Applied Mathematics

Q.1. Express the following matrices as a sum of Hermitian and skew-Hermission matrices

$$\begin{bmatrix} 2i & 1-i & 2+3i \\ 4 & 5+i & 3-i \\ 1+2i & 2-i & 4 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2i & 1-i & 2+3i \\ 4 & 5+i & 3-i \\ 1+2i & 2-i & 4 \end{bmatrix}$ $\quad \bar{A} = \begin{bmatrix} -2i & 1+i & 2-3i \\ 4 & 5-i & 3+i \\ 1-2i & 2+i & 4 \end{bmatrix}$

$$A^\theta = (\bar{A})^T = \begin{bmatrix} -2i & 4 & 1-2i \\ 1+i & 5-i & 2+i \\ 2-3i & 3+i & 4 \end{bmatrix}$$

$$A+A^\theta = \begin{bmatrix} 2i & 1-i & 2+3i \\ 4 & 5+i & 3-i \\ 1+2i & 2-i & 4 \end{bmatrix} + \begin{bmatrix} -2i & 4 & 1-2i \\ 1+i & 5-i & 2+i \\ 2-3i & 3+i & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5-i & 3+i \\ 5+i & 10 & 5 \\ 3-i & 5 & 8 \end{bmatrix}$$

$$A-A^\theta = \begin{bmatrix} 2i & 1-i & 2+3i \\ 4 & 5+i & 3-i \\ 1+2i & 2-i & 4 \end{bmatrix} - \begin{bmatrix} -2i & 4 & 1-2i \\ 1+i & 5-i & 2+i \\ 2-3i & 3+i & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4i & -3-i & 1+5i \\ -3-i & 2i & 1-2i \\ -1+5i & -1-2i & 0 \end{bmatrix}$$

$$A = \tfrac{1}{2}(A+A^\theta) + \tfrac{1}{2}(A-A^\theta)$$

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Q.2. Express the following matrices as a sum of Symmetric & skew-symmetric matrices

$$\begin{bmatrix} -2 & 1 & 0 \\ 5 & 3 & 6 \\ -1 & 0 & 4 \end{bmatrix}$$

Let $A = \begin{bmatrix} -2 & 1 & 0 \\ 5 & 3 & 6 \\ -1 & 0 & 4 \end{bmatrix}$ $\quad \therefore A^T = \begin{bmatrix} -2 & 5 & -1 \\ 1 & 3 & 0 \\ 0 & 6 & 4 \end{bmatrix}$

$$A+A^T = \begin{bmatrix} -2 & 1 & 0 \\ 5 & 3 & 6 \\ -1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 5 & -1 \\ 1 & 3 & 0 \\ 0 & 6 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 & -1 \\ 6 & 6 & 6 \\ 1 & 6 & 8 \end{bmatrix}$$

$$A-A^T = \begin{bmatrix} -2 & 1 & 0 \\ 5 & 3 & 6 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 5 & -1 \\ 1 & 3 & 0 \\ 0 & 6 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -1 \\ 4 & 0 & 6 \\ -1 & 6 & 0 \end{bmatrix}$$

$$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$$

$$A = \frac{1}{2}\begin{bmatrix} -4 & 6 & -1 \\ 6 & 6 & 6 \\ 1 & 6 & 8 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 4 & -1 \\ 4 & 0 & 6 \\ -1 & 6 & 0 \end{bmatrix}$$

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Q.3

$x+y+3z=0$

$x-y+z=0$

$x-y+2z=0$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 2 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Rank $[A:B] = 3$ Rank $[A] = 3$, $n = 3$

Rank $[A:B]$ = Rank $[A] = n$

the equation is consistent and has trivial solution

$x+y+3z = 0$ — (I)

$-2y - 2z = 0$ — (II)

$z = 0$ — (III)

Put z in II: $-2y = 0$, $y = 0$; Put z & y in I: $x+0+0=0$, $x = 0$

$x = y = z = 0$

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3) $4x - 2y + 6z = 8$, $x + y - 3z = -1$ & $15x - 3y + 9z = 21$

$$[A:B] = \left[\begin{array}{ccc|c} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 4 & -2 & 6 & 8 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

$R_2 - 4R_1$ & $R_3 - 15R_1$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -6 & 18 & 12 \\ 0 & -18 & 54 & 36 \end{array}\right]$$

$R_2\left(\frac{1}{6}\right)$ & $R_3\left(\frac{1}{18}\right)$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -1 & 3 & 2 \\ 0 & -1 & 3 & 2 \end{array}\right]$$

$R_3 = R_3 - R_2$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ — (1)}$$

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Rank $(A) = 2$; Rank $[A:B] = 2$ & $n = 3$
Rank $(A) =$ Rank $[A:B] < n$

system of eqn are consistent & it was infinite solution.

