Dynamic output stabilization of control systems: an unobservable kinematic drone model

Alain Ajami [a], Jean-Paul Gauthier [b], Ludovic Sachelli [c]

[a] *Université Saint Joseph de Beyrouth, Faculté d'ingégnierie, ESIB, Lebanon*

[b] *Université de Toulon, LIS, UMR CNRS 7020, Campus de la Garde, 83041 Toulon Cedex, France*

[c] *Department of Mathematics, Lehigh University, Bethlehem, Pennsylvania, USA*

Abstract

The problem of dynamic output stabilization is a very general and important problem in control theory. This problem is completely solved in the case where the system under consideration is strongly observable. However, usually, nonlinear systems do not share this property: in general, systems are observable or not depending upon the control as a function of time. In this general situation, very little is known about dynamic output stabilization.

In this paper, we solve the problem for a classical academic kinematic model for drones whose observability properties are especially bad.

Key words: Control systems, dynamic output stabilization, asymptotic stability

1 Introduction

In this paper, we consider an example of a controlled and observed nonlinear system within the following class: the systems that can be embedded into a state-affine one, satisfying the Fliess-Kubka criterion [1].

Our long-term program is the question of dynamic output stabilization within this class. Our example is the following academic kinematic model of a fixed wings drone (or UAV), flying at constant altitude, with constant velocity:

$$\begin{aligned} &\dot{x} = \cos(\theta), \quad \dot{y} = \sin(\theta), \quad \dot{\theta} = u, \\ &-u_{max} \leq u \leq u_{max}. \end{aligned} \tag{1}$$

Hence the (x, y) trajectories on the plane have a minimum possible radius of curvature being $r = \frac{1}{u_{\max}}$.

This model is inspired from the Dubins model [2]. In a series of previous works [3,4,5], we discussed some theoretical and practical facts about the control of equation (1). This classical academic example has been extensively studied, in particular in the context of modeling of vehicles and fixed wings drones [6,7,8].

In particular in [3,5], minimum time strategy to a target is studied. It could be thought that such minimum time strategy is quite obvious, and from practical point of view it is, more or less, but it is extremely nontrivial from academic point of view. On the topic, see also [2,9,10].

Here, we address the problem of dynamic output stabilization (to the target) of the drone, considering minimum information about the relative position of the target with respect to the drone. It turns out that this problem has extremely bad observability properties.

The problem of dynamic output stabilization has been studied a lot, see (among a huge number of authors) [11] for a general discussion of this problem. However the question of output stabilization using a state feedback law coupled with a nonlinear observer for systems with weak observability properties still needs investigation. One of the main results on this question being for instance Coron's result [12] relying on time-varying state feedback laws. Here, we present a nice academic-looking problem, which is a simple but nontrivial example of these issues, solved using a general state feedback law. In [13], another example of the same kind of problem is treated with similar methodology, and the general matter is discussed in [14].

The paper is organized as follows: in Section 2, we

Email address: `alain.ajami@usj.edu.lb` (Alain Ajami).

present the problem, and several of its reductions, for the purpose of output stabilization using observers, together with our output-stabilization result.

In Section 3, we present the proof of this result, which is simple but not completely elementary.

In Section 4, we show some simulation results, together with some conclusions and perspectives.

2 The problem, the reductions of the problem, the result.

Notation 1 *In the following, for any X matrix or vector, X' denotes its transpose.*

The issue at hand is to stabilize System (1) at a fixed target. But what does this mean for a drone with constant velocity? In fact, it is required here that the drone reaches a limit motion of turning around the target achieving a circle of minimal radius $r = \frac{1}{u_{\max}}$.

This leads to the following reduction of the model. We define the target (traveled counter-clockwise) $\mathcal{T}$ by:

$$\mathcal{T} = \{(x, y, \theta) \mid x = r\sin\theta, y = -r\cos\theta\}, \tag{2}$$

and we set

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \tag{3}$$

In these new UAV-based coordinates $(\tilde{x}, \tilde{y}, \theta)$, System (1) can be rewritten as:

$$\frac{d}{dt} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} u\,\tilde{y} + 1 \\ -u\,\tilde{x} \end{pmatrix}. \tag{4}$$

For a non-zero $u \in [-u_{\max}, u_{\max}]$, System (4) possesses a single equilibrium $(0, -1/u)$. In particular for $u = u_{\max}$ and $u = -u_{\max}$, they have equilibria $(0, -r)$ and $(0, r)$.

