Introduction

This experiment aims to determine the optimal combination of broth, garlic, and tomato sauce in Mexican rice for enhanced flavor and texture. By manipulating the quantity of broth, tomato sauce, and garlic, the experiment seeks to identify which combination yields the most palatable and visually appealing dish. For this experiment, I choose three factors: garlic, chicken broth, and tomato sauce. I chose these factors because they are three very prominent ingredients in this recipe. The primary response variable will be the "Overall Quality" of the rice, which includes assessments of texture, flavor, and visual appeal. This will be measured using a taste test conducted by a panel of judges who will score the rice based on these criteria. I would use a scale of 1 to 10 to assess how each rice tastes. Since the "Overall Quality" of the rice is rated using a 1 to 10 scale, it is considered ordinal data. In order to reduce variability, I used the same pan and measured each ingredient to ensure the cooking conditions and ingredient proportions were consistent across all trials. In addition, for each trial, I used the same heat on the stove, cooked the rice for the same amount of time.

Each of the three factors in the design has two levels:

Factor	Variable	+	-
Garlic	A	Garlic	No Garlic
Chicken Broth	B	1 cup	½ cup
Tomato Sauce	C	1 cup	½ cup

To test these factors, I designed a 2^3 full factorial design with ___ replications. I set up the following design:

Run	Garlic	Chicken Broth	Tomato Sauce
1	-	-	-
2	-	-	+
3	-	+	-
4	-	+	+
5	+	-	-
6	+	-	+
7	+	+	-
8	+	+	+

This design is effective because it includes only three factors, each with two levels, making a full factorial design practical. This approach provides a complete set of data that can be used to determine the significance of the factorial effects.

Data Collection

To collect the data, I cooked in my kitchen using an electric stove top, a timer, and various measurement tools. I used the same brand of rice, tomato sauce, and chicken broth for every trial.

For each run, I took the measured ingredients and applied them to the same heat of level 4 for 25 minutes. First I took ½ tbsp of oil and placed the onions in the clean pan for 5 minutes. I then added 1 cup of rice in ½ tbsp of all seasoning. I would then lower the heat to level 4, and place in the desired amount of chicken broth and tomato sauce depending on the run. I set the timer for 25 minutes and let the rice cook. I then made all my roommates rate the rice out of 10 and came up with an accurate rating.

Analysis

After I collected this data, I analyzed my results in R. I used a full factorial regression model including all main effects (Garlic, Tomato_Sauce, and Chicken_Broth), all two-way interactions, and the three-way interaction. The model is expressed as:

$$\text{Rating} = \beta 0 + \beta 1(\text{Garlic}) + \beta 2(\text{Tomato\_Sauce}) + \beta 3(\text{Chicken\_Broth}) +$$
$$\beta 12(\text{Garlic}\times\text{Tomato\_Sauce}) + \beta 13(\text{Garlic}\times\text{Chicken\_Broth}) +$$
$$\beta 23(\text{Tomato\_Sauce}\times\text{Chicken\_Broth}) + \beta 123(\text{Garlic}\times\text{Tomato\_Sauce}\times\text{Chicken\_Broth}) + \varepsilon$$

This full factorial regression model is appropriate because I am studying three categorical factors (Garlic, Tomato_Sauce, and Chicken_Broth), each at two levels. By including all main effects, two-way interactions, and the three-way interaction, the model captures not only the individual effects of each factor but also how the factors may work together to influence the rice rating.

Using a full factorial design ensures that I can investigate possible synergies or antagonistic interactions between ingredients, which is important in cooking experiments where ingredient combinations often have non-additive effects. Additionally, since the experiment was relatively simple to conduct and involved only eight runs, it was feasible to fit the full model without omitting important interaction terms.

Although the model perfectly fits the data (resulting in a saturated model with no degrees of freedom for residual error), this is acceptable given the experimental context and the small number of observations.

The output for the regression analysis looks as follows:

```
Call:
lm.default(formula = Rating ~ Garlic * Tomato_Sauce * Chicken_Broth,
    data = rice_data)

Coefficients:
                               (Intercept)
                                       7.5
                                 GarlicYes
                                       0.5
                           Tomato_SauceLow
                                       1.5
                          Chicken_BrothLow
                                      -3.5
                 GarlicYes:Tomato_SauceLow
                                      -3.0
                GarlicYes:Chicken_BrothLow
                                      -0.5
          Tomato_SauceLow:Chicken_BrothLow
                                      -3.5
GarlicYes:Tomato_SauceLow:Chicken_BrothLow
                                       3.0
```

In this model, the intercept of 7.5 represents the predicted rice rating when Garlic is "No," Tomato_Sauce is "High," and Chicken_Broth is "High" — the baseline levels.

The main effect of Garlic being "Yes" increases the predicted rating by 0.5 points, while using a "Low" level of Tomato_Sauce increases the rating by 1.5 points. In contrast, using a "Low" level of Chicken_Broth decreases the rating by 3.5 points.

The interaction terms reveal that combinations of factors further influence the rating: for example, having both Garlic = "Yes" and Tomato_Sauce = "Low" reduces the rating by an additional 3.0 points. The three-way interaction between Garlic, Tomato_Sauce, and Chicken_Broth increases the rating by 3.0 points when all three non-baseline levels are present.