from (i) $x + y - 3z = -1$ & $-y + 3z = 2$
Put $z = t$, $t \in R$

$$-y + 3t = 2$$
$$y = 3t = -2$$

Now, $x + y - 3z = -1$
$$x + 3t - 2 - 3t = -1$$
$$\boxed{x = 1}$$

$x = 1$, $y = 3t - 2$, $z = t$
where $t \in R$

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$$\begin{bmatrix} 8 & 0 \\ 1 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & 0 \\ 1 & 9 \end{bmatrix}$$

$$(A - I\lambda) = \begin{bmatrix} 8 & 0 \\ 1 & 9 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{vmatrix} 8-\lambda & 0 \\ 1 & 9-\lambda \end{vmatrix} = (8-\lambda)(9-\lambda) \cdot 0 = 0$$
$$= 72 - 8\lambda - 9\lambda - \lambda^2 = 0$$
$$= \lambda^2 - 17\lambda - 72 = 0$$

$$A^2 - 17A - 72I = 0$$

$$A^2 = A \cdot A = \begin{bmatrix} 8 & 0 \\ 1 & 9 \end{bmatrix} \cdot \begin{bmatrix} 8 & 0 \\ 1 & 9 \end{bmatrix} = \begin{bmatrix} 64 & 0 \\ 17 & 81 \end{bmatrix}$$

$$17A = 17 \begin{bmatrix} 8 & 0 \\ 1 & 9 \end{bmatrix} = \begin{bmatrix} 136 & 0 \\ 17 & 153 \end{bmatrix}$$

$$72I = \begin{bmatrix} 72 & 0 \\ 0 & 72 \end{bmatrix}$$

To check $A^2 - 17A - 72I$

$$\begin{bmatrix} 64 & 0 \\ 17 & 81 \end{bmatrix} - \begin{bmatrix} 136 & 0 \\ 17 & 153 \end{bmatrix} - \begin{bmatrix} 72 & 0 \\ 0 & 72 \end{bmatrix} = 0$$

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find the eigen values & eigen vector of the following matrices.

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}$$

for char eqn $(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(3-\lambda) - 8 = 0$$
$$3 - \lambda - 3\lambda + \lambda^2 - 8 = 0$$
$$\lambda^2 - 4\lambda - 5 = 0$$
$$\lambda = 5 \text{ or } \lambda = -1$$

eigen values are 5 and -1

for $\lambda = 5$
Suppose $(A - \lambda I)x = 0$

$$\begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$$-4x + 2y = 0$$
$$2y = 4x$$
$$y = \frac{4}{2}x \quad \therefore y = 2x$$

Put $x = t$ $\therefore y = 2t$

$$x = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ 2t \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Eigen vector for $\lambda = 5$ is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\lambda = -1$
Suppose $(A - \lambda I)x = 0$

$$\begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x + 2y = 0$$
$$2y = -2x$$
$$y = -x$$
$$x = t \quad \therefore y = -t$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ -t \end{bmatrix} = t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

eigen vector for $\lambda = -1$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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Simplify i) $\frac{(\cos 5\theta - i\sin 5\theta)^2 \cdot (\cos 7\theta - i\sin 7\theta)^3}{(\cos 4\theta - i\sin 4\theta)^9 \cdot (\cos\theta + i\sin\theta)^5}$

$$\Rightarrow \frac{(\cos 5\theta - i\sin 5\theta)^2 \cdot (\cos 7\theta - i\sin 7\theta)^3}{(\cos 4\theta - i\sin 4\theta)^9 \cdot (\cos\theta + i\sin\theta)^5}$$

$$= \frac{(\cos\theta + i\sin\theta)^{-5\times 2} \cdot (\cos\theta + i\sin\theta)^{-7\times 3}}{(\cos\theta + i\sin\theta)^{-4\times 9} \cdot (\cos\theta + i\sin\theta)^5}$$

$$= \frac{(\cos\theta + i\sin\theta)^{-10} \cdot (\cos\theta + i\sin\theta)^{-21}}{(\cos\theta + i\sin\theta)^{-36} \cdot (\cos\theta + i\sin\theta)^5}$$

$$= (\cos\theta + i\sin\theta)^{-10-21+36+5}$$

$$= (\cos\theta + i\sin\theta)^0$$

$$= 1$$

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Q Evaluate $\left(\frac{1}{\sqrt2} + i\frac{1}{\sqrt2}\right)^{10} + \left(\frac{1}{\sqrt2} - i\frac{1}{\sqrt2}\right)^{10}$

