Main Ecosim Equation Consumption Estimate Bibliography

### BREAKING DOWN ECOSIM

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# Main Equation PDE

The main equations of Ecosim model are the next.

$$\frac{\partial B_i}{\partial t} = g_i \sum_j Q_{ji} - \sum_j Q_{ij} + I_i - (M_i + F_i + e_i) B_i \quad (1)$$

Where  $B_i$  is the Biomass of the ith functional group,  $g_i$  is the net growth efficiency,  $Q_{ij}$  is the j consumption over group i,  $I_i$  is the net immigration of group i, and  $M_i$ ,  $F_i$ ,  $e_i$  are the natural and other mortality rate, Fishing rate and emigration rate of group i respectively.

## Main Equation

#### Dimensional Analysis

If we go through a dimensional Analysis of every term in the equation, from a naive approach.

$$\frac{\partial B_i}{\partial t} = g_i \sum_j Q_{ji} - \sum_j Q_{ij} + I_i - (M_i + F_i + e_i) B_i$$
 (2)

$$[MT^{-1}] = [MM^{-1}][M] - [M] + [M] - ([T^{-1}] + [T^{-1}] + [T^{-1}])[M]$$
(3)

Please note that  $g_i$  is just  $(Q/B)_i$ , and also that this is a naive approximation because units make no sense. So consumption and Immigration ought be related to a time unit.

## Main Equation

#### Types of Functional Groups

-Producers, in which the consumption (firs term in the RHS) is replaced by a production term :

$$f_i = \frac{r_i B_i}{(1 + B_i h_i)} \tag{4}$$

Where  $h_i = (r_i(P/B)_i - 1)/B_i$ , and  $r_i$  is an user given maximum (P/B) relation per functional group.

- -Consumers, consumption rates must be determined.
- -Detritus, which have consumption over and where mortality and unassimilated fractions goes.

#### First Approach

The first approach is just the primitive Lotka-Volterra one

$$Q_{i,j}(B_i, B_j) = a_{ij}B_iB_j \tag{5}$$

Where  $a_{ij}$  is the instantaneous mortality rate on prey i caused by one unit of predator j biomass, and it could be estimated from diet matrix and Ecopath biomasses and consumptions results.

$$a_{ij} = Q_{ij}/(B_i B_j) \tag{6}$$

By doing that, the time unit must be the same as the Ecopath one.

#### First approach results

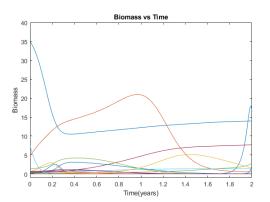


Figure - Plain Lotka-Volterra Approximation

#### First approach comments

- -The assumption here is that the preys are always available for the predators, and that they can eat as fast as light, which is not true.
- -That's why there are a lot of oscillations in the plot, and that causes also a numerical stability problem, because the time step must be really small to ensure it  $(1E^{-6} \text{ years})$ .
- -This kind of approximations are good for chemical processes that takes little time to come about, but not for fishes which must deal with searching for preys, catch and eat them.

#### Second Approach

The second approach is a Lotka-Volterra modified to include a vulnerability factor on every relation prey-predator. Let's call it Plain Foraging Arenas. The assumption is that every prey pool  $B_i$  have an available component to each consumer j,  $V_{ij}$ 

$$\frac{V_{ij}}{dt} = \tilde{\nu}_{ij}(B_i - V_{ij}) - \nu_{ij}V_{ij} - a_{ij}V_{ij}B_j \tag{7}$$

"That is,  $V_{ij}$  gains biomass from the currently unavailable pool  $(B_i - V_{ij})$  at rate  $\nu_{ij}$ , biomass returns to the unavailable state at rate  $\nu_{ij}V_{ij}$ , and biomass is removed from  $V_{ij}$  by the consumer at a mass-action encounter rate  $a_{ij}V_{ij}B_j$ " (Walters, 97).

