地球データ駆動科学シンポジウム JAMSTEC-東大-筑波大-NIMS 合同セミナー

粒子データ駆動流体力学

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中尾篤史

2008〜2012 筑波大学生命環境学群地球学類
　研究テーマ：地震波を用いた断層の破壊過程の推定手法の開発（指導教員：八木勇治）

2012〜2017 東京工業大学大学院理工学研究科地球惑星科学専攻
　研究テーマ：マントル内部水輸送シミュレーション（指導教員：岩森光）

2017〜2019 東京大学地震研究所 特任研究員
　研究テーマ：多相系の大規模シミュレーション（受入研究者：鈴木雄治郎）

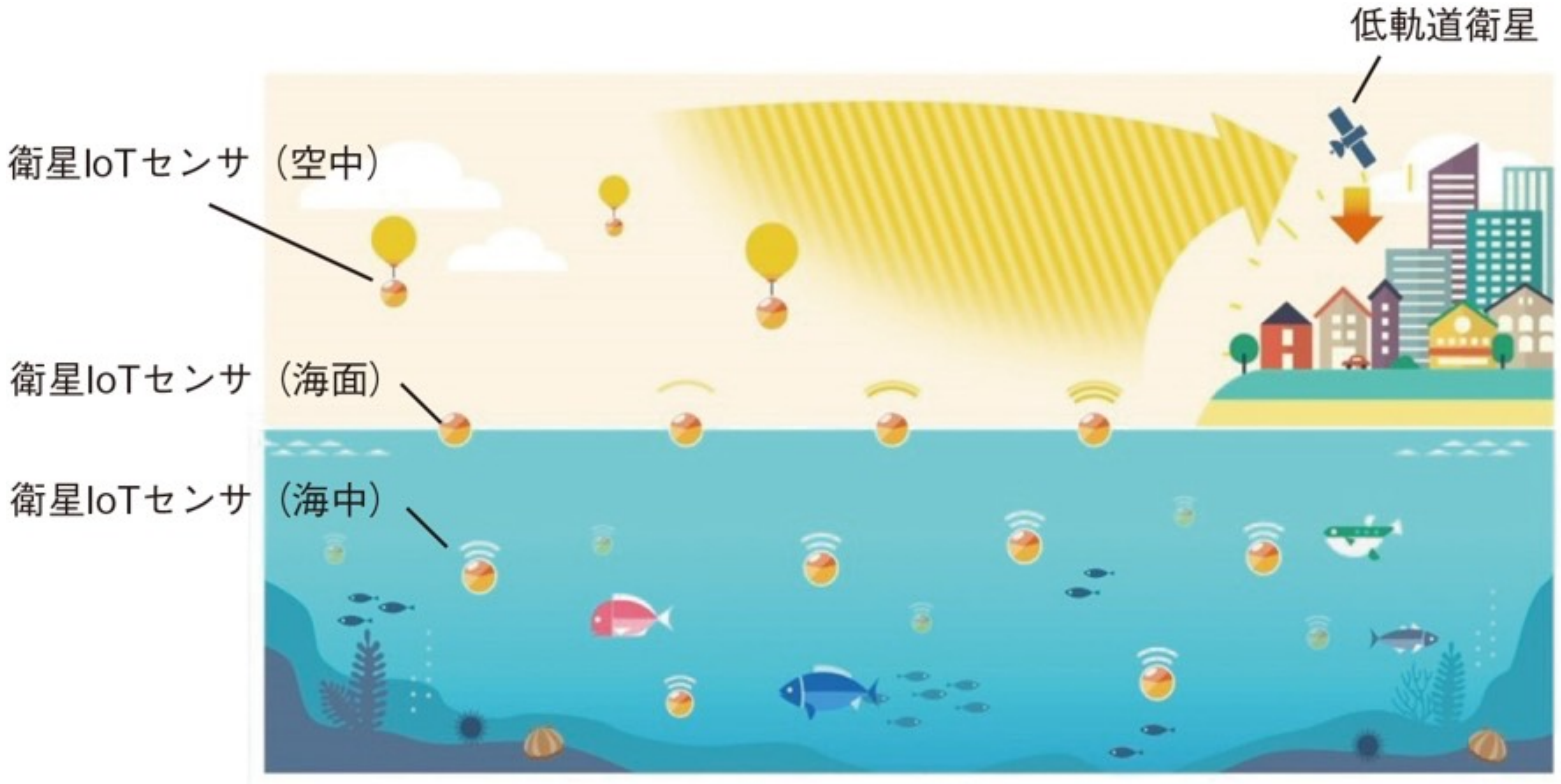
2019〜2020 香港大学地球科学系 ポストドクトラルフェロー
　研究テーマ：マントル内部微量元素輸送シミュレーション（受入研究者：中川貴司）

2021〜2023 JAMSTEC 海域地震火山部門 ポストドクトラル研究員
　研究テーマ：沈み込み帯パラメタの統計学的解析（受入研究者：桑谷立, 岡田CREST）

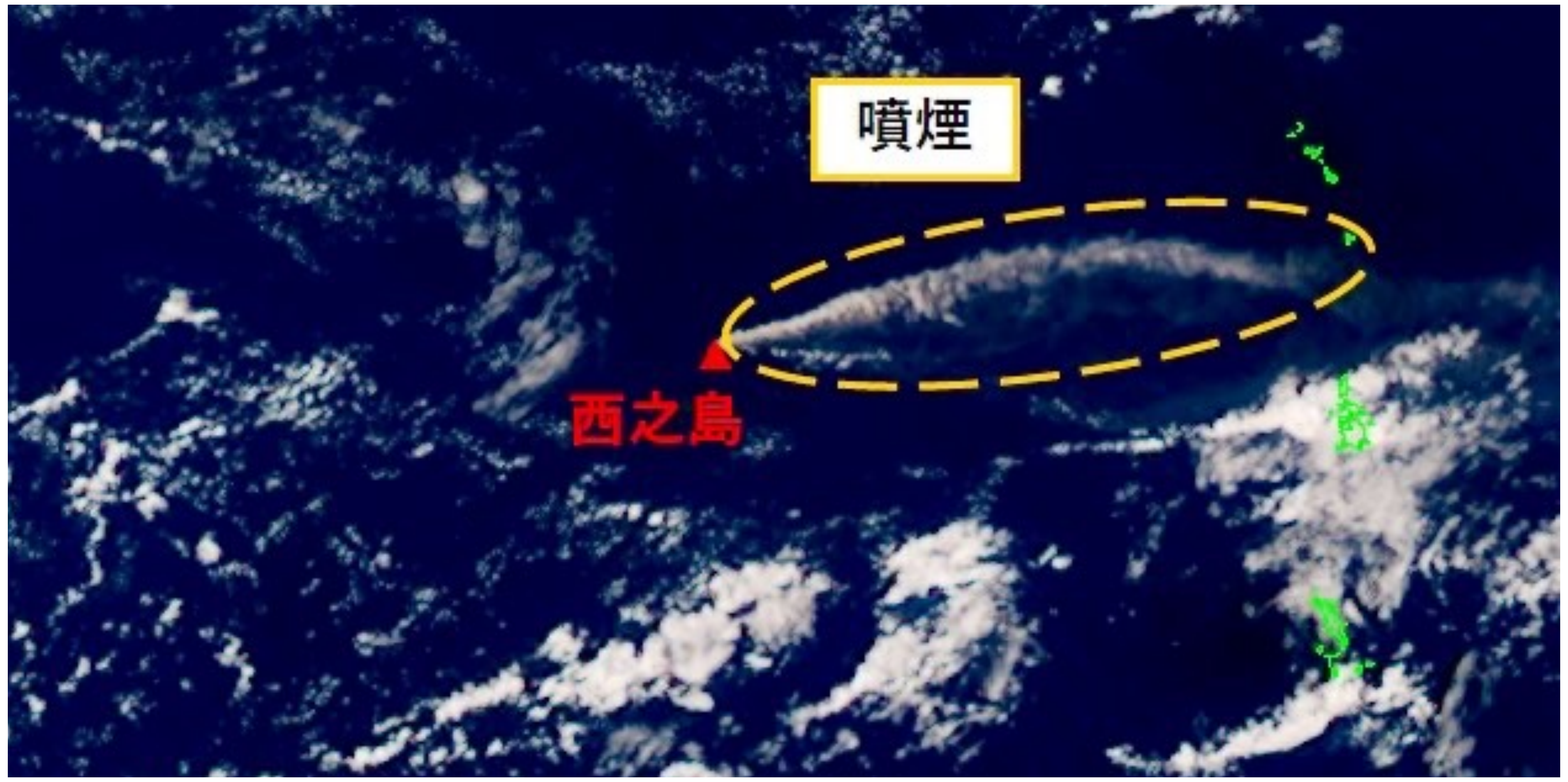
2023〜2025 秋田大学大学院理工学研究科 助教 (現職)
　研究テーマ：地球内部ダイナミクスを推定するためのデータ同化手法開発