They correspond to the target $\mathcal{T}$ being browsed counter-clockwise and clockwise respectively. If u is changed for $-u$, the two equilibria are exchanged so that the set of equilibria is unchanged. It means that we can indifferently consider one among the two equilibria positions.

Changing the $(\tilde{x}, \tilde{y})$ coordinates for $(\bar{x}, \bar{y}) = (\tilde{x}, \tilde{y} + r)$, the stabilization problem to $\mathcal{T}$ is reformulated in these variables as the stabilization problem to the submanifold $\{(\bar{x}, \bar{y}, \theta) | \bar{x} = \bar{y} = 0\}$. Equivalently, we will refer to the convergence of $(\bar{x}, \bar{y})$ to the point $(0, 0)$ of the reduced state space. The reader can easily check that coordinates $(\bar{x}, \bar{y})$ obey the following equations:

$$\begin{cases} \dfrac{d\bar{x}}{dt} = u\,\bar{y} + 1 - r\,u \\ \dfrac{d\bar{y}}{dt} = -u\bar{x} \end{cases} \tag{5}$$

Now, we shall consider for the three systems (1), (4), (5), the following **"minimum information output"**, i.e. the square distance to the target:

$$\rho^2 = x^2 + y^2 = \tilde{x}^2 + \tilde{y}^2 = \bar{x}^2 + (\bar{y} - r)^2. \tag{6}$$

For $t \in [0, T]$, if $(x(t), y(t), \theta(t))$ is a trajectory of (1), corresponding to the arbitrary control $u(t)$ and the output $\rho^2 = x^2 + y^2$, it is clear that $(x(t), g(t), \theta(t) + \theta_0)$ is also a trajectory of (1) corresponding to the same control and output.

This means that system (1) is not even locally weakly observable in the sense of [15]: close to any (x, y, θ), there is a continuum of points that are indistinguishable to (x, y, θ) by the observations, whatever the input $u(\cdot)$.

On their side, both systems (4), (5) are locally weakly observable (they are just the quotient system (1) by the weak indistinguishability relation from [15]), but they are not strongly observable in the sense of [11]: for the constant control $u \equiv 0$, $\tilde{x}$ (resp. $\bar{x}$) only can be reconstructed from the observation ρ^2 while $\tilde{y}$ can be reconstructed up to sign only.

Remark 1 *Besides [15], one can also consult [16,17,18] for the general theory of quotienting through unobservability.*

At this point, another important fact appears: this system (4) (resp. (5)) can be embedded into a state-affine one: following [1], its observation space is finite-dimensional. Actually, setting $z = (z_1, z_2, z_3)$, $z_1 = x^2 + y^2 = \tilde{x}^2 + \tilde{y}^2$, $z_2 = \tilde{x}$, $z_3 = \tilde{y}$, and denoting the output by s, we get the following system:

$$\begin{cases} \dot{z} = Az + uBz + b, \\ s = Cz, \quad u \in [-u_{\max}, u_{\max}] \end{cases} \tag{7}$$

with $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$.

The first very natural idea that comes to mind when facing such an output stabilization problem is the use of a classical Kalman filter or a Kalman-like observer, as in [11] for instance, but in this situation, it seems that it is not a good idea: the system is not strongly observable, and hence it is not guaranteed that the Riccati matrix will remain invertible.

Moreover in this case, we exhibit a simpler observer that completes this purpose, and we prove the theorem that follows.

The Kalman filter has a correction term which is time dependant through the solution of the Riccati matrix equation. Here we will consider for system (7) a "Luenberger-type" observer, i.e., with constant correction term K, i.e.,

$$\frac{d\hat{z}}{dt} = A\hat{z} + uB\hat{z} + b - K(C\hat{z} - s), \tag{8}$$

where $\hat{z}$ stands for the estimate of z in (7).