$\Rightarrow$ let $z = \frac{1}{\sqrt2} + i\frac{1}{\sqrt2}$

$a = \frac{1}{\sqrt2}$, $b = \frac{1}{\sqrt2}$

$r = \sqrt{a^2+b^2} = \sqrt{1/2 + 1/2} = \sqrt{1} = 1$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$ [$\because$ z lies in first quadrant]

$\tan^{-1}\left(\frac{1/\sqrt2}{1/\sqrt2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

$z = r(\cos\theta + i\sin\theta)$

$\left(\frac{1}{\sqrt2} + i\frac{1}{\sqrt2}\right)^{10} = 1.(\cos\theta + i\sin\theta)$

$\left(\frac{1}{\sqrt2} + i\frac{1}{\sqrt2}\right)^{10} = \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{10}$

$\left(\frac{1}{\sqrt2} + i\frac{1}{\sqrt2}\right)^{10} = \left(\cos 5\frac{\pi}{2} + i\sin 5\frac{\pi}{2}\right)^{10}$

$\left(\frac{1}{\sqrt2} + i\frac{1}{\sqrt2}\right)^{10} = 0 + i(1)$ —— (1)

$= i$

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Taking Conjugate

$\left(\frac{1}{\sqrt2} - i\frac{1}{\sqrt2}\right)^{10} = -i$ —— (2)

Adding (1) and (2)

$\left(\frac{1}{\sqrt2} + i\frac{1}{\sqrt2}\right)^{10} + \left(\frac{1}{\sqrt2} - i\frac{1}{\sqrt2}\right)^{10} = 0$

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i) $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

ii) $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\sin^3\theta\cos\theta$

Consider,

$(\cos\theta + i\sin\theta)^4 = (\cos\theta)^4.(i\sin\theta)^0 + 4\cos^3\theta.(i\sin\theta) + 6\cos^2\theta.(i\sin\theta)^2 + 4\cos\theta.(i\sin\theta)^3 + (\cos\theta)^0.(i\sin\theta)^4$

$= \cos^4\theta + 4\cos^3\theta.i\sin\theta - 6\cos^2\theta.\sin^2\theta - 4\cos\theta.i\sin^3\theta + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta.\sin^2\theta + \sin^4\theta + 4\cos^3\theta.i\sin\theta - 4\cos\theta.i\sin^3\theta$

$(\cos 4\theta + i\sin 4\theta) = (\cos^4\theta - 6\cos^2\theta.\sin^2\theta + \sin^4\theta) + i(4\cos^3\sin\theta - 4\cos\theta.\sin^3\theta)$

$\cos 4\theta = \cos^3\theta - 6\cos^2\theta.\sin^2\theta + \sin^4\theta$

$\sin 4\theta = 4\cos^3\sin\theta - 4\cos\theta.\sin^3\theta$

$\cos 4\theta = \cos^4\theta - 6\cos^2\theta.\sin^2\theta + \sin^2\theta$

& $\sin 4\theta = 4\cos^3\theta.\sin\theta - 4\sin^3.\cos\theta$

Hence Proved.

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If $5\sinh x - \cosh x = 5$ find $\tanh x$

$\Rightarrow$ $5\sinh x - \cosh x = 5$

$$5\left(\frac{e^{x}-e^{-x}}{2}\right) - \left(\frac{e^{x}+e^{-x}}{2}\right) = 5$$

$$5e^{x} - 5e^{-x} - e^{x} - e^{-x} = 10$$

$$4e^{x} - 6e^{-x} - 10 = 0$$

$$2e^{x} - 3e^{-x} - 5 = 0$$

Put $e^{x} = t$ $\therefore e^{-x} = 1/t$

$$2t - \frac{3}{t} - 5 = 0$$

$$2t^2 - 5t - 3 = 0$$

$$t = 3 \text{ or } t = -\frac{1}{2}$$

$e^{x} = 3$ $\therefore e^{-x} = -\frac{1}{2}$

$e^{-x} = -\frac{1}{2}$ is rejected $[e^{x} > 0$ for all $x]$

$e^{x} = 3$, $\therefore e^{-x} = \frac{1}{3}$

$$\tanh x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} = \frac{3 - 1/3}{3 + 1/3} = \frac{8/3}{10/3} = \frac{4}{5}$$

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Solve the equation $7\cosh x + 8\sinh x = 1$ for real values of x.