2025〜 筑波大学システム情報系 助教 (着任予定)
　情報科学 x 流動現象をテーマとした研究室を立ち上げたい

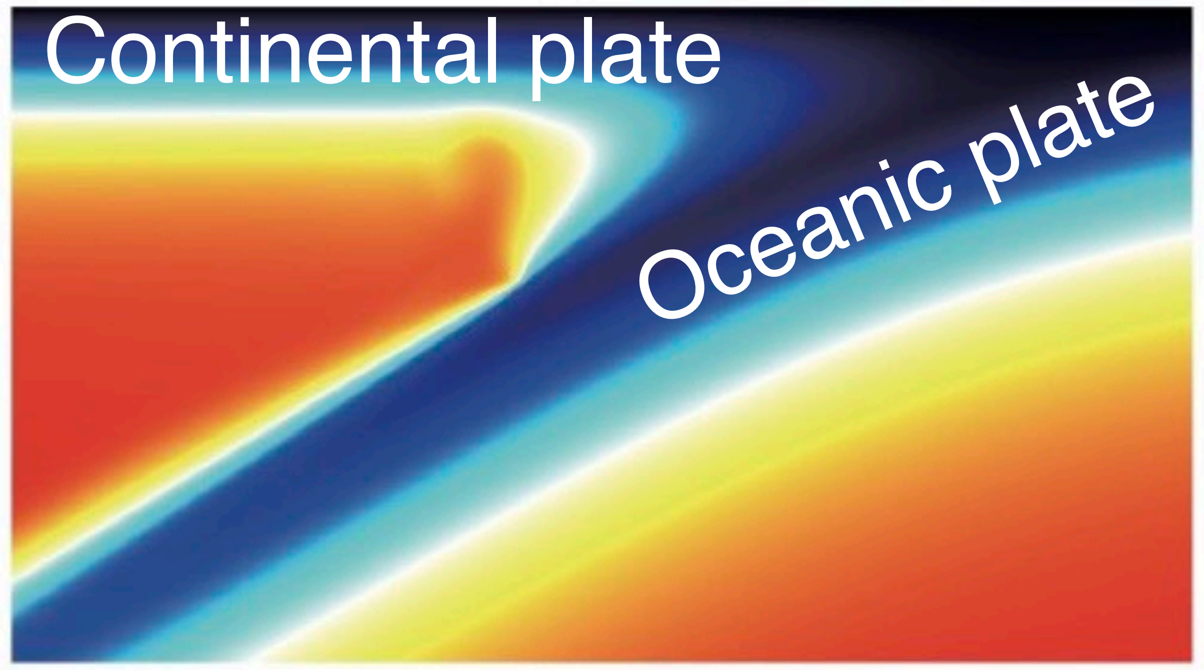
Tracer particles in geosciences

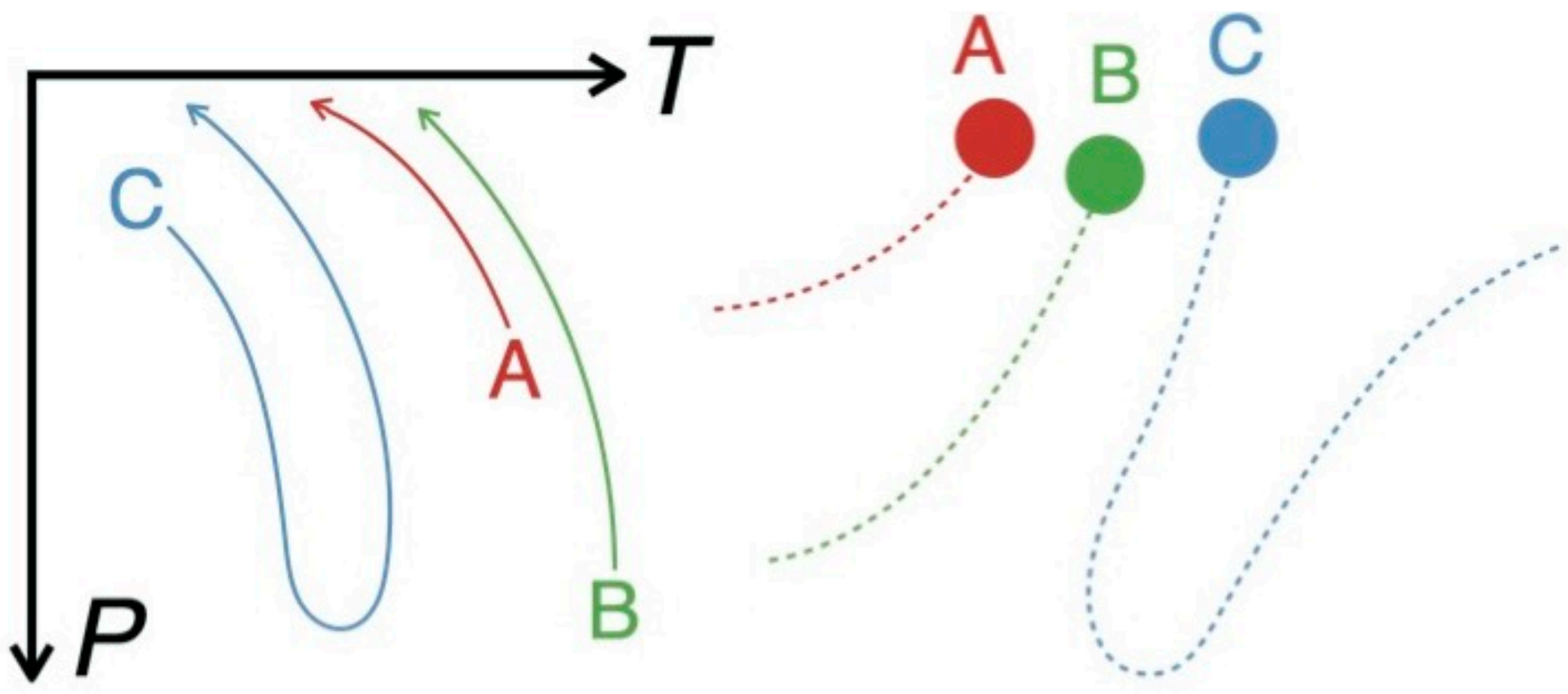


NTT journal



JMA





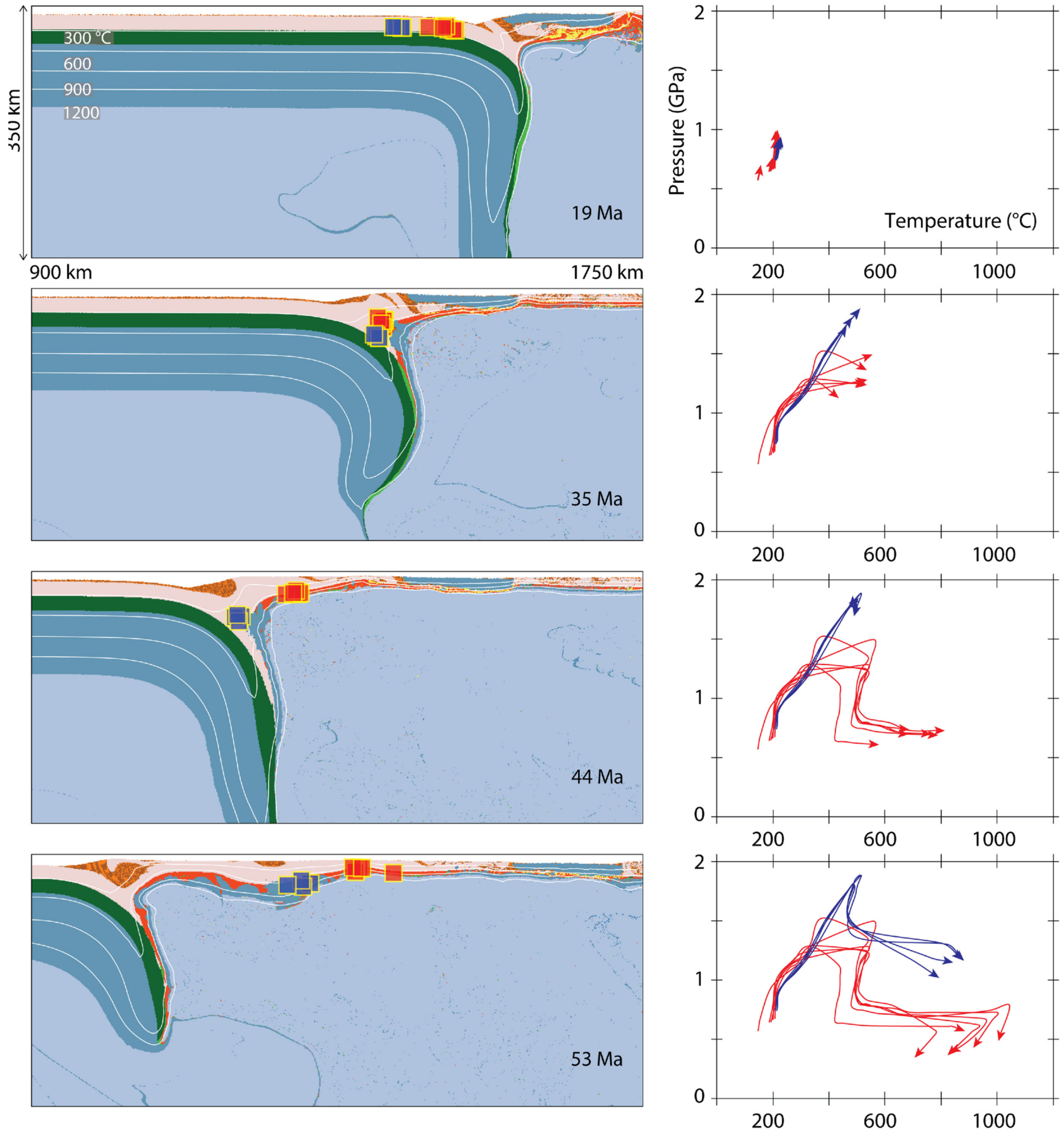
Unknown Eulerian Field