A smooth stabilizing feedback at the target for (4) is a smooth map $u : \mathbb{R}^2 \to [-u_{max}, u_{max}]$ such that the vector field $(u(x,y)y+1)\frac{\partial}{\partial x} - u(x,y)x\frac{\partial}{\partial y}$ admits a globally asymptotically stable equilibrium at $(0,-r)$. Likewise, a smooth stabilizing feedback at the target for (5) is a smooth map $u : \mathbb{R}^2 \to [-u_{max}, u_{max}]$ such that the vector field $(u(x,y)y + 1 - ru(x,y))\frac{\partial}{\partial x} - u(x,y)x\frac{\partial}{\partial y}$ admits a globally asymptotically stable equilibrium at $(0,0)$.

Theorem 1 *For any smooth stabilizing feedback at the target, for systems* (4), (5), *there is a Luenberger-type observer for system* (7), *that is, a choice of* K, *such that the coupled closed loop system is asymptotically stable at the target, with an arbitrarily large basin of attraction.*

Remark 2 *(1) We were not able to prove that the coupling with a Kalman filter works. We were also unable up to now, to prove any general result of dynamic output stabilization for bilinear systems* ***that are not strongly observable*** *using Kalman-like observers. We consider that such a result would be very important.*

(2) In the paper [5], Theorem 2.2, we have exhibited a smooth stabilizing feedback control law for system (5) *that may be used in the applications of our theorem, see Section 4.*

(3) It would be interesting to analyze the behavior of the coupling of the observer with the time optimal synthesis, that has been computed in [5] (see also [3]). Note that the minimum time optimal synthesis is not smooth (it is not even continuous).

(4) See [13] for a similar example of a dynamic-output stabilization problem, where moreover the target point is not observable.

(5) It is known for long (see for instance [19]) that global feedback stabilization plus strong observability does not in general imply the possibility of global dynamic output stabilization. Here, the situation is even worse: the system (4) *is not even strongly observable. However, we obtain semi-global output stabilization.*

3 Proof of the main result

3.1 The Luenberger-type observer

As we said, our observer is a standard Luenberger-type observer for System (7), given in (8) above. In the following, we pick $K^{'} = (\alpha, 2, 0)$ for some arbitrary $\alpha > 0$.

Therefore, the coupled system feedback-observer is (for System (4)):

$$\begin{aligned} \frac{d\hat{z}}{dt} &= A\hat{z} + u(\hat{z})\ B\hat{z} + b - K(C\hat{x} - z_2^2 - z_3^2), \\ \frac{dz_2}{dt} &= z_3 u(\hat{z}) + 1, \\ \frac{dz_3}{dt} &= -z_2 u(\hat{z}). \end{aligned} \tag{9}$$

This system (9) also meets:

$$\begin{aligned} \frac{d\hat{z}}{dt} &= A\hat{z} + u(\hat{z})\ B\hat{z} + b - KC(\hat{x} - z), \\ \frac{dz}{dt} &= Az + u(\hat{z})\ Bz + b, \end{aligned} \tag{10}$$

in which the state z is not arbitrary, but living inside the (invariant) manifold $\mathcal{Z} = \{z \mid z_1 = z_2^2 + z_3^2\}$.

Nevertheless, solutions of (9) satisfy, in the error-estimate coordinates:

$$\begin{aligned} \frac{d\epsilon}{dt} &= (A + u(\hat{z})B - KC)\epsilon, \\ \frac{d\hat{z}_2}{dt} &= \hat{z}_3 u(\hat{z}) + 1 - 2C\epsilon, \\ \frac{d\hat{z}_3}{dt} &= -\hat{z}_2 u(\hat{z}), \end{aligned} \tag{11}$$

where ϵ is the estimation error, $\epsilon = \hat{z} - z$, and the function $u(\cdot)$ is the stabilizing feedback law. In particular one could use the feedback law proposed in [5].

