DYNAMIC OUTPUT STABILIZATION OF HALE DRONES

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Abstract. In this paper, we adress the problem of stabilization of a HALE drone (high altitude, low endurance) around a given fixed target, when the "primary information" only is measured, i.e. the distance of the drone to the target, and no information about the orientation of the drone is available.

It turns out that this question leads to a very interesting small problem in geometric control theory. The problem is particularly interesting due to very bad observability properties.

1. Introduction

A HALE drone (High altitude, low endurance) is a drone moving in general at a fixed altitude, with minimum speed (that may be considered to be constant as a first approximation but this is of no importance in the present work). Typical use of such drones is within the domain of military oversight or tracking of fixed or moving targets. Typical constants for such a HALE drone are velocity of 50 miles per hour, altitude 3 miles.

A very rough but in general sufficient for practical applications is the following simple kinematic model:

$$\dot{x} = \cos(\theta),\ \dot{y} = \sin(\theta), \dot{\theta} = u, \qquad -u_{\max} \leq u \leq u_{\max}, \tag{1.1}$$

which expresses that the drone moves on the (x, y) plane, with constant velocity 1, with maximum possible curvature radius r of trajectories being $r = \frac{1}{u_{\max}}$.

This model is inspired from the Dubins model [8]. It has been discussed in a lot of papers, and we just give the elements of bibliography that are absolutely necessary in the present work.

In a series of previous works [1, 2, 3, 4], we discussed some theoretical and practical facts about the control of these drones, in the perspective of autonomy (and maybe self decision) of the drones regarding military missions. Nowadays, the situation is quite rough: drones are just driven from the headquarter by pilots, according to a prescribed flight plan. The main issue, for long flights in military missions is of course fuel consumption. It is why minimum time strategy is in general considered.

It could be thought that such minimum time strategy is quite obvious, and from practical point of view it is, more or less, but it is extremely nontrivial from academic point of view. These issues are discussed in our paper [1]. See also [2, 8, 13].

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Here, we address the problem of dynamic output stabilization (to the target) of the drone, considering minimum information about the relative position of the target w.r.t the drone. This problem is of interest in case of failure of instruments, but also in other circumstances: for instance, in case of nuclear attack, where for psychological reasons, the pilot at the headquarter must know as less as possible about the target.

Although the problem of dynamic output stabilization has been studied a lot, one of the main results being for instance Coron's result [6], we present here a very nice academic-looking problem, which is a nontrivial simple example of output stabilization using a state feedback law coupled with a nonlinear observer. See (among a huge number of authors) [7] for a general discussion of this problem.

The paper is organized as follows: in the next section 2, we present the problem, and several of its reductions, for the purpose of output stabilization using observers, together with our output-stabilization result.

In Section 3, we present the proof of this result, which is simple but not completely elementary.

In Section 4, we show some simulation results, together with some conclusions and perspectives.

2. The problem, the reductions of the problem, the result.

The problem is to stabilize System 1.1 at the origin, but what does this mean, since the drone has constant (or at least minimum) velocity? In fact, in the case of a fixed target, it is required that at the end of the mission the pilot turns around the target achieving a circle of minimal radius $r = \frac{1}{u_{\max}}$, waiting for the order to drop his charge.

This leads to the following reduction of the model. We define the target (travelled counter-clockwise) $\mathcal{T}$ by:

$$\mathcal{T} = \{(x, y, \theta) \mid x = r\sin\theta, y = -r\cos\theta\}, \tag{2.1}$$

and we set

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \tag{2.2}$$

In these new UAV-based coordinates $(\tilde{x}, \tilde{y}, \theta)$, System (1.1) can be rewritten as:

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} u\,\tilde{y} + 1 \\ -u\,\tilde{x} \end{pmatrix}. \tag{2.3}$$

For a non-zero $u \in [-u_{\max}, u_{\max}]$, System (2.3) possesses a single equilibrium $(0, -1/u)$. In particular for $u = u_{\max}$ and $u = -u_{\max}$, we denote $e_1 = (0, -r)$, $e_2 = (0, r)$ their respective equilibria.