e.g., Temperature and flow of the surrounding mantle

Lagrangian Particle Data

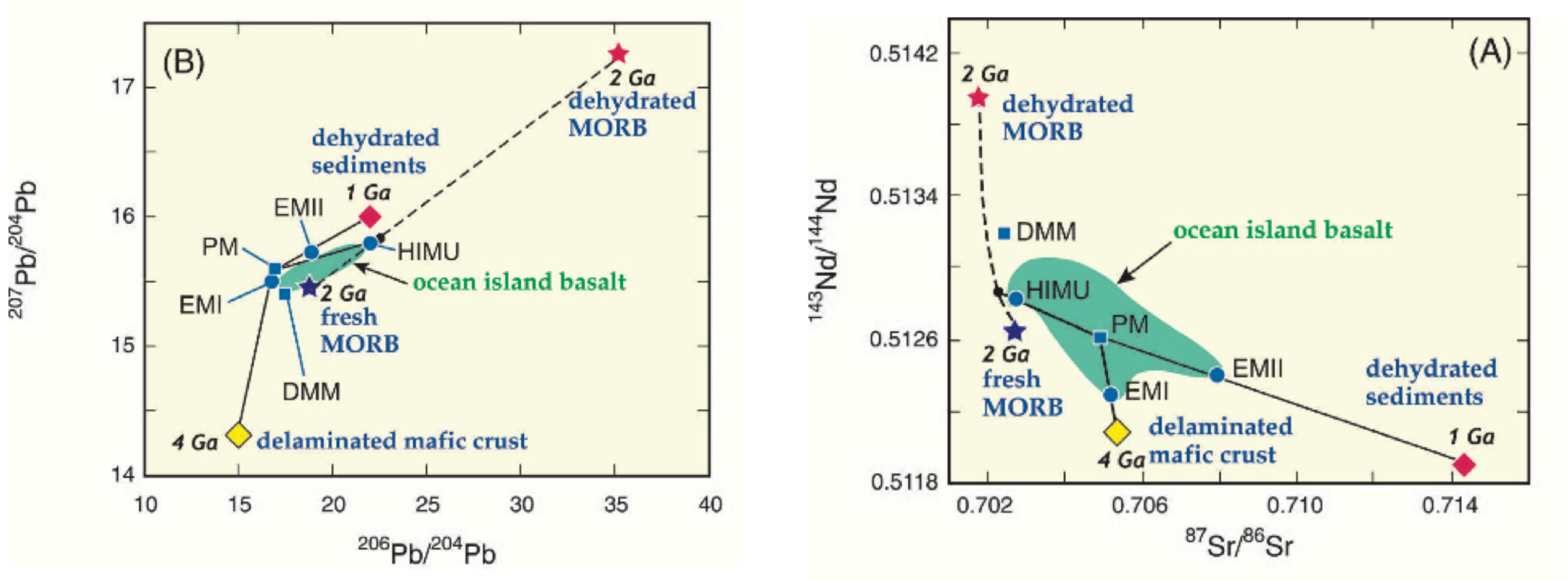
e.g., *P*–*T* and age of rock samples

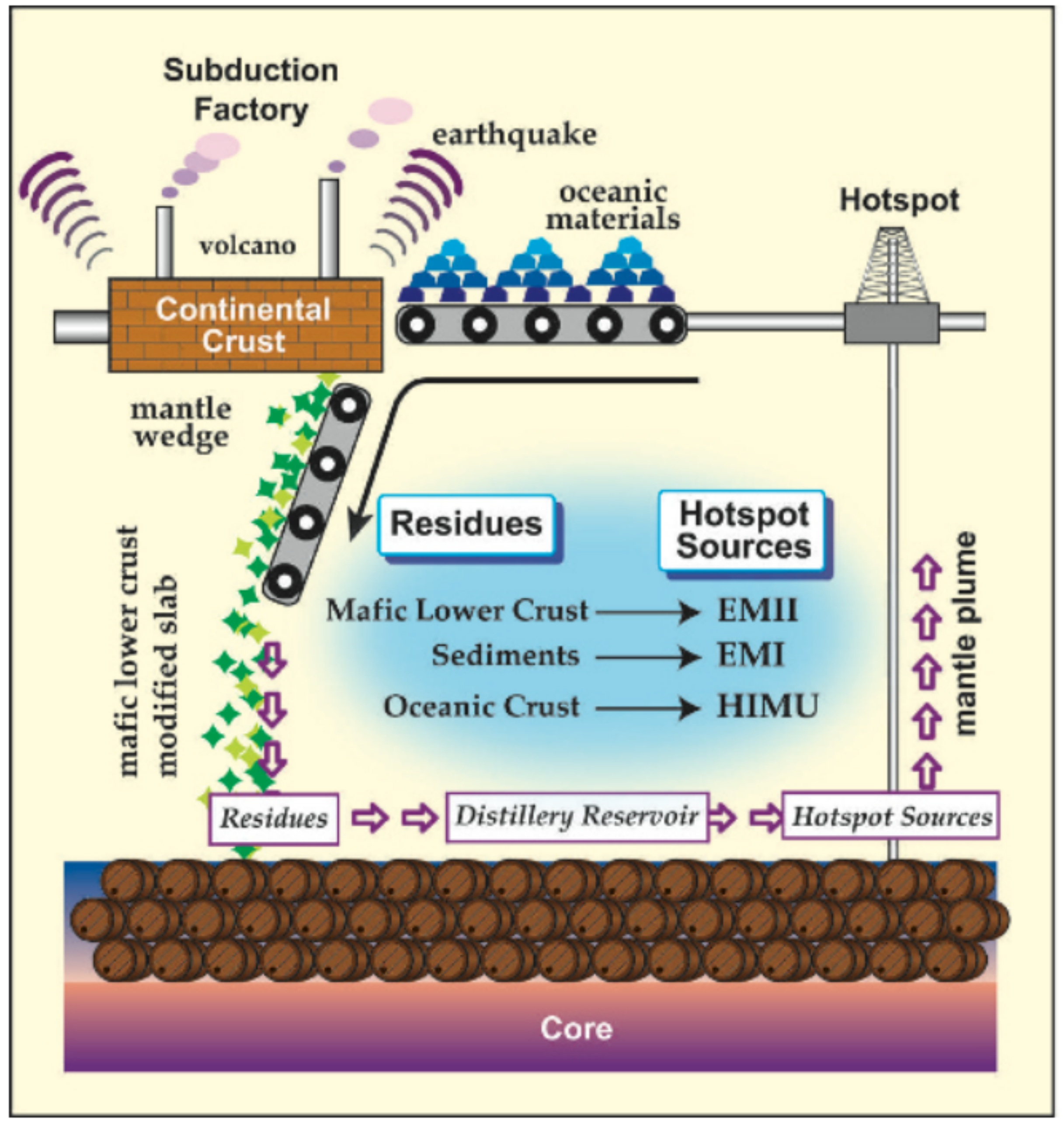
Forward simulation for rock P-T



Chowdhury et al. (2021)