The question, in the remaining of this paper, is the stability of this system at $P^* \in \mathbb{R}^5$, where P^* is the point of coordinates $\{\epsilon = 0, \hat{z}_2 = 0, \hat{z}_3 = -r\}$ corresponding to the control $u = u_{\max} = \frac{1}{r}$.

There are three steps to the proof of the (semi) global asymptotic stability of this coupled System (11): proof of local asymptotic stability, proof that bounded trajectories go to the target, proof that all trajectories starting in a given compact set are bounded.

3.2 Local asymptotic stability

We follow a classical scheme of proof.

- At the target point $\{\epsilon = 0, \hat{z}_2 = 0, \hat{z}_3 = -r\}$ the linearized system is lower triangular.

  Since $KC = A' + \alpha e_{11}$, the linearization relative to the ϵ-part of the system is given by $\dot{\epsilon} = (A - A' + u_{\max}B - \alpha e_{11})\epsilon$ where e_{11} denotes the 3×3 matrix with coefficient in position $(1, 1)$ set to 1, and others to 0. Then $\|\epsilon\|^2$ is a Lyapunov function for the subsystem, as
  $$\frac{1}{2}\frac{d}{dt}\|\epsilon\|^2 = -\alpha\epsilon_1^2.$$
  Since the pair $(C, (A - A' + u_{\max}B - \alpha e_{11}))$ is observable, this implies that 0 is an asymptotically stable equilibrium by Lasalle's theorem. In particular, this implies that the eigenvalues relative to the ϵ-part all have negative real part.
- Notice that the $\hat{z}$-diagonal block of the linearization of System (11) (i.e. forgetting about ϵ) coincides with the linearization of the system
  $$\begin{aligned} \frac{d\hat{z}_2}{dt} &= \hat{z}_3 u(\hat{z}) + 1, \\ \frac{d\hat{z}_3}{dt} &= -\hat{z}_2 u(\hat{z}). \end{aligned} \tag{12}$$
  Since u is a feedback law stabilizing (12) at $(0, -r)$, its linearized can only have eigenvalues of non-positive real part.

  Furthermore, the asymptotic stability of (12) implies the existence of a center manifold for (12) at $(0, -r)$ that we denote $\mathcal{C}$ (possibly empty if both eigenvalues of the linearized system have strictly negative real part).
- If $\mathcal{C}$ is nonempty, $\mathcal{C}$ is an invariant manifold inside the invariant manifold $\{\epsilon = 0\}$ for the coupled system (11). Since all other eigenvalues have strictly negative real part, $\mathcal{C}$ is then also a (stable) center manifold for the full coupled system (11).

Hence we conclude at the asymptotic stability of the system at P^*.

3.3 Bounded trajectories converge to the target

First, along a bounded trajectory $(\epsilon(t), \hat{z}(t))$ of System (11), $C\epsilon(t)$ tends to zero. Indeed

$\frac{1}{2}\frac{d}{dt}(\|\epsilon\|^2) = \epsilon' A\epsilon - \epsilon' KC\epsilon = -\alpha(\epsilon_1)^2.$

Therefore, $C\epsilon(t) = \epsilon_1(t)$ is a square summable function over $\mathbb{R}+$. Moreover, $C\epsilon(t)$ has bounded derivative since $\frac{d\epsilon_1}{dt} = -\alpha\epsilon_1 + 2\epsilon_2$ and we are considering a bounded trajectory. An $\mathbb{L}^2$ function with bounded derivative tends to zero.

Looking at the $\hat{z}$-equation in (11), we see that in the ω-limit set Ω of the trajectory $(\epsilon(t), \hat{z}_2(t), \hat{z}_3(t))$ we have

$$\begin{aligned} \frac{d\hat{z}_2}{dt} &= \hat{z}_3 u(\hat{z}) + 1, \\ \frac{d\hat{z}_3}{dt} &= -\hat{z}_2 u(\hat{z}). \end{aligned}$$

This is the equation of the feedback system, which is globally asymptotically stable by assumption. Hence, by the general fact of invariance and closure of the ω-limit set, Ω contains at least one trajectory such that $\hat{z} \equiv 0, \hat{z} \equiv r$.