They correspond to the target $\mathcal{T}$ being browsed counter-clockwise and clockwise respectively. If u is changed for $-u$, the two equilibria are exchanged so that the set of equilibria is unchanged. It means that we can indifferently consider one among the two equilibria positions.

Changing the $(\tilde{x},\tilde{y})$ coordinates for $(\bar{x},\bar{y})=(\tilde{x},\tilde{y}+r)$, the stabilization problem to $\mathcal{T}$ is reformulated in these variables as the stabilization problem to the submanifold $\{(\bar{x},\bar{y},\theta)|\bar{x}=\bar{y}=0\}$. Equivalently, we will refer to the convergence of $(\bar{x},\bar{y})$ to the point $(0,0)$ of the reduced state space. The reader can easily check that coordinates $(\bar{x},\bar{y})$ obey the following equations:

$$\begin{cases} \dfrac{\mathrm{d}\bar{x}}{\mathrm{d}t} = u\,\bar{y} + 1 - r\,u \\ \dfrac{\mathrm{d}\bar{y}}{\mathrm{d}t} = -u\,\bar{x} \end{cases} \tag{2.4}$$

Now, we shall consider for the three systems (1.1), (2.3), (2.4), the following **"minimum information output"**, i.e. the square distance to the target:

$$\rho^2 = x^2 + y^2 = \tilde{x}^2 + \tilde{y}^2 = \bar{x}^2 + (\bar{y}-r)^2. \tag{2.5}$$

With this minimum information, it is clear that System (1.1) is not even weakly observable in the sense of [9]: changing θ for $\theta+\theta_0$ leaves both the system and the observation invariant.

On their side, both systems (2.3), (2.4) are weakly observable (they are just the quotient system (1.1) by the "weak indistinguishability" relation from [9]), but they are not strongly observable in the sense of [7]: for the constant control $u \equiv 0$, $\tilde{x}$ (resp. $\bar{x}$) only can be reconstructed from the observation ρ^2: $\tilde{y}$ can be reconstructed up to sign only.

Remark 1. *Besides* [9]*, one can also consult* [15, 16, 17] *for the general theory of quotienting through unobservability.*

At this point, another important fact appears: this system (2.3) (resp. (2.4)) can be embedded into a state-affine one: following [10], its "observation space" is finite-dimensional. Actually, setting $z=(z_1,z_2,z_3)$, $z_1 = x^2+y^2 = \tilde{x}^2+\tilde{y}^2$, $z_2=\tilde{x}$, $z_3=\tilde{y}$, and denoting the output by s, we get the following system:

$$\begin{cases} \dot{z} = Az + uBz + b, \\ s = Cz, \quad u \in [-u_{\max}, u_{\max}] \end{cases} \tag{2.6}$$

with $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$.

The first very natural idea that comes to mind when facing such an output stabilization problem is the use of a classical Kalman filter or a Kalman-like observer, as in [7] for instance, but in this situation, it is seems that it is not a good idea: the system is not strongly observable, and hence it is not guaranteed that the Riccati matrix will remain invertible.

Moreover in this case, we exhibit a simpler Luenberger-type observer that does the job, and we prove the theorem that follows.

A smooth stabilizing feedback at the target for (2.3) is a smooth map $u : \mathbb{R}^2 \to [-u_{\max}, u_{\max}]$ such that the vector field

$$(u(x,y)y+1)\frac{\partial}{\partial x} - u(x,y)x\frac{\partial}{\partial y}$$

admits a globally asymptotic equilibrium at $(0,-r)$. Likewise, a smooth stabilizing feedback at the target for (2.4) is a smooth map $u : \mathbb{R}^2 \to [-u_{\max}, u_{\max}]$ such that the vector field

$$(u(x,y)y+1-ru(x,y))\frac{\partial}{\partial x} - u(x,y)x\frac{\partial}{\partial y}$$

admits a globally asymptotic equilibrium at $(0,0)$.