Petrologists' inversion

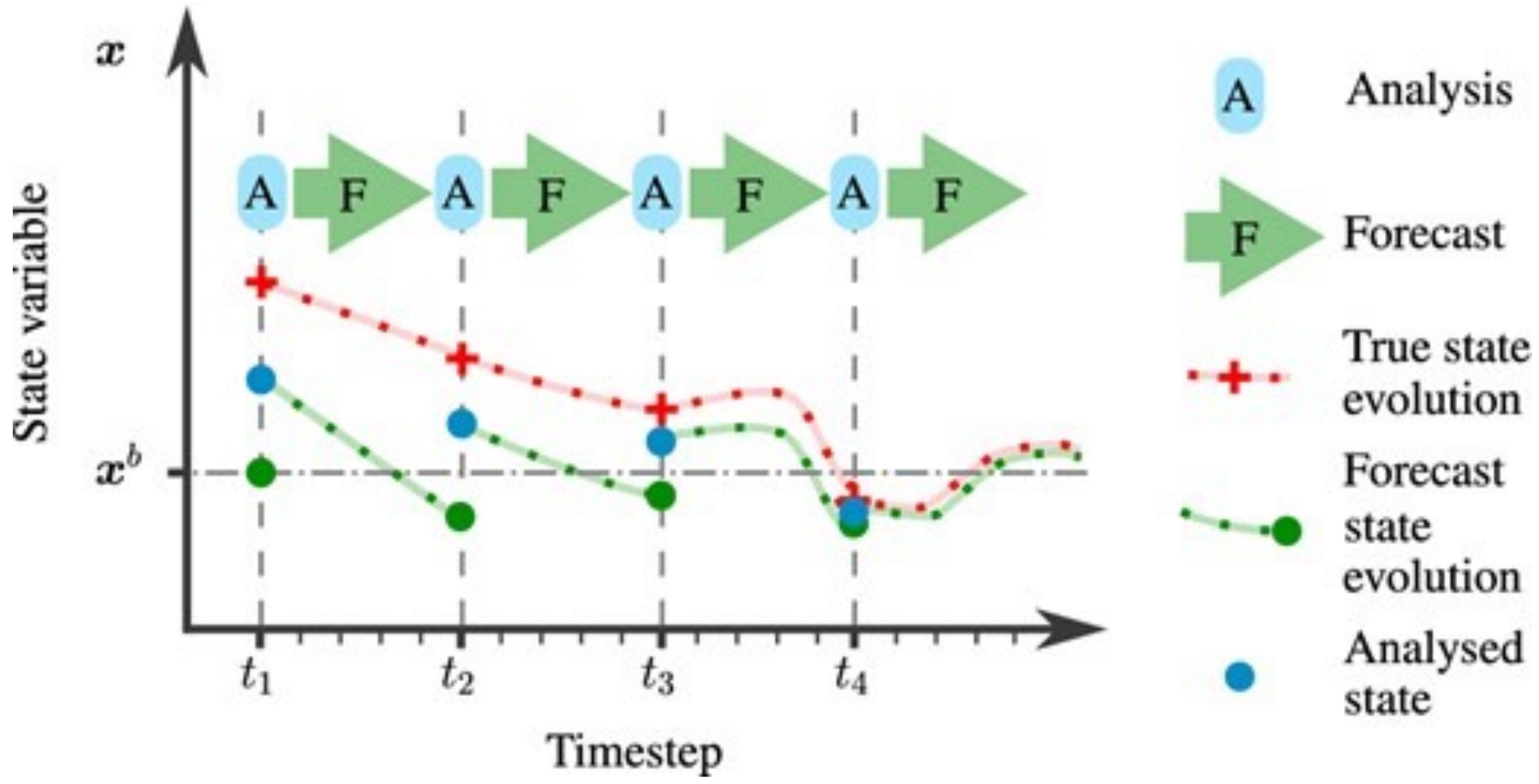




Tatsumi (2005)

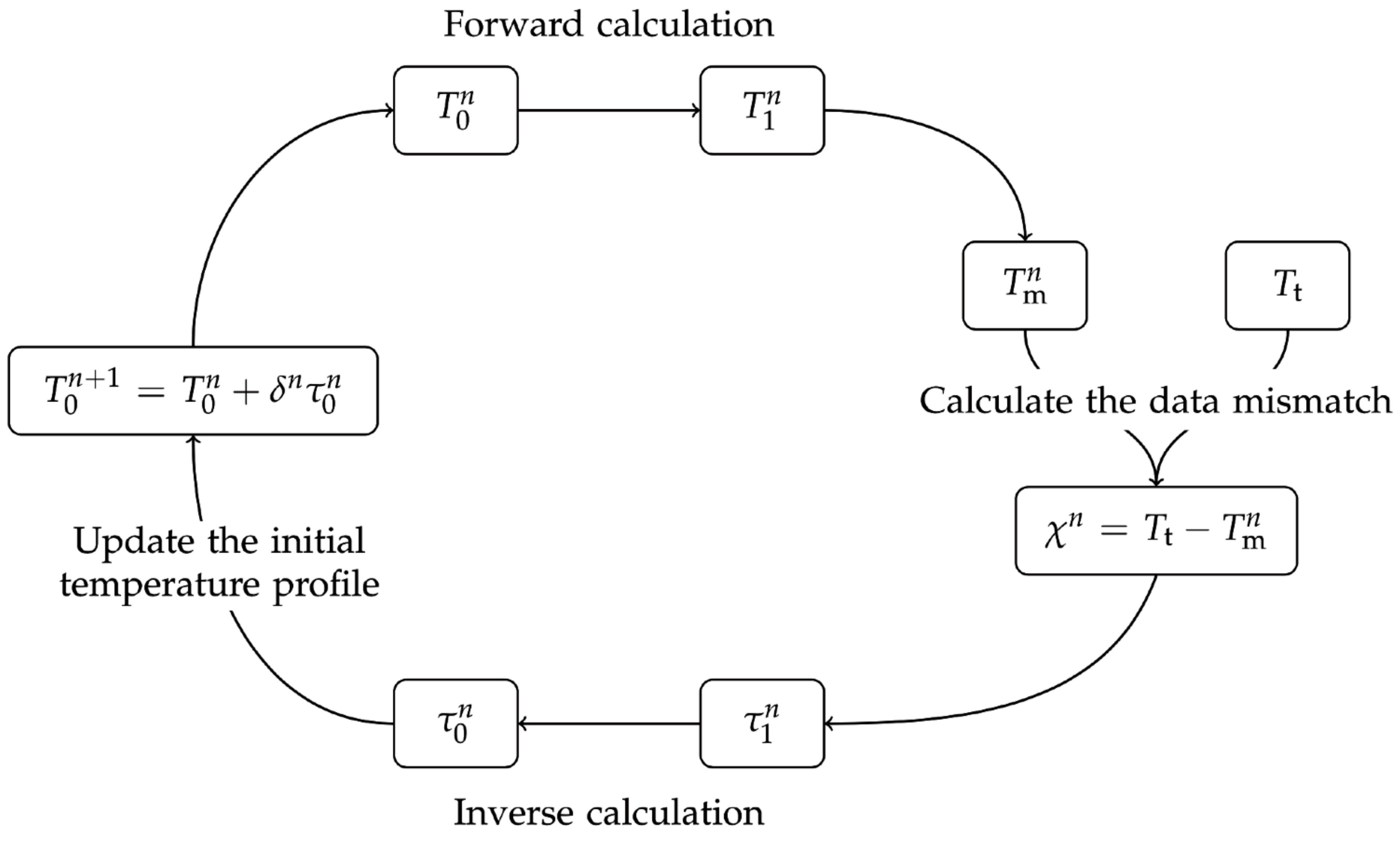
Two types of data assimilation

Sequential data assimilation
(e.g., Kalman filter)



Bocher et al. (2016 GJI)

Adjoint method (4D-Var)



Price & Davies (2017 GJI)

An **adjoint method** is more appropriate for geodynamic problems because geophysical observations are limited to the present.