Now, plugging $u(\hat{z}) = -u_{\max}$ in the equation of ϵ, we see that $C\epsilon \equiv 0$ (preserved along trajectories in Ω) can only be achieved if $\epsilon \equiv 0$ by observability of the system for $-u_{\max}$. Thus $(0, (0, r))$ belongs to Ω and, therefore, the trajectory $(\epsilon(t), \hat{z}(t))$ enters in finite time in the basin of attraction of P^*.

3.4 All semi-trajectories are bounded

As shown in Section 3.3, we have

$$\frac{1}{2}\frac{d}{dt}(\|\epsilon\|^2) = -\alpha(\epsilon_1)^2,$$

which implies that ϵ is bounded and $C\epsilon(t) = \epsilon_1(t)$ tends to zero. But, $\dot{\epsilon}_1 = -\alpha\epsilon_1 + 2\epsilon_2$. Hence

$$\epsilon_1(t) = e^{-\alpha t}\epsilon_1(0) + 2\int_0^t e^{-\alpha(t-s)}\epsilon_2(s)ds$$

and

$$\begin{aligned} |\epsilon_1(t)| &\leq e^{-\alpha t}|\epsilon_1(0)| + 2\int_0^t e^{-\alpha(t-s)}\|\epsilon(0)\|ds \\ &\leq e^{-\alpha t}|\epsilon_1(0)| + \frac{2\|\epsilon(0)\|}{\alpha}(1 - e^{-\alpha t}). \end{aligned}$$

Therefore, we have proved the following.

Lemma 2 *for $\|\epsilon(0)\| \leq \psi$ ($\psi > 0$, an arbitrary constant) $|\epsilon_1(t)|$ can be made arbitrarily small in arbitrary short time, by increasing α. That is, for all $\tau > 0$ and all $\eta > 0$ there exists $\alpha_\tau > 0$ such that a semi-trajectory $(\epsilon, \hat{z})$ of (11), with $\alpha > \alpha_\tau$, and $\|\epsilon(0)\| < \psi$, satisfies*

$$|\epsilon_1(t)| < \eta, \qquad \forall t > \tau.$$

Now, let us consider the equation of $\hat{z}$ in (11):

$$\begin{aligned} \frac{d\hat{z}_2}{dt} &= \hat{z}_3 u(\hat{z}) + 1 - 2\epsilon_1, \\ \frac{d\hat{z}_3}{dt} &= -\hat{z}_2 u(\hat{z}), \end{aligned} \tag{13}$$

Let $V(\tilde{x}_1, \tilde{x}_2)$ be a strict proper Lyapunov function for the feedback system (13) in which $\epsilon_1 \equiv 0$. Such a Lyapunov function can be obtained by applying inverse Lyapunov's theorems (see, for instance, [20,21]).

Let K be any compact subset of $\mathbb{R}^5$ (the state space of (11)) and let $\Pi : \mathbb{R}^5 \to \mathbb{R}^2$ be the projection on the two last components $(\hat{z}_2, \hat{z}_3)$ (that are the estimates of $(\tilde{x}_1, \tilde{x}_2)$ in (4)). Let $k \in \mathbb{N}$ be a large enough integer such that $\Pi(K) \subset D_k$, where we denote

$$D_\delta = \{(\tilde{x}_1, \tilde{x}_2) \mid V(\tilde{x}_1, \tilde{x}_2) \leq \delta\}.$$

the level sets of V.

Let $\psi > 0$ be such that $K \subset [-\psi, \psi]^3 \times D_k$. Notice that the vector field $(u(x, y)y + 1 - 2\epsilon_1)\dfrac{\partial}{\partial x} - u(x, y)x\dfrac{\partial}{\partial y}$ is uniformly bounded with respect to $|\epsilon_1| \leq \psi$ on D_{k+1}. We denote by $R > 0$ a uniform bound on the norm of the vector field. Then by setting

$$\tau = \frac{1}{R+1} \mathrm{dist}(D_k, D_{k+1}) > 0,$$

we have that if ζ is a semi-trajectory of (11) starting in K, then $\Pi(\zeta(t))$ remains in the interior of D_{k+1} for all $t \in [0, \tau]$, since the norm R of the velocity vector $(\dot{\tilde{x}}_1, \dot{\tilde{x}}_2)$ is small enough for that.