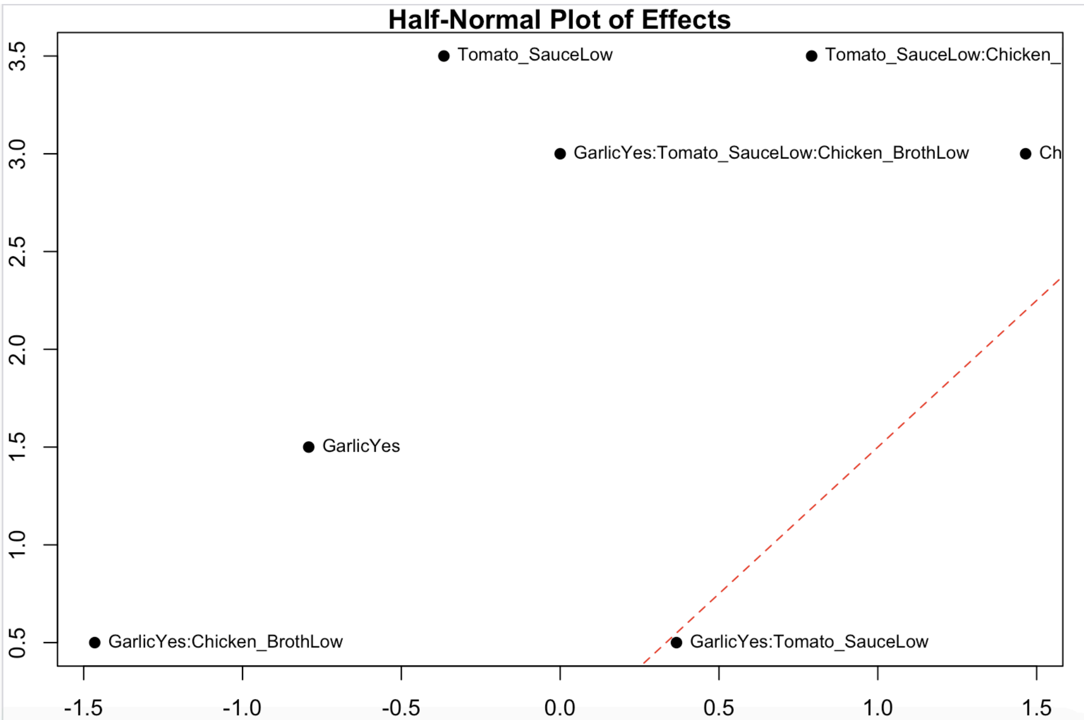
Since the model perfectly fits the data (saturating all degrees of freedom), no residual error remains for formal hypothesis testing. However, the estimated coefficients still provide insight into which factors and combinations had the largest impact on rice ratings.

In order to formally assess which effects were large enough to be considered significant, I applied Lenth's method. First, I calculated the absolute effects from the fitted full factorial model and used them to estimate the pseudo standard error (PSE). The trimming step removed any effects larger than 2.5 times the initial s0s_0s0, and the final PSE was used to compute standardized effect estimates (tPSE values). Using an individual error rate (IER) critical value of 2.3, none of the main effects, two-way interactions, or the three-way interaction had a tPSE value greater than 2.3. Here's the output:

```
                                                                           Effect
GarlicYes                                                               GarlicYes
Tomato_SauceLow                                                   Tomato_SauceLow
Chicken_BrothLow                                                 Chicken_BrothLow
GarlicYes:Tomato_SauceLow                                GarlicYes:Tomato_SauceLow
GarlicYes:Chicken_BrothLow                              GarlicYes:Chicken_BrothLow
Tomato_SauceLow:Chicken_BrothLow                  Tomato_SauceLow:Chicken_BrothLow
GarlicYes:Tomato_SauceLow:Chicken_BrothLow GarlicYes:Tomato_SauceLow:Chicken_BrothLow
                                           Estimate Abs_Effect      tPSE
GarlicYes                                       0.5        0.5 0.1111111
Tomato_SauceLow                                 1.5        1.5 0.3333333
Chicken_BrothLow                               -3.5        3.5 0.7777778
GarlicYes:Tomato_SauceLow                      -3.0        3.0 0.6666667
GarlicYes:Chicken_BrothLow                     -0.5        0.5 0.1111111
Tomato_SauceLow:Chicken_BrothLow               -3.5        3.5 0.7777778
GarlicYes:Tomato_SauceLow:Chicken_BrothLow      3.0        3.0 0.6666667
> ier.crit = 2.3
> results$Significant = abs(results$tPSE) > ier.crit
> print(results$Significant)
[1] FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

Therefore, according to Lenth's method, no effects were statistically significant in this experiment. This result is not unexpected given that the model is saturated and the sample size is small, limiting the ability to detect significant effects even if they exist.

To visually assess the magnitude of the effects, I created a half-normal plot of the absolute effect estimates:



The half-normal plot of the absolute effects shows that while some effects, such as Tomato_SauceLow and the interaction between Tomato_SauceLow and Chicken_BrothLow, appear larger than others, none of the points deviate dramatically from the overall trend line. This observation is consistent with the results of Lenth's method, which indicated that no effects were statistically significant.

Results:

Based on the results of the regression analysis, the fitted model for this data is:

Rating = 7.5 + 0.5(GarlicYes) + 1.5(Tomato_SauceLow) − 3.5(Chicken_BrothLow) − 3.0(GarlicYes×Tomato_SauceLow) − 0.5(GarlicYes×Chicken_BrothLow) −

$3.5(Tomato\_SauceLow \times Chicken\_BrothLow) +$

$3.0(GarlicYes \times Tomato\_SauceLow \times Chicken\_BrothLow)$

The goal of this experiment was to figure out the optimal combination of ingredients to make the best Mexican rice. By plugging in all possible factor level combinations into the fitted regression model, the combination of Garlic = "No," Tomato_Sauce = "Low," and Chicken_Broth = "High" produced the highest predicted rating of 9.0. Because my model is saturated, the fitted values from the model exactly match the ratings I collected.