Forward Model

2-D, inconpressible, slow motion, constant viscosity, and pure thermal convection

Motion

$$\nabla^4 \psi = \mathrm{Ra}\frac{\partial T}{\partial x} \qquad \mathbf{u} = (u, w)^\top = \left(\partial \psi/\partial z, -\partial \psi/\partial x\right)^\top$$

$$\mathrm{Ra} = \rho_0 \alpha \Delta T g h^3/\eta\kappa$$

Heat $$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = \nabla^2 T$$

Particle $$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}(\mathbf{x}_i(t), t)$$

Cost Function

$$J = \int_{t_0}^{t_1} dt \sum_i \frac{1}{2}\alpha\left(\mathbf{x}_i - \mathbf{x}_i^{\mathrm{obs}}\right)^\top\left(\mathbf{x}_i - \mathbf{x}_i^{\mathrm{obs}}\right) + \int_{t_0}^{t_1} dt \sum_i \frac{1}{2}\beta\left(\mathbf{u}_i - \mathbf{u}_i^{\mathrm{obs}}\right)^\top\left(\mathbf{u}_i - \mathbf{u}_i^{\mathrm{obs}}\right)$$

$x \in [x_0, x_1]$

$z \in [z_0, z_1]$

$t \in [t_0, t_1]$

i : Particle ID

Adjoint Model

Motion

$$\nabla^4\varphi + (\nabla\tau)\times(\nabla T) = \nabla\times\sum_{i\in dxdz}\beta(\mathbf{u}_i - \mathbf{u}_i^{\mathrm{obs}})$$

Heat

$$-\frac{\partial \tau}{\partial t} - \mathbf{u}\cdot\nabla\tau = \nabla^2\tau - \mathrm{Ra}\frac{\partial\varphi}{\partial x}$$

Particle

$$\frac{d\boldsymbol{\lambda}_i}{dt} = \alpha\left(\mathbf{x}_i - \mathbf{x}_i^{\mathrm{obs}}\right) - \boldsymbol{\lambda}_i^\top(\nabla_i\mathbf{u})$$

Gradient of cost function

$$\frac{\partial J}{\partial T(\mathbf{x}, t_0)} = -\tau(\mathbf{x}, t_0)$$

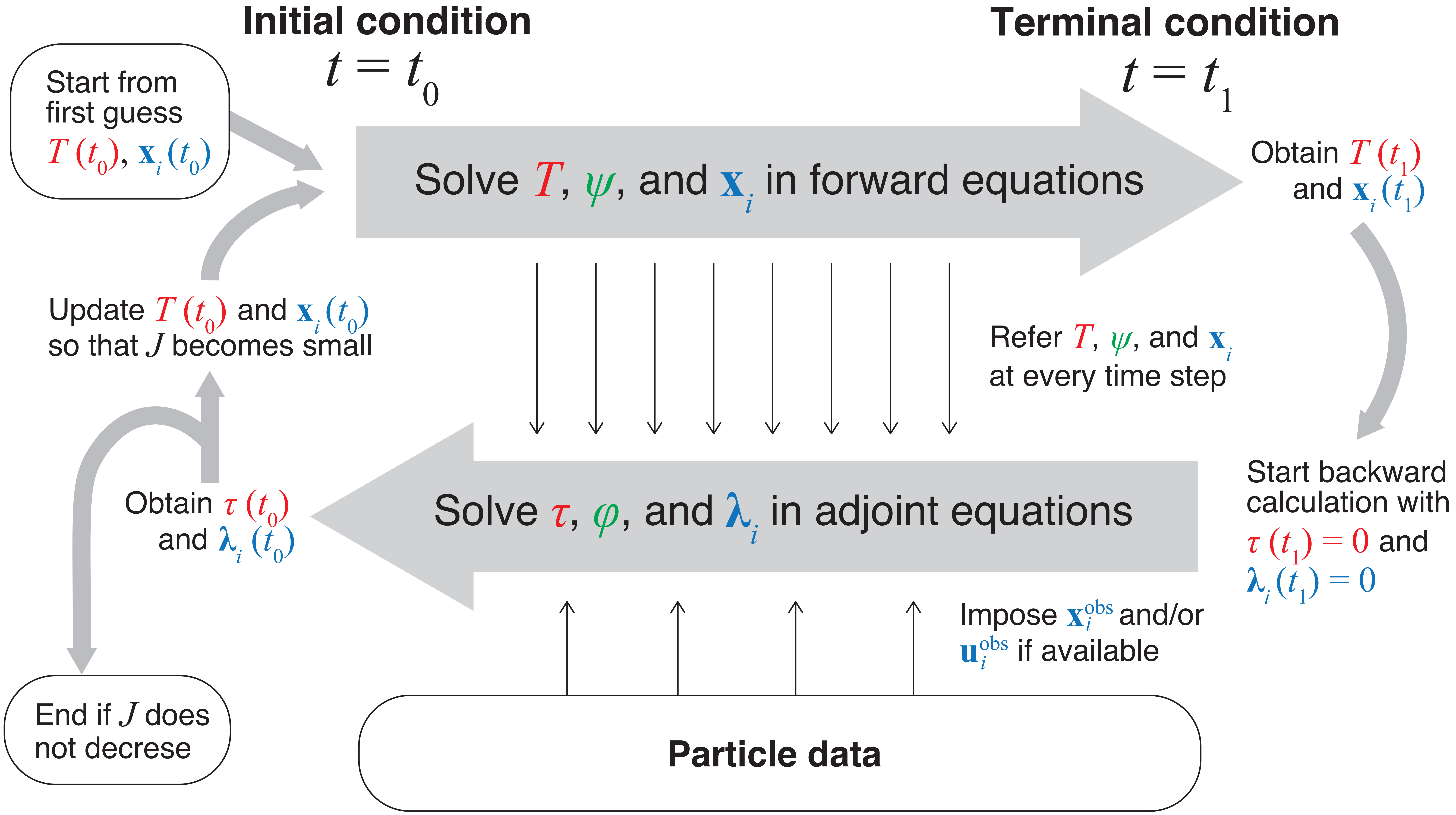
$$\frac{\partial J}{\partial \mathbf{x}_i(t_0)} = -\boldsymbol{\lambda}_i(t_0)$$

Terminal condition

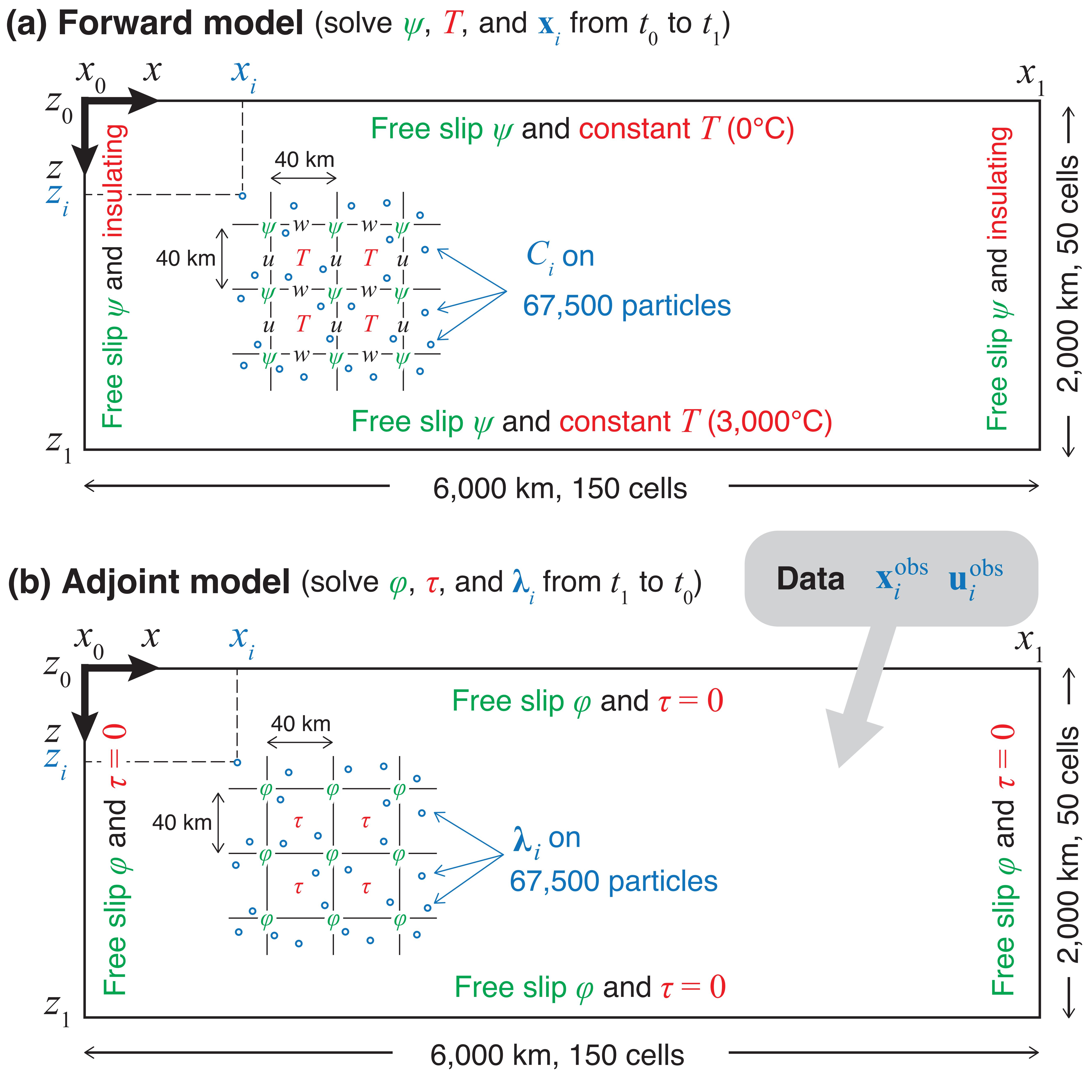
$$\tau(\mathbf{x}, t_1) = 0$$

$$\boldsymbol{\lambda}_i(t_1) = \mathbf{0}$$

ϕ: Adjoint stream function
τ: Adjoint temperature
λ_i : Adjoint particle position



Nakao et al. (JGR:ML, in press)



Physical parameters used to calculate the thermal Rayleigh number and/or to dimensionalize the simulation results.