Note that this fact is independent on the choice of α since the bound R is uniform and ϵ_1 is decreasing.

Denoting $f(x, y) = (u(x, y)y + 1)\dfrac{\partial}{\partial x} - u(x, y)x\dfrac{\partial}{\partial y}$ and L_f the Lie-derivative with respect to the vector field f, let

$$m = \inf_{D_{k+1} \setminus D_k} |L_f V| > 0.$$

As a consequence of Lemma (2), we can choose $\alpha > 0$ such that any semi-trajectory ζ of (11) with $\zeta(0) \in K$, satisfies

$$2|\epsilon_1(t)| \sup_{D_{k+1} \setminus D_k} \left| \frac{\partial V}{\partial \tilde{x}_1} \right| < m, \qquad \forall t > \tau.$$

This implies that if $\Pi(\zeta(t)) \in D_{k+1} \setminus D_k$ at $t > \tau$, then $\frac{d}{dt} V(\hat{z}_2, \hat{z}_3) < 0$

However, if there exists $t > 0$ such that $\Pi(\zeta(t)) \notin D_{k+1}$, this implies the existence of a time $t' \in (\tau, t)$ such that $\Pi(\zeta(t')) \in D_{k+1}$ and

$$\frac{d}{dt} V(\hat{z}_2, \hat{z}_3)_{|t=t'} > 0,$$

which we excluded. Therefore, $(\hat{z}_2, \hat{z}_3)$ remains in D_{k+1} for ever. We already know that $\epsilon(t)$ is bounded. Hence, the full trajectory is bounded.

This ends the proof of Theorem (1).

4 Results, conclusion and perspectives

We show two simulations in which we used the smooth stabilizing feedback control law from [5], Theorem 2.2, relative to System (4). The first simulation is with large α ($\alpha = 30$), see Figure 1, the second with $\alpha = 0.5$, see Figure 2. Both were taken with initial conditions $(x_0, y_0, \theta_0) = (3, 5, \pi/4)$ and $\hat{z}_0 = (-7, 5, 4)$.

On the left side of the figures are represented the trajectory of the drone (1) in the (x, y)-plane. On the right side are represented the trajectories of the reduced system (4) together with the corresponding observer state estimate.

Finally, as an illustration of our method, Figure 3 shows a motion of the drone with a slowly moving target: at each time instant, we apply the strategy assuming a fixed target.

Remark 3 *One can see on Figures 2-3 that the value of the control u crosses the strong inobservability value u=0. Actually, u is the curvature of the plane curve (x, y), that clearly changes sign.*

An interesting question is the coupling with the minimum time optimal synthesis. This is the purpose of a forthcoming paper.

To finish, we would like to repeat that we consider as a challenge the question of practical output stabilization of control systems, in the unobservable case. Here we have treated a case where the target point is an observable point, but we refer to [13] for a case where the target control makes the system unobservable. It is particularly challenging to consider, for unobservable bilinear or bilinearizable systems, the coupling of a stabilizing feedback law with a Kalman-type observer. Up to now, we were not able to derive any (global or semi-global) result. Although, a local result is easy to prove provided that the target point is observable.