Theorem 1. *For any smooth stabilizing feedback at the target, for systems* (2.3), (2.4)*, there is a Luenberger-type observer for system* (2.6)*, such that the coupled closed loop system is asymptotically stable at the target, with an arbitrarily large basin of attraction.*

Remark 2. *1. We were not able to prove that the coupling with a Kalman filter works. We were also unable up to now, to prove any general result of dynamic output stabilization for bilinear systems* ***that are not strongly observable*** *using Kalman-like observers. We consider that such a result would be very important.*

2. In the paper [1]*, we have exhibited a smooth stabilizing feedback control law for system* (2.4) *that may be used in the applications of our theorem, see Section 4.*

3. It would be interesting to analyze the behavior of the coupling of the observer with the time optimal synthesis, that has been computed in [1] *(see also* [2]*). Note that the minimum time optimal synthesis is not smooth (it is not even continuous).*

4. See [3] *for a similar example of a dynamic-output stabilization problem, where moreover the target point is not observable.*

5. It is known for long (see for instance [14]*) that global feedback stabilization plus strong observability does not in general imply the possibility of global dynamic output stabilization. Here, the situation is even worse: the system* (2.3) *is not even strongly observable. However, we obtain semi-global output stabilization.*

3. Proof of the main result

3.1. **The Luenberger-type observer.** Our observer is the following standard Luenberger-type observer for System (2.6):

$$\frac{\mathrm{d}\hat{z}}{\mathrm{d}t} = A\hat{z} + u\,B\hat{z} + b - K(C\hat{z} - s) \tag{3.1}$$

with $K^{'} = (\alpha, 2, 0)$ for some arbitrary $\alpha > 0$. Here $\hat{z}$ stands for the estimate of z in (2.6).

Therefore, the coupled system feedback-observer is (for system (2.3)):

$$\begin{aligned}
\frac{\mathrm{d}\hat{z}}{\mathrm{d}t} &= A\hat{z} + u(\hat{z})\,B\hat{z} + b - K(C\hat{x} - z_2^2 - z_3^2),\\
\frac{\mathrm{d}z_2}{\mathrm{d}t} &= z_3 u(\hat{z}) + 1,\\
\frac{\mathrm{d}z_3}{\mathrm{d}t} &= -z_2 u(\hat{z}).
\end{aligned} \tag{3.2}$$

This system (3.2) also meets:

$$\begin{aligned} \frac{\mathrm{d}\hat{z}}{\mathrm{d}t} &= A\hat{z} + u(\hat{z})\ B\hat{z} + b - KC(\hat{x} - z), \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= Az + u(\hat{z})\ Bz + b, \end{aligned} \tag{3.3}$$

in which the state z not arbitrary, living inside the (invariant) manifold $\mathcal{Z} = \{z \mid z_1 = z_2^2 + z_3^2\}$.

Nevertheless, solutions of 3.2 satisfy, in the "error-estimate" coordinates:

$$\begin{aligned} \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} &= (A + u(\hat{z})B - KC)\varepsilon, \\ \frac{\mathrm{d}\hat{z}_2}{\mathrm{d}t} &= \hat{z}_3 u(\hat{z}) + 1 - 2C\varepsilon, \\ \frac{\mathrm{d}\hat{z}_3}{\mathrm{d}t} &= -\hat{z}_2 u(\hat{z}), \end{aligned} \tag{3.4}$$

where ε is the estimation error, $\varepsilon = \hat{z} - z$, and the function $u(\cdot)$ is the stabilizing feedback law. In particular one could use the feedback law proposed in [1].