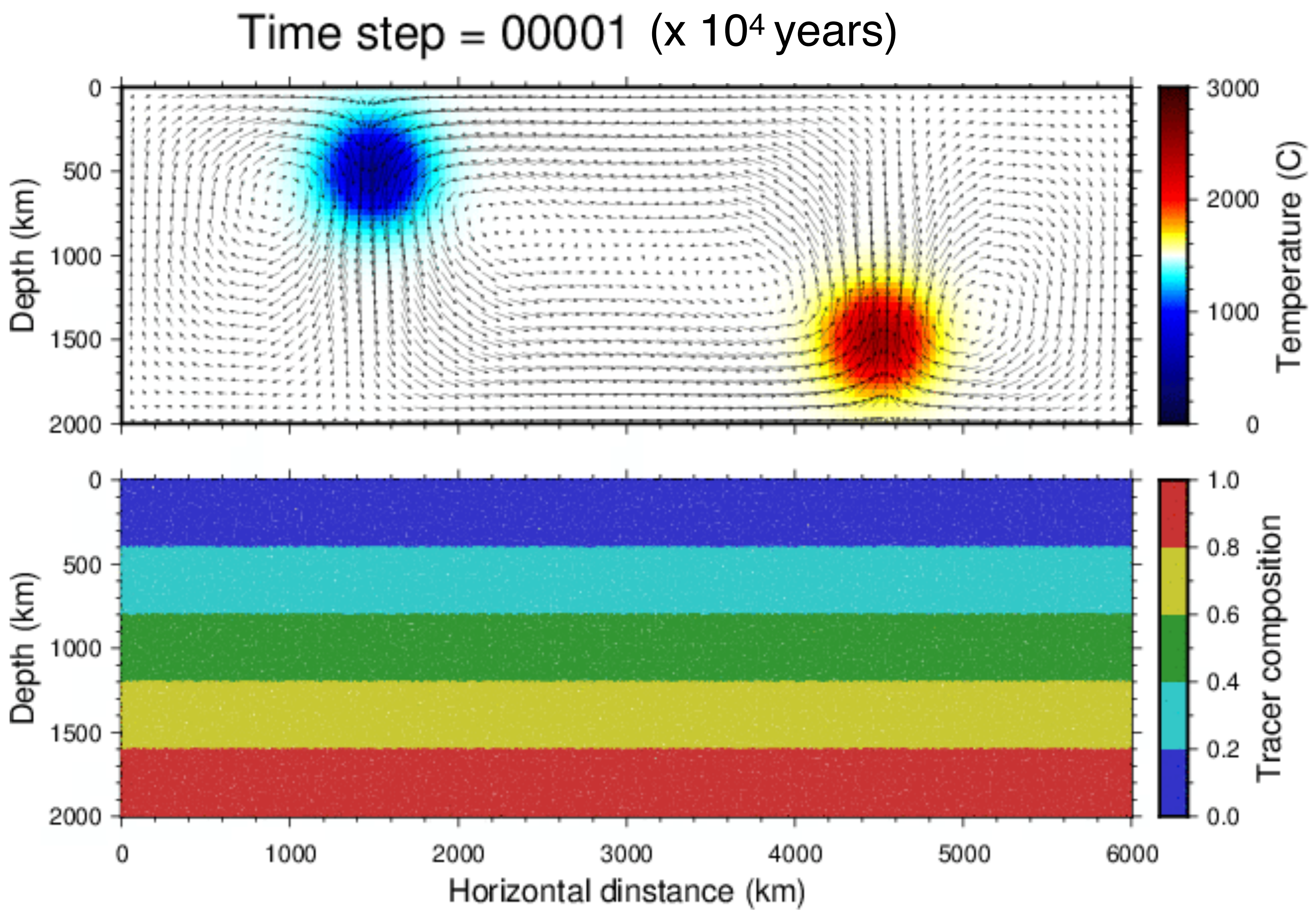
Symbol	Explanation	Value	Unit
g	Gravitational acceleration	10	m s^{-2}
h	Thickness of the convective layer	2×10^6	m
ΔT	Temperature difference between upper and lower boundaries	3×10^3	K
α_T	Thermal expansivity	2.5×10^{-5}	K^{-1}
η	Viscosity	10^{22}	Pa s
κ	Thermal diffusivity	10^{-6}	m^2 s^{-1}
ρ_0	Reference density	3.9×10^3	kg m^{-3}

$$\mathrm{Ra} = \frac{\rho_o \alpha_T \Delta T g h^3}{\eta \kappa} = 2.34 \times 10^6$$

Nakao et al. (JGR:ML, in press)