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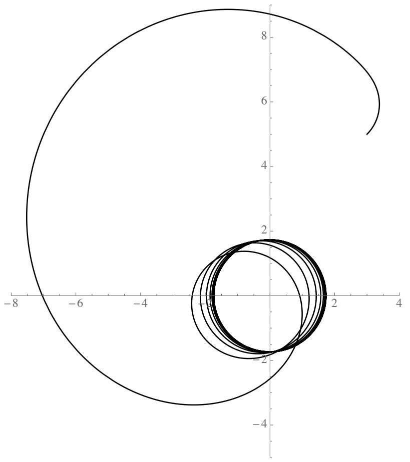
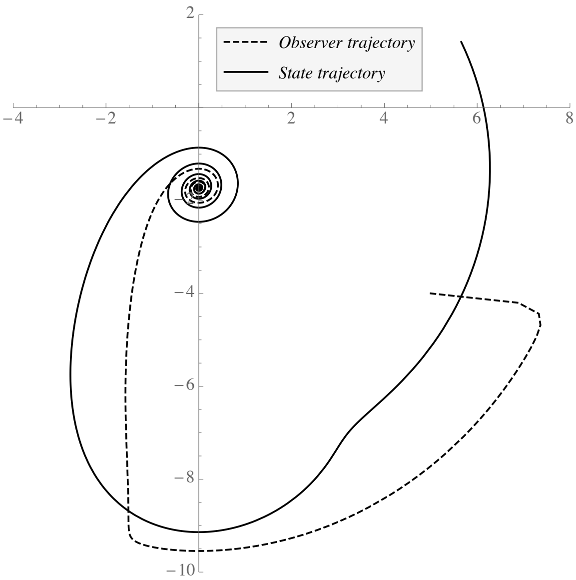
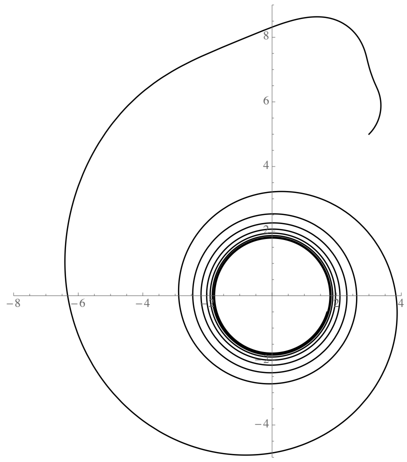


(a) Original coordinates

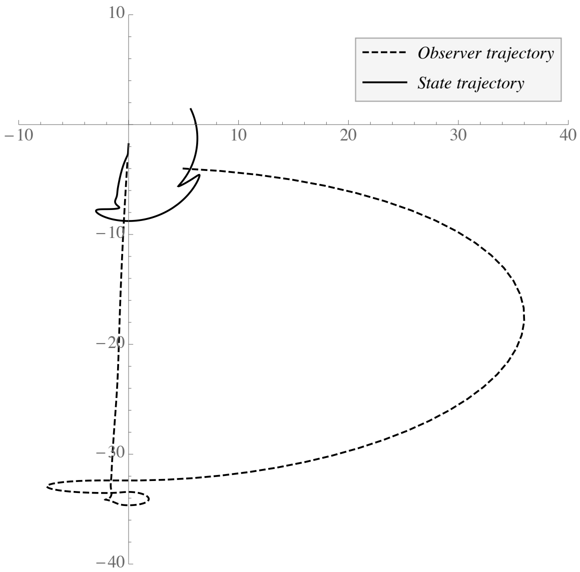


(b) Moving frame coordinates

Fig. 1. Trajectory of the drone with $\alpha = 30$.



(a) Original coordinates



(b) Moving frame coordinates

Fig. 2. Trajectory of the drone with $\alpha = 0.5$.

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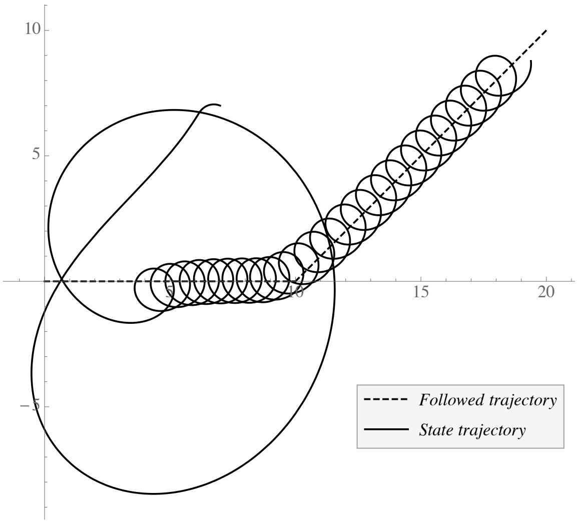

Fig. 3. Trajectory with a slowly moving target

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