The question, in the remaining of this paper, is the stability of this system at $P^* \in \mathbb{R}^5$, where P^* is the point of coordinates $\{\varepsilon = 0, \hat{z}_2 = 0, \hat{z}_3 = -r\}$ corresponding to the control $u = u_{\max} = \frac{1}{r}$.

There are three steps to the proof of the (semi) global asymptotic stability of this coupled system (3.4): proof of local asymptotic stability, proof that bounded trajectories go to the target, proof that all trajectories starting in a given compact set are bounded.

3.2. **Local asymptotic stability.** We follow a classical scheme of proof.

— At the target point $\{\varepsilon = 0, \hat{z}_2 = 0, \hat{z}_3 = -r\}$ the linearized system is lower triangular. By construction, the linearization relative to the ε-part of the system is given by

$$\dot{\varepsilon} = (A - A' + u_{\max}B - \alpha e_{11})\varepsilon,$$

where e_{11} denotes the 3×3 matrix with coefficient in position $(1,1)$ set to 1, and others to 0. Then $\|\varepsilon\|^2$ is a Lyapunov function for the sub-system, as

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\varepsilon\|^2 = -\alpha\varepsilon_1^2.$$

Since the pair $(C, (A - A' + u_{\max}B - \alpha e_{11}))$ is observable, this implies that 0 is an asymptotically stable equilibrium by Lasalle's theorem. In particular, this implies that the eigenvalues relative to the ε-part being all with negative real part.

— Notice that the diagonal block of the $\hat{z}$ part coincides with the linearization of the system

$$\begin{aligned} \frac{\mathrm{d}\hat{z}_2}{\mathrm{d}t} &= \hat{z}_3 u(\hat{z}) + 1, \\ \frac{\mathrm{d}\hat{z}_3}{\mathrm{d}t} &= -\hat{z}_2 u(\hat{z}). \end{aligned} \tag{3.5}$$

Since u is a feedback law stabilizing (3.5) at $(0, -r)$, its linearized can only have eigenvalues of non-positive real part.

Furthermore, the asymptotic stability of (3.5) implies the existence of a center manifold for (3.5) at $(0, -r)$ that we denote $\mathcal{C}$ (possibly empty if both eigenvalues of the linearized system have strictly negative real part).

— If $\mathcal{C}$ is nonempty, $\mathcal{C}$ is an invariant manifold inside the invariant manifold $\{\varepsilon = 0\}$ for the coupled system (3.4). Since all other eigenvalues have strictly negative real part, $\mathcal{C}$ is then also a (stable) center manifold for the full coupled system (3.4).

Hence we conclude at the asymptotic stability of the system at P^*.

3.3. **Bounded trajectories converge to the target.** First, along a bounded trajectory $(\varepsilon(t), \hat{z}(t))$ of System (3.4), $C\varepsilon(t)$ tends to zero. Indeed

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|\varepsilon\|^2) = \varepsilon' A\varepsilon - \varepsilon' KC\varepsilon = -\alpha(\varepsilon_1)^2.$$

Therefore, $C\varepsilon(t) = \varepsilon_1(t)$ is a square summable function over $\mathbb{R}+$. Moreover, $C\varepsilon(t)$ has bounded derivative since

$$\frac{\mathrm{d}\varepsilon_1}{\mathrm{d}t} = -\alpha\varepsilon_1 + 2\varepsilon_2$$

and we are considering a bounded trajectory. An $\mathbb{L}^2$ function with bounded derivative tends to zero.

Looking at the $\hat{z}$-equation in (3.4), we see that in the ω-limit set Ω of the trajectory $(\varepsilon(t), \hat{z}_2(t), \hat{z}_3(t))$ we have

$$\begin{aligned}\frac{\mathrm{d}\hat{z}_2}{\mathrm{d}t} &= \hat{z}_3 u(\hat{z}) + 1,\\ \frac{\mathrm{d}\hat{z}_3}{\mathrm{d}t} &= -\hat{z}_2 u(\hat{z}).\end{aligned}$$

This is the equation of the feedback system, which is globally asymptotically stable by assumption. Hence, by the general fact of invariance and closure of the ω-limit set, Ω contains at least one trajectory such that $\hat{z}_2 \equiv 0, \hat{z}_3 \equiv r$.