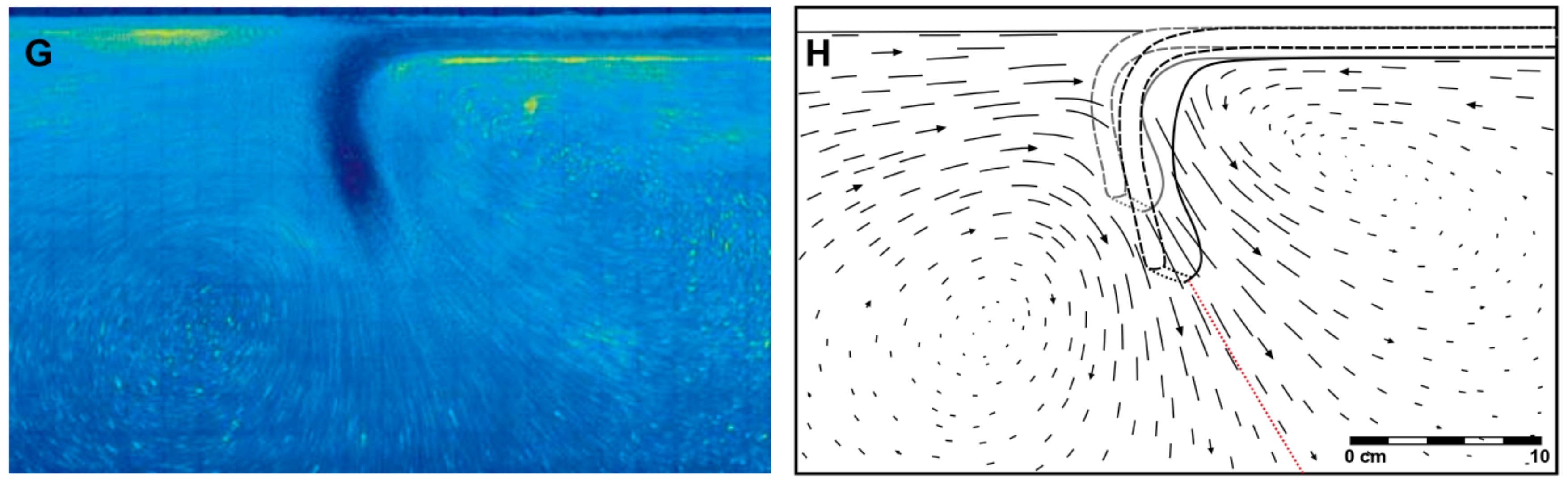
Case 1: Noisy “water tank” data

Reference Forward Model

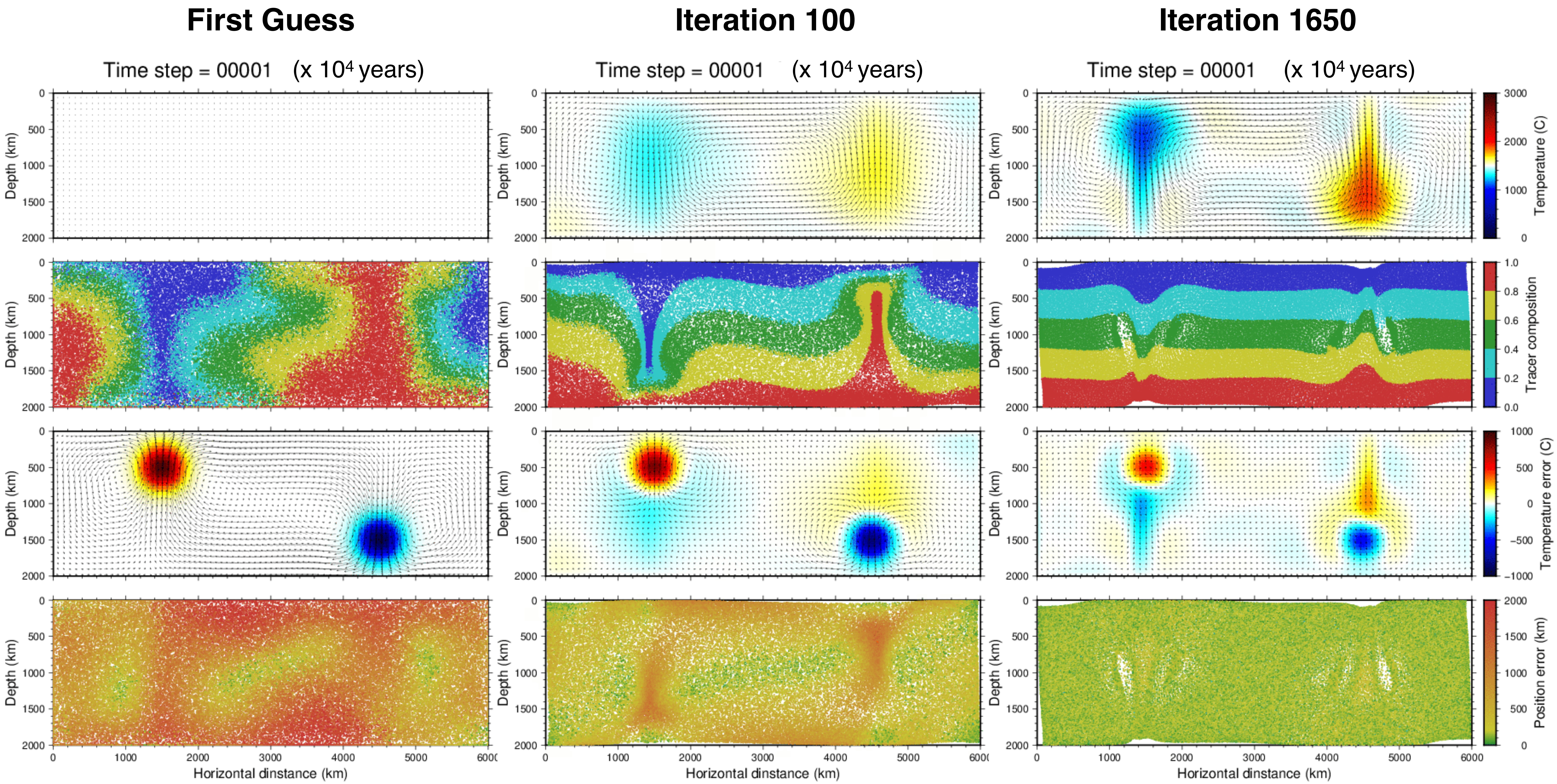


Data Generation

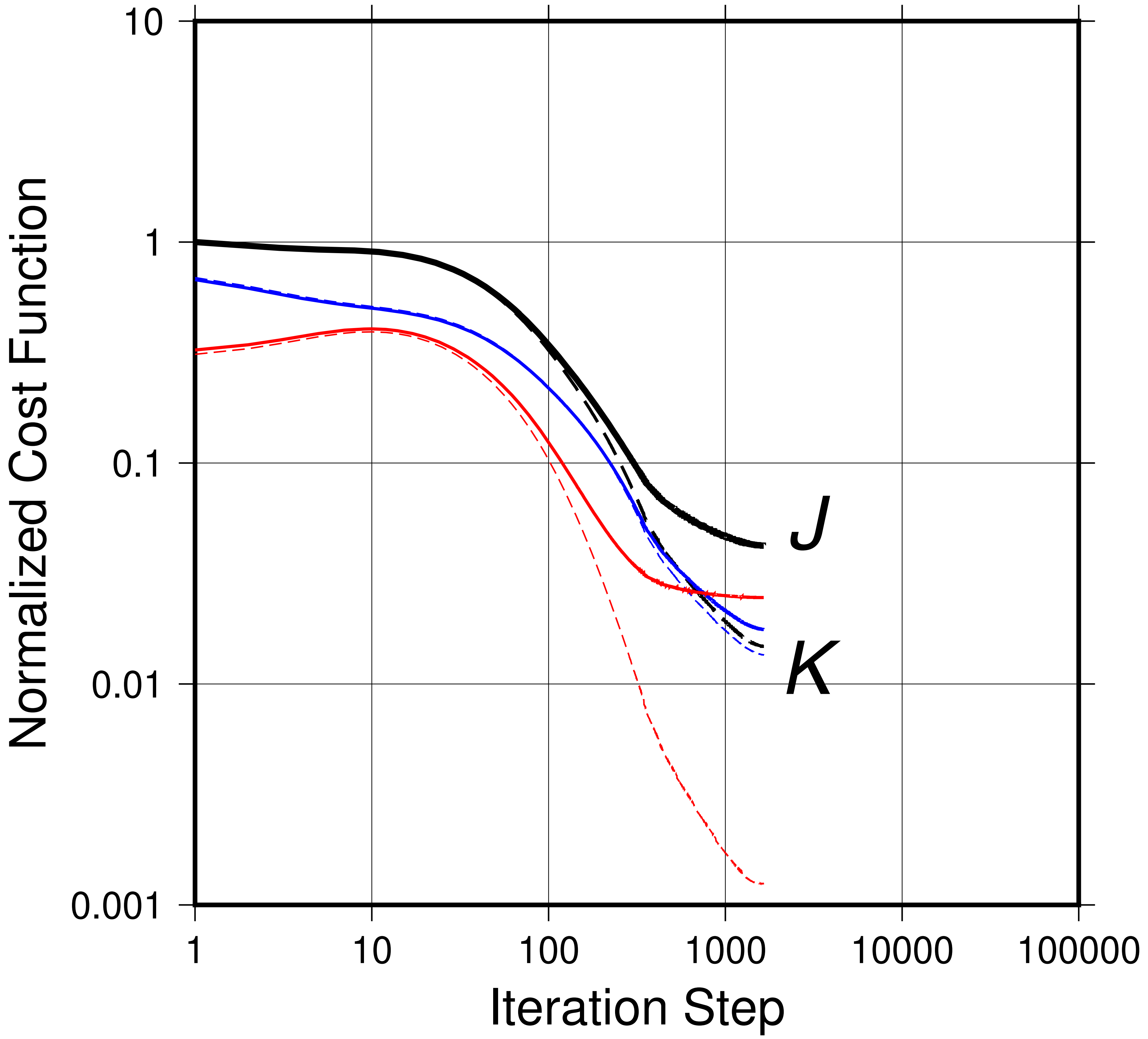
- Sampling time series of positions of all 67,500 particles
- Add random noise (200 km at maximum) to the true particle positions



Silicone putty subduction into syrop in laboratory experiments (Shellart, 2008)



Nakao et al. (Nakao et al. (JGR:ML, in press)



Solid lines = Residual between **observation** and estimation

$$J = \int_{t_0}^{t_1} dt \sum_{i=1}^{N} \frac{1}{2}\alpha \left(\boldsymbol{x}_i^{\mathrm{est}} - \boldsymbol{x}_i^{\mathrm{obs}}\right)^{\top} \left(\boldsymbol{x}_i^{\mathrm{est}} - \boldsymbol{x}_i^{\mathrm{obs}}\right) + \int_{t_0}^{t_1} dt \sum_{i=1}^{N} \frac{1}{2}\beta \left(\boldsymbol{u}_i^{\mathrm{est}} - \boldsymbol{u}_i^{\mathrm{obs}}\right)^{\top} \left(\boldsymbol{u}_i^{\mathrm{est}} - \boldsymbol{u}_i^{\mathrm{obs}}\right)$$