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#### Second Approach

Taking an equilibrium assumption

$$V_{ij} = \frac{\tilde{\nu}_{ij}B_i}{(\nu_{ij} + \tilde{\nu}_{ij} + a_{ij}B_j)} \tag{9}$$

And that rates  $\nu$  and  $\tilde{\nu}_{ij}$  can be assumed the same, then

$$V_{ij} = \frac{\nu_{ij}B_i}{(2\nu_{ij} + a_{ij}B_j)} \tag{10}$$

#### Second Approach

The flow rates are

$$\nu_{ij} = \frac{(Q/B)_j B_j D_{ij} \chi_{ij}}{B_i} = \frac{Q_{ij} \chi_{ij}}{B_i} \tag{11}$$

Where  $(Q/B)_i$  is the consumption-biomass Ecopath relation of predator, B are the biomasses,  $D_{ij}$  is the diet relation between prey-predator,  $\chi_{ij}$  is the user given vulnerability.

#### Second Approach

Then  $a_{i,j}$  can be computed as follow

$$a_{ij} = \frac{2Q_{ij}\nu_{ij}}{(\nu_{ij}B_iB_j - Q_{ij}B_j)} \tag{12}$$

Where  $(Q/B)_i$  is the consumption-biomass Ecopath relation of predator, B are the biomasses,  $D_{i,j}$  is the diet relation between prey-predator,  $\chi_{i,j}$  is the user given vulnerability. In other words, the relation between net consumption times vulnerability factors and prey biomasses.

#### Second Approach

And finally consumptions are

$$C_{ij} = \frac{a_{ij}\nu_{ij}B_iB_j}{(2\nu_{ij} + a_{ij}B_j)} \tag{13}$$

Here there is a limitation in the available biomass for consumption based on vulnerabilities. The term "arena" is nothing but every relation prey-predator.

#### First approach results

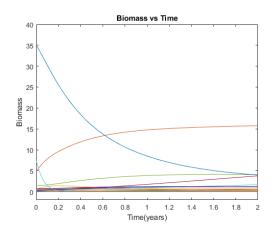


Figure – Plain Foraging-Arena Approximation

Third Approach

The third approach adds some modifications: a feeding time Ftime limitation and switching relation Rs, i.e., prey selection of predator based on abundance.  $a_{ij}$  and  $v_{ij}$  are modified in the next way

$$\tilde{a}_{ij} = a_{ij}Ftime_jRs_{ij} \tag{14}$$

$$\tilde{\nu}_{ij} = \nu_{ij} Ftime_j \tag{15}$$

# Consumption Third Approach

And consumption is:

$$C_{ij} = \frac{\tilde{a}_{ij}\tilde{\nu}_{ij}B_iB_j}{(\nu_{ij}hden_j + \tilde{\nu}_{ij}hden_j + \tilde{a}_{ij}B_j)}$$
(16)

Where  $hden_j$  is a limitation to the maximum consumption of every predator, and is updated every time step; as well as Ftime, but in Ecosim is updated every month.

#### Third Approach

Updating feeding time:

$$Ftime_{j}^{m+1} = 0.1 + 0.9Ftime_{j}^{m}(1 - Fadj_{j} + Fajd_{j}Qopt_{j}/CBlast_{j})$$

$$(17)$$

Where Fadj is an input parameter for adjust Feeding Times, Qopt is the optimal consumption, and CBlast is the actual consumption over biomass rate. And in differential form (deducted for updating every time step):

$$\frac{\partial Ftime}{\partial t} = 0.1(1 - Ftime_j) - 0.9Ftime_j Fadj_j \left(1 + \frac{Qopt_j}{CBlast_j}\right)$$
(18)

#### Third Approach

Updating hden:

$$hden_j = 0.5(1 + htime_j hdent_j) + 0.5hden_j$$
 (19)

Where  $htime_j$  is a handling time factor for predators, and hdent:

$$hdent_j = \sum_i \tilde{a}_{ij} V biomij$$
 (20)

Where  $Vbiom_{ij}$  is the vulnerable biomass in every arena.

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