need a Better Way to Test for Trend!

Estimation procedures exist for deciding whether a trending behavior in data is due to either a

- Deterministic component

  or

- Random/correlation driven component

  or both

These include:

- Cochrane-Orcutt
- ML techniques
- Bootstrap techniques
- ...

We illustrate the ***Cochrane-Orcutt*** method next

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Cochrane-Orcutt Method

Cochrane-Orcutt Method

Consider: $X_t = a + bt + Z_t$ where $(1 - \varphi_1 B)Z_t = a_t$.

1. Fit a linear regression to the original data and find estimates $\hat{a}$ and $\hat{b}$.
2. Find $\hat{Z}_t = X_t - \hat{a} - \hat{b}\mathrm{t}$ which should be approximately AR(1).
3. Estimate $\hat{\varphi}_1$ by fitting an AR(1) model to $\hat{Z}_t$
4. Calculate $\hat{Y}_t = (1 - \hat{\varphi}_1 B)X_t, \quad t = 2, \ldots, n$
5. Note that $\hat{Y}_t = (1 - \hat{\varphi}_1 B)X_t = (1 - \hat{\varphi}_1 B)(a + bt + Z_t)$

$$= a(1 - \hat{\varphi}_1) + bt - b\hat{\varphi}_1(t - 1) + (1 - \hat{\varphi}_1 B)Z_t$$
$$= c + bt_{\hat{\varphi}_1} + g_t, \text{ where } t_{\hat{\varphi}_1} = t - \hat{\varphi}_1(\mathrm{t} - 1).$$

where $\mathrm{c} = a(1 - \hat{\varphi}_1)$, $t_{\hat{\varphi}_1} = t - \hat{\varphi}_1(\mathrm{t} - 1)$, and
$g_t = (1 - \hat{\varphi}_1 B)Z_t$

light board

Why did we do that?

Logic behind the Cochrane-Orcutt Method

We transformed: $X_t = a + bt + Z_t$ where $\boxed{(1 - \varphi_1 B)Z_t = a_t.}$

To the new model: same slope

$\boxed{\Rightarrow Z_t = (1 - \varphi_1 B)^{-1} a_t}$

$\hat{Y}_t = (1 - \hat{\varphi}_1 B)X_t = c + b\, t_{\hat{\varphi}_1} + g_t$, where

$c = a(1 - \hat{\varphi}_1)$, $t_{\hat{\varphi}_1} = t - \hat{\varphi}_1(t - 1)$, and $\boxed{g_t = (1 - \hat{\varphi}_1 B)Z_t}$

?

new dependent variable

Let's look at g_t

new constant

new independent variable

$g_t = (1 - \hat{\varphi}_1 B)Z_t$

$$g_t = (1 - \hat{\varphi}_1 B)Z_t = (1 - \hat{\varphi}_1 B)\boxed{(1 - \varphi_1 B)^{-1} a_t} \leftarrow \boxed{Z_t}$$

The two factors: $(1 - \hat{\varphi}_1 B)$ and $(1 - \hat{\varphi}_1 B)$ nearly cancel each other out

- i.e. $g_t \approx a_t$ so g_t should be "nearly uncorrelated"

Finally: Test $H_0: b = 0$ in this "new" model

- Using standard regression methods

light board

Cochrane-Orcutt Test

Test H_0: $b = 0$ using the test statistic

$$t_{\text{co}} = \frac{\hat{b}_{\text{co}}}{\widehat{SE}(\hat{b}_{\text{co}})}$$

where $\hat{b}_{\text{co}}$ is the least squares (regression) estimate of slope b in the "new" model, and $\widehat{SE}(\hat{b}_{\text{co}})$ is its usual standard error.

Note:

On the next slide we discuss the performance of the Cochrane-Orcutt method on simulated data

light board

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Cochrane-Orcutt in R (Orcutt Package)