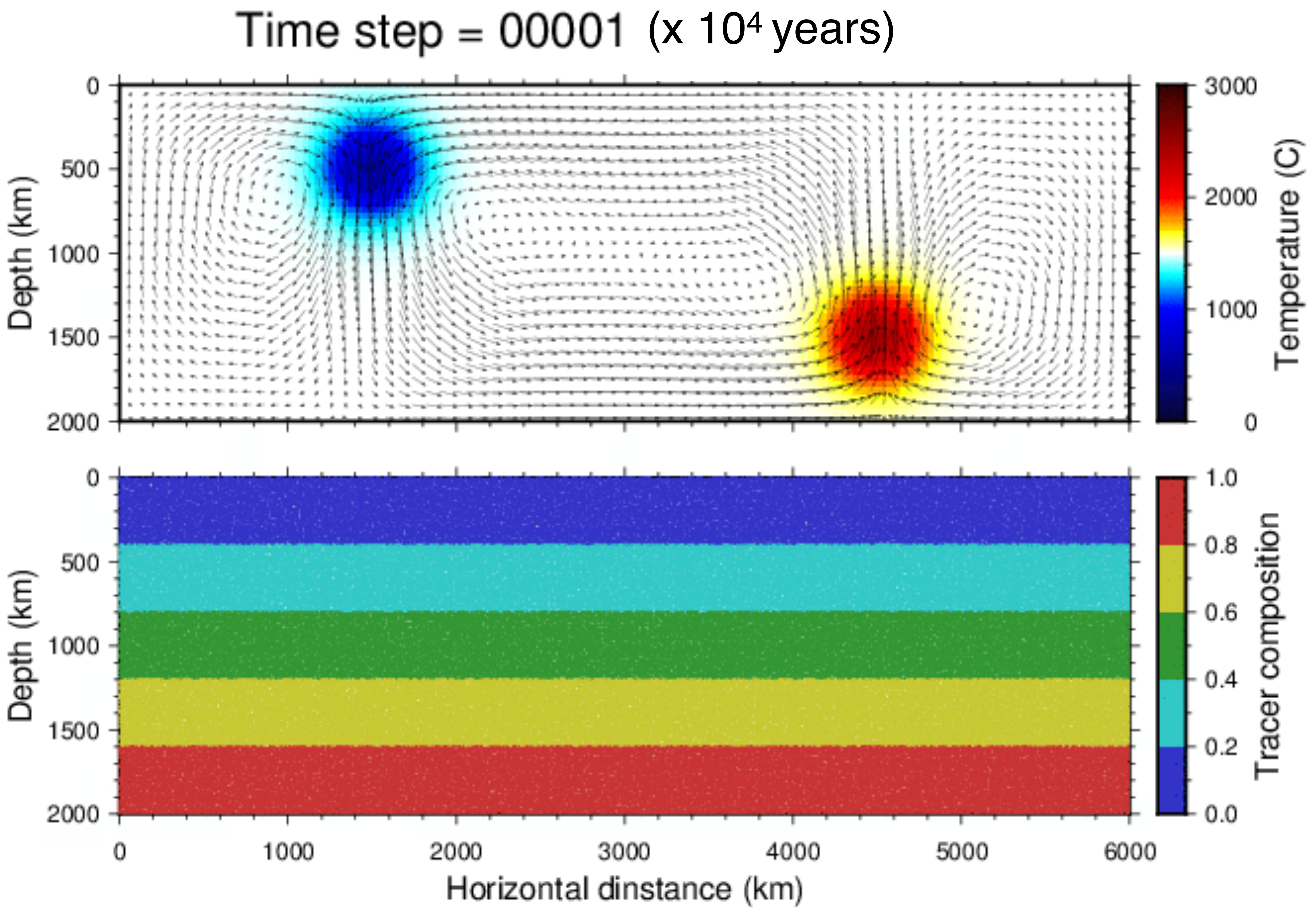
Dashed lines = Residual between **true** and estimation

$$K = \int_{t_0}^{t_1} dt \sum_{i=1}^{N} \frac{1}{2}\alpha \left(\boldsymbol{x}_i^{\mathrm{est}} - \boldsymbol{x}_i^{\mathrm{true}}\right)^{\top} \left(\boldsymbol{x}_i^{\mathrm{est}} - \boldsymbol{x}_i^{\mathrm{true}}\right) + \int_{t_0}^{t_1} dt \sum_{i=1}^{N} \frac{1}{2}\beta \left(\boldsymbol{u}_i^{\mathrm{est}} - \boldsymbol{u}_i^{\mathrm{true}}\right)^{\top} \left(\boldsymbol{u}_i^{\mathrm{est}} - \boldsymbol{u}_i^{\mathrm{true}}\right)$$

Nakao et al. (Nakao et al. (JGR:ML, in press)

Case 2: “Xenolith” (捕獲岩) data

Reference Forward Model



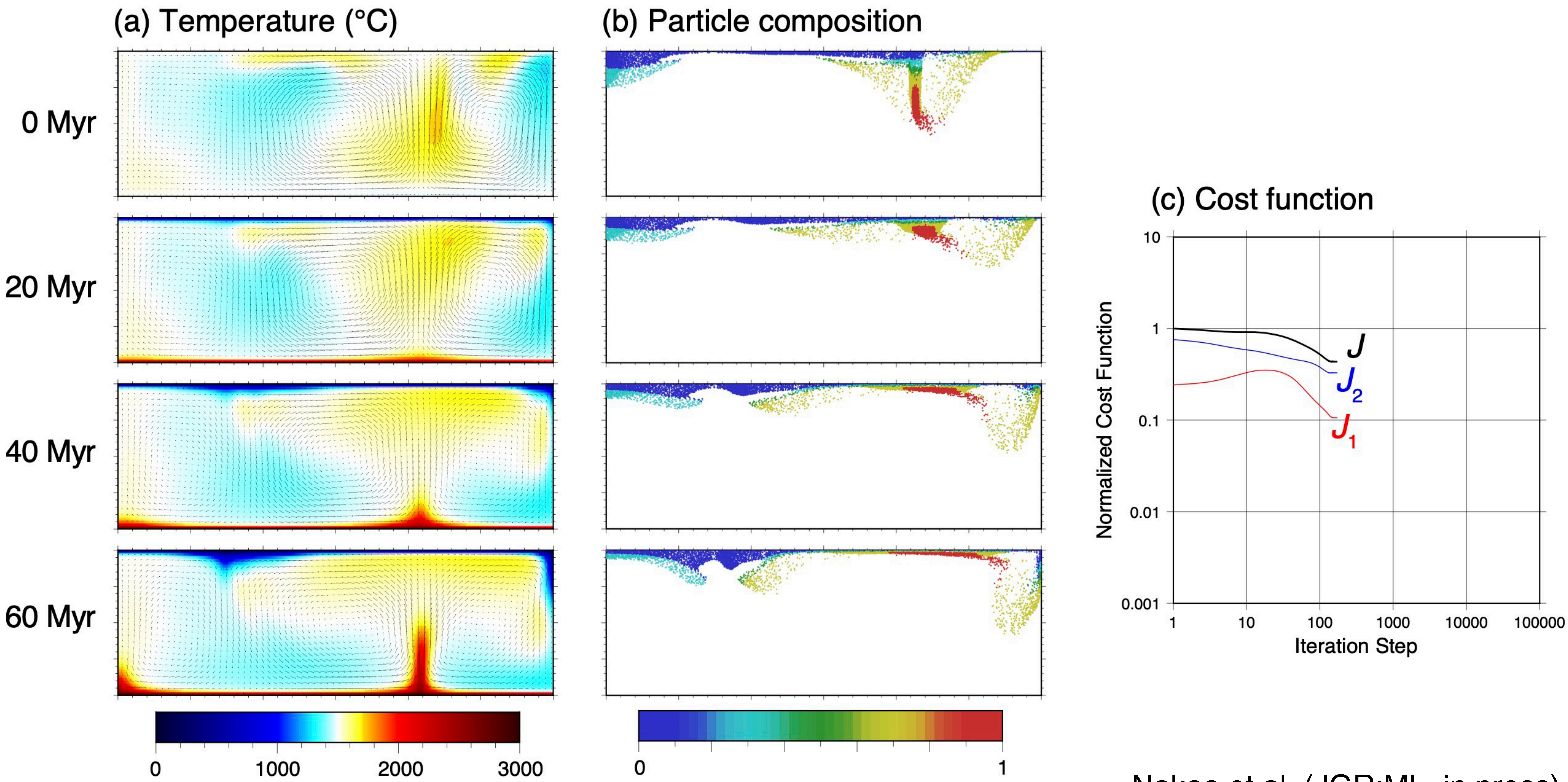
Data Generation

(1) Sample only data from particles that are at <200 km depth at the last time step

(2) Lateral particle positions are only recorded at the depth of <200 km (analogous to paleomagnetic and chronological oservations of the ocean floor)

(3) Vertical particle positions are recorded at any depth (analogous to geobarometers)

(4) Particle positions can bee recorded with a probability of 10% at each time step, except the last time step



Nakao et al. (JGR:ML, in press)

Summary

We have developed an adjoint-based marker-in-cell scheme to reconstruct the thermal and flow processes of thermal convection using only particle position-track datasets.

This is the first framework to assimilate petrological data from the Earth's interior to quantitatively constrain past mantle thermal convection.

We have confirmed that the methods are applicable to real data obtained from laboratory experiments to constrain the thermal and flow processes, as well as unknown parameters, such as the thermal Rayleigh number.

The framework will be useful for other various systems, such as the simultaneous estimation of volcanic source parameters and eruption process.