Now, plugging $u(\hat{z}) = -u_{\max}$ in the equation of ε, we see that $C\varepsilon \equiv 0$ (preserved along trajectories in Ω) can only be achieved if $\varepsilon \equiv 0$ by observability of the system for $-u_{\max}$. Thus $(0, (0, r))$ belongs to Ω and, therefore, the trajectory $(\varepsilon(t), \hat{z}(t))$ enters in finite time in the basin of attraction of P^*.

3.4. **All semi-trajectories are bounded.** As shown in section 3.3, we have

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|\varepsilon\|^2) = -\alpha(\varepsilon_1)^2,$$

which implies that ε is bounded and $C\varepsilon(t) = \varepsilon_1(t)$ tends to zero. But, $\dot{\varepsilon}_1 = -\alpha\varepsilon_1 + 2\varepsilon_2$. Hence

$$\varepsilon_1(t) = e^{-\alpha t}\varepsilon_1(0) + 2\int_0^t e^{-\alpha(t-s)}\varepsilon_2(s)\mathrm{d}s$$

and

$$\begin{aligned}|\varepsilon_1(t)| &\le e^{-\alpha t}|\varepsilon_1(0)| + 2\int_0^t e^{-\alpha(t-s)}\|\varepsilon(0)\|\mathrm{d}s\\ &\le e^{-\alpha t}|\varepsilon_1(0)| + \frac{2\|\varepsilon(0)\|}{\alpha}(1 - e^{-\alpha t}).\end{aligned}$$

Therefore, we have proved the following.

Lemma 1. *for $\|\varepsilon(0)\| \leq A$ ($A > 0$, an arbitrary constant) $|\varepsilon_1(t)|$ can be made arbitrarily small in arbitrary short time, by increasing α. That is, for all $\tau > 0$ and all $\eta > 0$ there exists $\alpha_\tau > 0$ such that a semi-trajectory $(\varepsilon, \hat{z})$ of (3.4), with $\alpha > \alpha_\tau$, and $\|\varepsilon(0)\| < A$, satisfies*

$$|\varepsilon_1(t)| < \eta, \qquad \forall t > \tau.$$

Now, let us consider the equation of $\hat{z}$ in (3.4):

$$\begin{aligned} \frac{\mathrm{d}\hat{z}_2}{\mathrm{d}t} &= \hat{z}_3 u(\hat{z}) + 1 - 2\varepsilon_1, \\ \frac{\mathrm{d}\hat{z}_3}{\mathrm{d}t} &= -\hat{z}_2 u(\hat{z}), \end{aligned} \tag{3.6}$$

Let $V(\tilde{x}_1, \tilde{x}_2)$ be a strict proper Lyapunov function for the feedback system (2.3). Such a Lyapunov function can be obtained by applying inverse Lyapunov's theorems (see, for instance, [11, 12]).

Let K be any compact subset of $\mathbb{R}^5$ (the state space of (3.4)) and let $\Pi : \mathbb{R}^5 \to \mathbb{R}^2$ be the projection onto the two last components $(\hat{z}_2, \hat{z}_3)$ (that are the estimates of $(\tilde{x}_1, \tilde{x}_2)$ in (2.3)). Let $k \in \mathbb{N}$ be an large enough integer such that $\Pi(K) \subset D_k$, where we denote

$$D_\delta = \{(\tilde{x}_1, \tilde{x}_2) \mid V(\tilde{x}_1, \tilde{x}_2) \leq \delta\}.$$

the level sets of V.