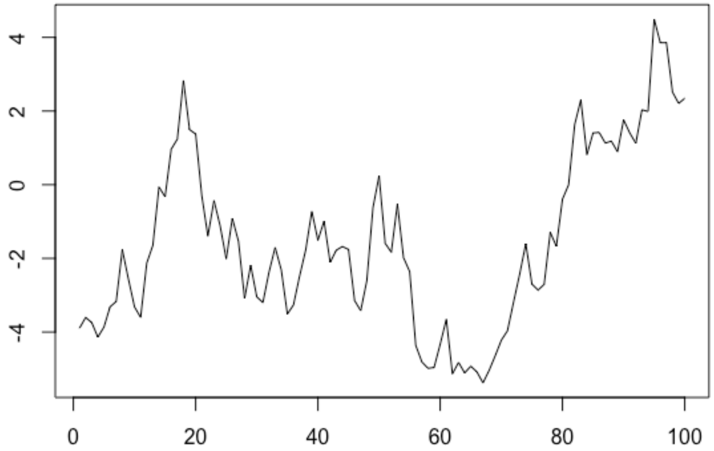
Cochrane-Orcutt Test in R (in Orcutt Package)

```
install.packages("orcutt")
library(orcutt)

x = gen.sigplusnoise.wge(100, b0 = 0, b1= 0, phi= .95, sn = 21)

t = seq(1,100,1)
df = data.frame(x = x, t= t)
fit = lm(x~t, data = df)
summary(fit)

cfit = cochrane.orcutt(fit)
summary(cfit)
```




```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.078211   0.450925  -6.826 7.31e-10 ***
t            0.030428   0.007752   3.925 0.000161 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.360341   2.811634  -1.195   0.2349
t            0.041244   0.040656   1.014   0.3129
```

Mini-Simulation: Using Cochran-Orcutt

- We consider the model
  $X_t = a + bt + Z_t$ where $(1 - \varphi_1 B)Z_t = a_t$ ***where b=0***
  - We test at the $.05$ level of significance
- 10 realizations were generated from the model above
  - For $\varphi_1 = .9,\ .95,\ .975$
  - Realization lengths n=100 and 200
- In each case we record the number of times out of 10 we ***(incorrectly)*** rejected $H_0: b = 0$

	$\varphi_1 = .9$	$\varphi_1 = .95$	$\varphi_1 = .975$
n=100	2/10	4/10	4/10
n=200	1/10	2/10	2/10

Note: Results are much better than those using simple regression

- Still higher than 5%   ??

More Extensive Cochrane-Orcutt Simulation Results

$Y_t = a + bt + Z_t$ where $(1 \quad {}_1B)Z_t = a_t$

- $b = 0$ (i.e. null hypothesis of zero slope is true)
- 1000 replicates generated from each model

Observed significance levels for tests $H_0 : b = 0$

n	${}_1 = .8$	${}_1 = .9$	${}_1 = .95$
50	18.4	27.2	37.2
100	16.0	20.0	28.4
250	8.0	12.4	17.6
1000	4.8	8.4	9.6

Conclusion: Cochrane-Orcutt has significance levels inflated over 5%

Comments about Inflated Rejection Rates

Recall:

Type I error rate: Probability of rejecting the null when it is true.

- We typically run hypothesis tests controlling the Type I error rate to be small (commonly $\alpha = .05$)
- We want there to be a small chance of rejecting the null *when it is true*
  - In our case we want there to be a small chance of declaring there is a deterministic trend ($b \neq 0$) when it is actually zero
  - i.e. when the only trending in the data is correlation-based random trends

Testing for Trend: Summary

$$X_t = a + bt + Z_t$$

- It's a difficult problem when Z_t has a positive correlation structure
  - That produces "random trends"
  - And are difficult to distinguish from "deterministic trends"
- Ignoring the correlation structure in the residuals and running a ***standard regression analysis is a very bad idea***
- The Cochrane-Orcutt technique helps adjust for the correlation structure but still rejects the null hypothesis more often than it should
- Other methods such as ML (used by SAS) have similar problems with inflated significance levels
  - See Woodward, Bottone, and Gray (1993) for more discussion of these issues

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Bonus Material

Bootstrap Method

Woodward, Bottone, and Gray (1997). *Journal of Agricultural, Biological, and Environmental Statistics (JABES)*, 403-416.

These authors devised a method for improving the observed significance levels

- Based on a bootstrap = based technique

Bivin: let's talk

Simulation Results

$Y_t = a + bt + Z_t$ where $(1 \quad {}_1B)Z_t = a_t$

- $b = 0$ (i.e. null hypothesis of zero slope is true)
- 1000 replicates generated from each model

Observed significance levels for tests $H_0 : b = 0$

n	${}_1 = .8$		${}_1 = .9$		${}_1 = .95$	
50	18.4	*5.9*	27.2	*8.1*	37.2	*10.0*
100	16.0	*6.4*	20.0	*5.3*	28.4	*7.6*
250	8.0	*3.8*	12.4	*5.1*	17.6	*6.4*
1000	4.8	*4.1*	8.4	*3.8*	9.6	*6.1*