$$7\cosh x + 8\sinh x = 1$$

$$7\left(\frac{e^{x}+e^{-x}}{2}\right) + 8\left(\frac{e^{x}-e^{-x}}{2}\right) = 1$$

$$\frac{7e^{x} + 7e^{-x} + 8e^{x} - 8e^{-x}}{2} = 1$$

$$\frac{7e^{x} + 8e^{x} + 7e^{-x} - 8e^{-x}}{2} = 1$$

$$15e^{x} - e^{-x} = 2$$

Put $e^{x} = t$ $\therefore e^{-x} = 1/t$

$$15t - \frac{1}{t} = 2$$

$$15t^2 - 1 = 2t$$

$$15t^2 - 2t - 1 = 0$$

$$t = \frac{1}{3} \text{ or } t = -\frac{1}{5}$$

Resubstitute

$e^{x} = \frac{1}{3}$ or $e^{-x} = -\frac{1}{5}$

$e^{-x} = -\frac{1}{5}$ is rejected $[e^{x} > 0$ for all $x]$

$e^{x} = \frac{1}{3}$ $\quad x = \log\left(\frac{1}{3}\right)$

$x = \log 1 - \log 3$

$x = 0 - \log 3$

$x = -\log 3$

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Find the sixth roots of unity

$\Rightarrow$ $1^{1/6}$

let $z = 1 + 0i$

$a = 1$, $b = 0$

$r = \sqrt{a^2+b^2} = \sqrt{1^2+0^2} = 1$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = \tan^{-1}(0) = 0$

$\theta = 0$

$z = r(\cos\theta + i\sin\theta)$

$1^{1/6} = (\cos(0) + i\sin(0))^{1/6}$

$$1^{1/6} = \cos\left(\frac{0+2k\pi}{6}\right) + i\sin\left(\frac{0+2k\pi}{6}\right)$$

$$1^{1/6} = \cos\left(\frac{k\pi}{3}\right) + i\sin\left(\frac{k\pi}{6}\right)$$

$k = 0, 1, 2, \ldots n.$

To find 6th root.

put $k=0$ 1st root $= \cos(0) + i\sin(0) = 1+0$

put $k=1$ 2nd root $= \cos(\pi/3) + i\sin(\pi/3) = \frac{1}{2} + \frac{\sqrt{3}}{2}$

put $k=2$ 3rd root $= \cos(2\pi/3) + i\sin(2\pi/3) = -\frac{1}{2} - \frac{\sqrt{3}}{2}$

put $k=3$ 4th root $= \cos(\pi) + i\sin(\pi) = -1+0$

put $k=4$ 5th root $= \cos(4\pi/3) + i\sin(4\pi/3) = -\frac{1}{2} - \frac{\sqrt{3}}{2}$

Put $k=5$ 6th root $= \cos(0\pi) + i\sin(0\pi) = 1+0.$

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Unit II. Q.B

1. $3e^{x}\tan y\,dx + (1-e^{x})\sec^2 y\,dy = 0$

$3e^{x}\tan y\,dx = -(1-e^{x})\sec^2 y\,dy$

$$\frac{3e^{x}\,dx}{-(1-e^{x})} = \frac{\sec^2 y\,dy}{\tan y}$$

$$3\int \frac{e^{x}}{(e^{x}-1)}\,dx = \int \frac{\sec^2 y}{\tan y}\,dy$$

$3\log(e^{x}-1) = \log(\tan y) + \log c$

$\log(e^{x}-1)^3 = \log(\tan y \cdot c)$

$(e^{x}-1)^3 = c\cdot\tan y$

2. $\cos x\cdot\cos y\,dy - \sin x\cdot\sin y\,dx = 0$

$\therefore \cos x\cdot\cos y\,dy = \sin x\,\sin y\,dx$

$$\therefore \frac{\cos y}{\sin y}\,dy = \frac{\sin x}{\cos x}\,dx$$

$$\therefore \int\frac{\cos y}{\sin y}\,dy = -\int\frac{-\sin x}{\cos x}\,dx$$

$\therefore \log(\sin y) = -\log(\cos x) + \log c$

$\therefore \log(\sin y) + \log(\cos x) = \log c$

$\therefore \log(\sin y\cdot\cos x) = \log c$

$\therefore \sin y\cdot\cos x = c$

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Q.3 $(2x-y+1)dx + (2y-x-1)dy = 0$

$M = 2x - y + 1$ & $N = 2y - x - 1$

$\frac{\partial M}{\partial y} = -1$ & $\frac{\partial N}{\partial x} = -1$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

It is exact D.E

Solution

$\int M.dx + \int \left(\begin{matrix}\text{Terms in N}\\ \text{free from } x\end{matrix}\right) dy = c$

$\int (2x - y + 1)\, dx + \int (2y - 1)\, dy = c$

$\therefore \frac{\cancel{2}x^2}{\cancel{2}} - xy + x + \frac{\cancel{2}y^2}{\cancel{2}} - y = c$