Let $A > 0$ be such that $K \subset [-A, A]^3 \times D_k$. Notice that the vector field $(u(x,y)y + 1 - 2\varepsilon_1)\frac{\partial}{\partial x} - u(x,y)x\frac{\partial}{\partial y}$ is uniformly bounded with respect to $|\varepsilon_1| \leq A$ on D_{k+1}. We denote by $R > 0$ a uniform bound on the norm of the vector field. Then by setting

$$\tau = \frac{1}{R+1}\mathrm{dist}(D_k, D_{k+1}) > 0,$$

we have that if ζ is a semi-trajectory of (3.4) starting in K, then $\Pi(\zeta(t))$ remains in the interior of D_{k+1} for all $t \in [0, \tau]$.

Note that this fact is independent on the choice of α since the bound R is uniform and ε_1 is decreasing.

Denoting $f(x,y) = (u(x,y)y + 1)\frac{\partial}{\partial x} - u(x,y)x\frac{\partial}{\partial y}$ and L_f the Lie-derivative with respect to the vector field f, let

$$m = \inf_{D_{k+1} \setminus D_k} |L_f V| > 0.$$

As a consequence of Lemma 1, we can choose $\alpha > 0$ such that any semi-trajectory ζ of (3.4) with $\zeta(0) \in K$, satisfies

$$2|\varepsilon_1(t)| \sup_{D_{k+1} \setminus D_k} \left| \frac{\partial V}{\partial \tilde{x}_1} \right| < m, \qquad \forall t > \tau.$$

This implies that if $\Pi(\zeta(t)) \in D_{k+1} \setminus D_k$ at $t > \tau$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\hat{z}_2, \hat{z}_3) < 0.$$

However, if there exists $t > 0$ such that $\Pi(\zeta(t)) \notin D_{k+1}$, this implies the existence of a time $t' \in (\tau, t)$ such that $\Pi(\zeta(t')) \in D_{k+1}$ and

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\hat{z}_2, \hat{z}_3)_{|t=t'} > 0,$$

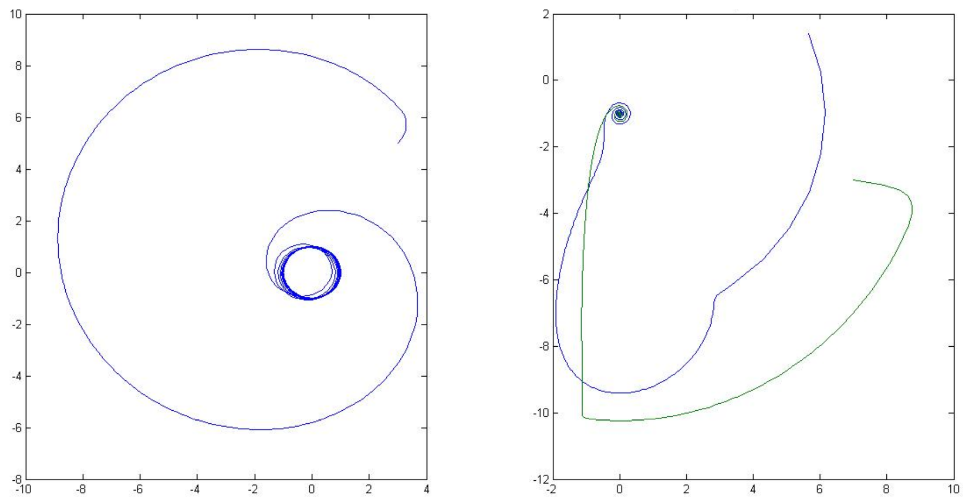

FIGURE 1. Trajectory of the drone with $\alpha = 30$.

which we excluded. Therefore, $(\hat{z}_2, \hat{z}_3)$ remains in D_{k+1} for ever. We already know that $\varepsilon(t)$ is bounded. Hence, the full trajectory is bounded.

This ends the proof of Theorem 1.

4. Results, conclusion and perspectives

We show two simulations in which we used the smooth stabilizing feedback control law from [1], relative to System (2.3). The first simulation is with large α ($\alpha = 30$), see Figure 1, the second with $\alpha = 0.1$, see Figure 2.