Black: Cochrane-Orcutt

Red italics: Bootstrap

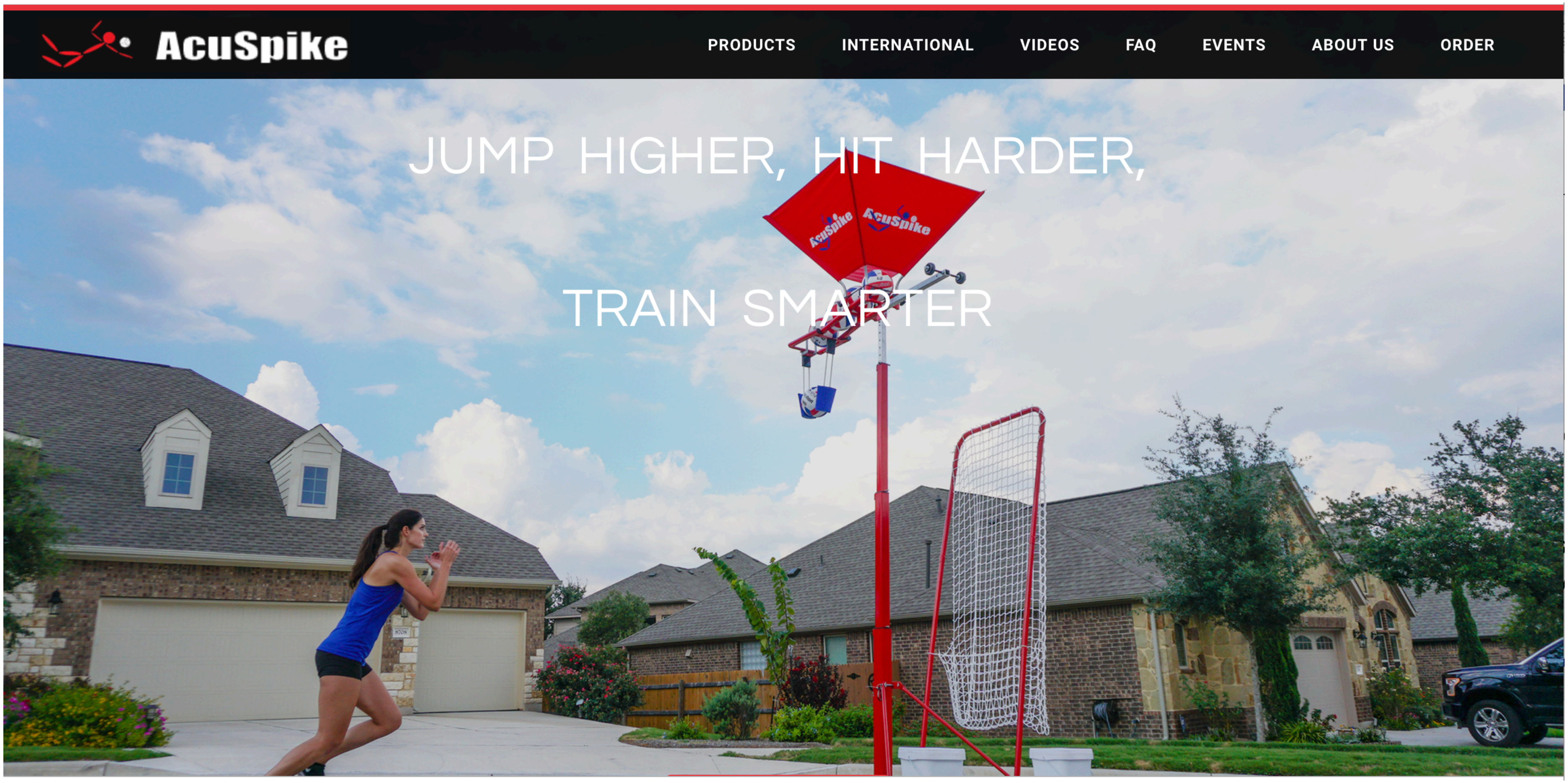
Final Remarks

We have discussed the problem of detecting a deterministic trend in time series data

- Even if a deterministic trend is detected, caution should be exercised for all but short term forecasts
  - Unless there is sufficient evidence that current conditions exist into the future
  - This is essentially the extrapolation problem in the time series setting

We have not discussed the issues involved in testing for deterministic components other than linear trends.

- For example, cosine trends can be difficult to distinguish from cyclic nonstationary and near nonstationary behavior


AcuSpike
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AcuSpike
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