$x^2 - xy + y^2 + x - y = c$

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Q.5. $(x^2 - 3xy + 2y^2)\, dx + (3x^2 - 2xy)dy = 0$

let $M = x^2 - 3xy + 2y^2$ & $N = 3x^2 - 2xy$

$\frac{\partial M}{\partial y} = -3x + 4y$ & $\frac{\partial N}{\partial x} = 6x - 2y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

It is ~~not~~ non-exact D.E

$I.F = \frac{1}{Mx + Ny} = \frac{1}{x^3 - 3x^2y + 2xy^2 + 3x^2y - 2xy^2}$

$= \frac{1}{x^3}$

Now exact D.E

$\frac{(x^2 - 3xy + 2y^2)}{x^3} dx + \frac{(3x^2 - 2xy)}{x^3} dy = 0$

$\left(\frac{1}{x} - \frac{3y}{x^2} + \frac{2y^2}{x^3}\right) dx + \left(\frac{3}{x} - \frac{2y}{x^2}\right) dy = 0$

Solution:- $\int M.dx + \int \left(\begin{matrix}\text{Terms in N}\\ \text{free from } x\end{matrix}\right) dy = c$

$\therefore \int \left(\frac{1}{x} - \frac{3y}{x^2} + \frac{2y^2}{x^3}\right) dx + \int 0 \cdot dy = c$

$\therefore \int \left(\frac{1}{x} - 3y\, x^{-2} + 2y^2 x^{-3}\right) dx + 0 = c$

$\therefore \log x - 3y \frac{x^{-1}}{-1} + 2y^2 \frac{x^{-2}}{-2} = c$

$\log x + \frac{3y}{x} - \frac{y^2}{x^2} = c$

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Q.7 $(D^3 - 3D^2 + 3D - 1)\, y = e^{2x} \cdot \cosh x$

Auxillary eqn $\Rightarrow D^3 - 3D^2 + 3D - 1 = 0$

$D = 1, 1, 1$

$C.F = (c_1 + c_2 x + c_3 x^2) . e^x$

let $F(D) = D^3 - 3D^2 - 1$ & $X = e^{2x} \cdot \cosh x$

$= e^{2x}\left(\frac{e^x + e^{-x}}{2}\right)$

$= \left(\frac{e^{3x} + e^x}{2}\right)$

$= \frac{1}{2}\left(e^{3x} + e^x\right)$

$P.I = \frac{1}{f(D)} \cdot X$

$= \frac{1}{D^3 - 3D^2 + 3D - 1} \cdot \frac{1}{2} \cdot (e^{3x} + e^x)$

$P.I = \frac{1}{2}\left[\frac{1}{D^3 - 3D^2 + 3D - 1} \cdot e^{3x} + \frac{1}{D^3 - 3D^2 + 3D - 1} \cdot e^x\right]$

$= \frac{1}{2}\left[\frac{1}{27 - 27 + 9 - 1} e^{3x} + x . \frac{1}{3D^2 - 6D + 3} e^x\right]$

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$= \frac{1}{2}\left[\frac{1}{8} e^{3x} + x^2 . \frac{1}{6D - 6} e^x\right]$

$= \frac{1}{2}\left[\frac{1}{8} e^{3x} + x^3 . \frac{1}{6(1)} e^x\right]$

$= \frac{1}{16} \cancel{3x}\, e^{3x} + \frac{x^3}{12} e^x$

$y = C.f + P.I$

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Q.6 $(D^2+4)y = \cos 2x + e^{-x} + 3$

A.eqⁿ : $D^2+4=0$

$D = \pm 2i$

$D = \underset{\alpha}{0} \pm \underset{\beta}{2}i$

$C.F = e^{0x}(C_1\cos 2x + C_2 \sin 2x)$

$C.F = C_1\cos 2x + C_2\sin 2x$

Let $f(D) = D^2+4$ & $X = \cos 2x + e^{-x} + 3$

$P.I = \frac{1}{f(D)} \cdot X$

$= \frac{1}{D^2+4}(\cos 2x + e^{-x} + 3)$

$= \frac{1}{D^2+4}\cos 2x + \frac{1}{D^2+4} e^{-x} + \frac{1}{D^2+4}(3)$

$PI = x \cdot \frac{1}{2D}\cos 2x + \frac{1}{(1)^2+4} e^{-x} + \frac{1}{0+4}(3)$

$= \frac{x}{2}\frac{D}{D\times D}\cos 2x + \frac{1}{5}e^{-x} + \frac{3}{4}$

$= \frac{x}{2}\frac{-\sin 2x \cdot 2}{(-4)} + \frac{1}{5}e^{-x} + \frac{3}{4}$

$= \frac{x}{4}\sin 2x + \frac{1}{5}e^{-x} + \frac{3}{4}$

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Unit 4

Q.4. $(3y+2x^3)\,dx + (3x+y-1)\,dy = 0$

$M = 3y+2x^3$, $N = 3x+y-1$

$\frac{\partial M}{\partial y} = 3$, $\frac{\partial N}{\partial x} = 3$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Solution $\int M\,dx + \int \binom{\text{n in range}}{\text{free from } x} dy = 0$