On the left side of the figures, the evolution of the drone (1.1) on the plane is represented, while on the left side, one can see the trajectory of the reduced system (2.3), together with the evolution of the corresponding state estimate with our observer.

The figure 3 shows a motion of the drone with a slowly moving target.

An interesting question is the coupling with the minimum time optimal synthesis. This is the purpose of a forthcoming paper.

To finish, we would like to repeat that we consider as a challenge the question of practical output stabilization of control systems, in the unobservable case. Here we have treated a case where the target point is an observable point, but we refer to [3] for a case where the target control makes the system unobservable. It is particularly challenging to consider, for unobservable bilinear or bilinearizable systems, the coupling of a stabilizing feedback law with a Kalman-type observer. Up to now, we were not able to derive any (global or semi-global) result. Although, a local result is easy to prove provided that the target point is observable.

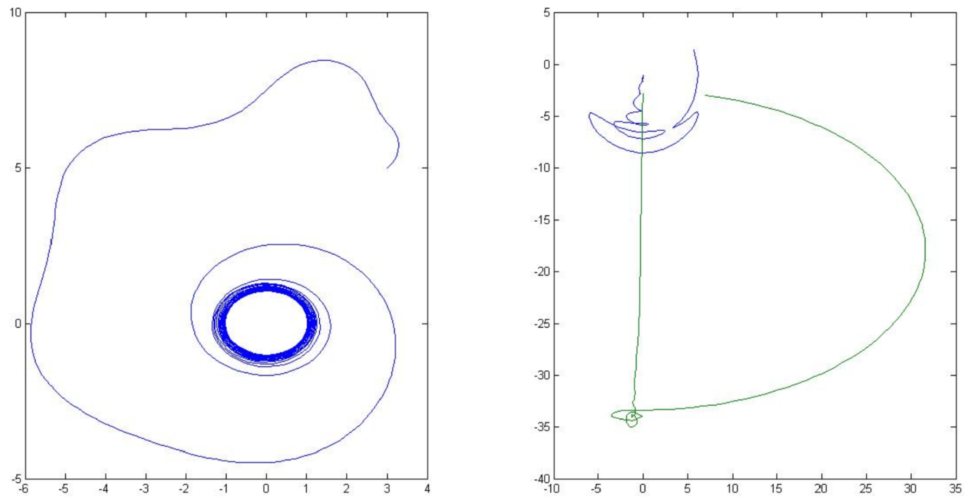

Figure 2. Trajectory of the drone with $\alpha = 0.1$.

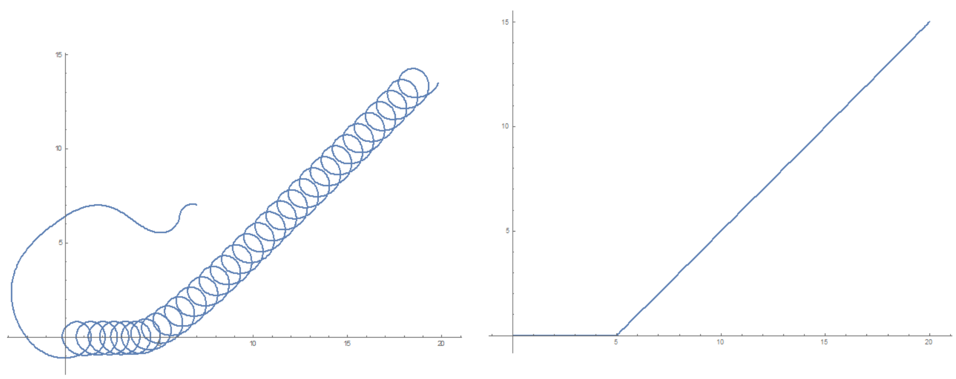

Figure 3. Trajectory with a slowly moving target

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