$\int 3y + 2x^3 + \int (y-1)\,dy = c$

$3yx + \frac{2x^4}{4} + \frac{y^2}{2} - y = c$

$6yx + x^4 + y^2 - 2y = 2$

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Unit 2

Q.6 $(x^4+y^4)\,dx - xy^3\,dy = 0$

$M = x^4+y^4$ $N = -xy^3\,dy$

$\frac{\partial M}{\partial y} = 4y^3$ $\frac{\partial N}{\partial y} = -y^3$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial y}$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y}}{N} = \frac{4y^3+y^3}{-xy^3} = \frac{5y^3}{-xy^3}$

$f(x) = \frac{5}{x}$

$I.f = e^{\int f(x)}$

$= e^{\int \cdot \frac{5}{x}dx}$

$= e^{-5\int \frac{1}{x}dx}$

$= e^{-5\log x}$

$= e^{\log x^{-5}}$

$x = \frac{1}{5}$

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Q.7 $(D^3 - 3D^2 + 3D - 1)y = e^{2x} \cdot \cosh x$

Auxillary eqⁿ $= D^3 - 3D^2 + 3D - 1 = 0$

$D = 1, 1, 1$

$C.F = (C_1 + C_2 x + C_3 x^2) \cdot e^x$

let $F(D) = D^3 - 3D^2 - 1$ & $X = e^{2x} \cdot \cosh x$

$= e^{2x}\left(\frac{e^x + e^{-x}}{2}\right)$

$= \left(\frac{e^{3x} + e^x}{2}\right)$

$= \frac{1}{2}(e^{3x} + e^x)$

$P.I = \frac{1}{f(D)} \cdot X$

$= \frac{1}{D^3 - 3D^2 + 3D - 1} \cdot \frac{1}{2} \cdot (e^{3x} + e^x)$

$P.I = \frac{1}{2}\left[\frac{1}{D^3 - 3D^2 + 3D - 1} \cdot e^{3x} + \frac{1}{D^3 - 3D^2 + 3D - 1} \cdot e^x\right]$

$= \frac{1}{2}\left[\frac{1}{27 - 27 + 9 - 1} e^{3x} + x \cdot \frac{1}{3D^2 - 6D + 3} e^x\right]$

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Q.7 $(D^3 - 3D^2 + 3D - 1)y = e^{2x}\cdot \cosh x$

Auxillary eqn $= D^3 - 3D^2 + 3D - 1 = 0$

$D = 1, 1, 1$

C.F $= (c_1 + c_2 x + c_3 x^2)\cdot e^x$

let $F(D) = D^3 - 3D^2 - 1$ & $X = e^{2x}\cdot \cosh x$

$$= e^{2x}\left(\frac{e^x + e^{-x}}{2}\right)$$

$$= \left(\frac{e^{3x} + e^x}{2}\right)$$

$$= \frac{1}{2}\left(e^{3x} + e^x\right)$$

$$P.I = \frac{1}{f(D)}\cdot X$$

$$= \frac{1}{D^3 - 3D^2 + 3D - 1}\cdot\frac{1}{2}\cdot\left(e^{3x} + e^x\right)$$

$$P.I = \frac{1}{2}\left[\frac{1}{D^3 - 3D^2 + 3D - 1}\cdot e^{3x} + \frac{1}{D^3 - 3D^2 + 3D - 1}\cdot e^x\right]$$

$$= \frac{1}{2}\left[\frac{1}{27 - 27 + 9 - 1}\, e^{3x} + x\cdot\frac{1}{3D^2 - 6D + 3}\, e^x\right]$$

2] $\sin^2 t$

$$L[\sin^2 t] = L\left[\frac{1 - \cos 2t}{2}\right]$$

$$= \frac{1}{2} L[1 - \cos 2t]$$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2 + 4}\right]$$

$$= \frac{1}{2}\left[\frac{s^2 + 4 - s^2}{s(s^2 + 4)}\right]$$

$$= \frac{2}{s(s^2 + 4)}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 - \cos 2\theta}{2}$$

3] $\cos^2 3t$

Unit 3

Q-3. Find the laplace of the following.

1) $\int_0^t e^{-2t}\cdot\frac{\sin t}{t}\, dt$

$$L\left[\frac{\sin t}{t}\right] = \cot^{-1}(s)$$

$$L\left[e^{-2t}\cdot \frac{\sin t}{t}\right] = \cot^{-1}(s+2)$$

$$L\left[\int_0^t e^{-2t}\cdot\frac{\sin t}{t}\cdot dt\right] = \frac{\cot^{-1}(s+2)}{s}$$

2) $e^{-5t}\cdot\int_0^t \frac{\sin 5t}{t}\cdot dt$

$$L\left[\frac{\sin 5t}{t}\right] = \int_s^\infty L[\sin 5t]\cdot ds$$

$$= \int_s^\infty \frac{5}{s^2 + 5^2}\, ds$$

$$= 5\cdot\frac{1}{5}\tan^{-1}\left(\frac{s}{5}\right)\Big|_s^\infty$$

$$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{5}\right)$$

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$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{5}\right)$$

$$= \cot^{-1}\left(\frac{s}{5}\right)$$

$$L\left[\int_0^t \frac{\sin 5t}{t}\, dt\right] = \frac{1}{s} L\left[\frac{\sin 5t}{t}\right] = \frac{1}{s}\cdot\cot^{-1}\left(\frac{s}{5}\right)$$

$$L\left[e^{-5t}\int_0^t \frac{\sin 5t}{t}\cdot dt\right] = L\left[\int_0^t \frac{\sin 5t}{t}\cdot dt\right]\Big|_{s \to (s+5)}$$

$$= \frac{\cot^{-1}(s/5)}{s}\Big|_{s \to (s+5)}$$

$$= \frac{\cot^{-1}\left(\frac{s+5}{5}\right)}{s+5}$$

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Unit 4

1) Evaluate $\int_0^1 \int_0^y \int_0^{xy} 5 \cdot dx \cdot dy \cdot dz$

$\Rightarrow \int_0^1 \int_0^y \int_0^{xy} 5 \cdot dx \cdot dy \cdot dz$

$= \int_0^1 \int_0^y \left[\int_0^{xy} 5\, dz \right] dx \cdot dy$

$= \int_0^1 \int_0^y \left[5 \cdot z \right]_0^{xy} dx \cdot dy$

$= \int_0^1 \int_0^y 5xy \cdot dx \cdot dy$

$= \int_0^1 \left[5 \cdot \frac{x^2}{2} \cdot y \right]_0^y dy$

$= \int_0^1 \left[\frac{5y^3}{2} \right] dy$

$= \frac{5}{2} \cdot \frac{y^4}{4} \Big|_0^1$

$= \frac{5}{8} [1-0]$

$= \frac{5}{8}$

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2) Evaluate $\int_0^{\log 2} \int_0^1 \int_0^1 e^{x+y+z}\, dx \cdot dy \cdot dz$

$\Rightarrow \int_0^{\log 2} \int_0^1 \int_0^1 e^{x+y+z}\, dx \cdot dy \cdot dz$

$\Rightarrow \int_0^{\log 2} e^z \cdot dz \cdot \int_0^1 e^y \cdot dy \cdot \int_0^1 e^x \cdot dx$

$= [e^z]_0^{\log 2} \cdot [e^y]_0^1 \cdot [e^x]_0^1$

$= [e^{\log 2} - e^0] \cdot [e^1 - e^0] \cdot [e^1 - e^0]$

$= [2-1] \cdot [e^1 - e^0] \cdot [e^1 - e^0]$

$= [2-1] \cdot [e-1][e-1]$

$= (e-1)^2$

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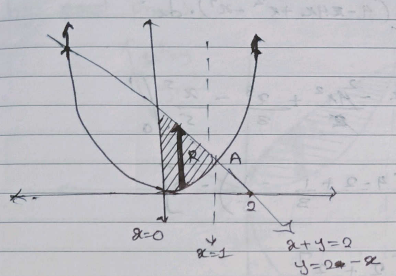
Unit 4

Q.3 Evaluate $\iint_R y \cdot dx \cdot dy$ where R in the region bounded by parabola $y = x^2$ & lines $x + y = 2$ and $x = 0$

$\rightarrow$

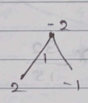


to find A,

$x + y = 2$ & $y = x^2$

$x + x^2 = 2$

$x^2 + x - 2 = 0$



$x = -2$ or $x = 1$

for A, $x = 1$

limits,

x: 0 to 1

y: x^2 to $2-x$

$\iint_R y \cdot dx \cdot dy = \int_0^1 \int_{x^2}^{2-x} y \cdot dy \cdot dx$

$= \int_0^1 \left[\frac{y^2}{2} \right]_{x^2}^{2-x} \cdot dx$

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$= \int_0^1 \left[\frac{(2-x)^2}{2} - \frac{x^4}{2} \right] dx$

$= \frac{1}{2} \int_0^1 (4 - 4x + x^2 - x^4) \cdot dx$?

$= \frac{1}{2} \left[4x - \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}} + \frac{x^3}{3} - \frac{x^5}{5} \right]_0^1$

$= \frac{1}{2} \left[\left(4 - 2 + \frac{1}{3} - \frac{1}{5} \right) - 0 \right]$

$= \frac{1}{2} \left[2 + \frac{2}{15} \right]$

$= \frac{1}{2} \times \frac{32}{15}$

$= \frac{16}{15}$

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(*) Change of order of Integral

unit 4

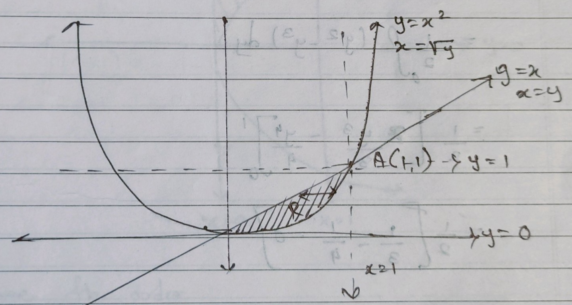
Q6 change the order of Integrational and hence evaluate.

1) $\int_0^1 \int_{x^2}^{x} xy \cdot dx \cdot dy$

$\Rightarrow$ let $I = \int_0^1 \int_{x^2}^{x} xy \cdot dy \cdot dx$

limits $x = 0$, $x = 1$, $y = x^2$, $y = x$

i.e. $x^2 = y$



To find A,

Solve, ~~y=x~~ $y = x^2$ & $y = x$

$x^2 = x$

$x = 1$

$A(1,1)$

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change the order of integration

limits

$y: 0$ to 1

$x: y$ to $\sqrt{y}$

$I = \int_0^1 \int_y^{\sqrt{y}} xy \cdot dx \cdot dy$

$= \int_0^1 y \cdot \frac{x^2}{2} \Big|_y^{\sqrt{y}} \cdot dy$

$= \frac{1}{2} \int_0^1 y \cdot \left[y - y^2 \right] \cdot dy$

$= \frac{1}{2} \int_0^1 (y^2 - y^3)\, dy$

$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^4}{4} \right]_0^1$

$= \frac{1}{2} \left[\frac{1}{3} - \frac{1}{4} - 0 \right]$

$= \frac{1}{2} \left[\frac{1}{12} \right]$

$= \frac{1}{24}$

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Q Using polar coordinate system, Evaluate the following.

1) $\int_0^1 \int_0^{\sqrt{4-y^2}} (x^2 + y^2) \cdot dx \cdot dy = I$

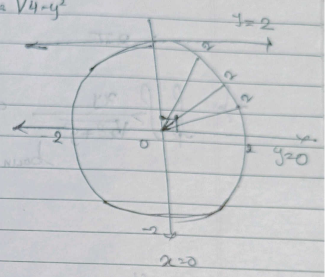
$\Rightarrow$ limits $y = 0$, $y = 2$, $x = 0$, $x = \sqrt{4-y^2}$

i.e. $x^2 = 4 - y^2$

$x^2 + y^2 = 4$

center $(0,0)$ &

radius $= 2$



Put $x = r \cdot \cos\theta$

$y = r \sin\theta$

$dx \cdot dy = r\, dr\, d\theta$

$x^2 + y^2 = r^2$

limits, $r: 0$ to 2

$\theta: 0$ to $\pi/2$

$I = \int_0^{\pi/2} \int_0^2 r^2 \cdot r \cdot dr \cdot d\theta$

$= \int_0^{\pi/2} \left[\frac{r^4}{4} \right]_0^2 \cdot d\theta$

$= \int_0^{\pi/2} (4 - 0) \cdot d\theta$

$= 4\theta \Big|_0^{\pi/2}$

$= 4 \left[\pi/2 - 0 \right]$

$= 2\pi$

2) $\int\int_R \frac{xy}{\sqrt{x^2+y^2}}\, dR$ where R in the region bounded by circles $x^2 + y^2 = 9$ &

$x^2 + y^2 = 25$

1st circle $= x^2 + y^2 = 9$

center $(0,0)$, $r = 3$

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To find A,

Solve $x^2 = 4y$ & $y^2 = 4x$

$y = \frac{x^2}{4}$ put in

$y^2 = 4x$

$\frac{x^4}{16} = 4x$

$x^3 = 64$

$x = 4$

for A, $x = 4$

limits $x : 0$ to 4

$y : \frac{x^2}{4}$ to $2\sqrt{x}$

Area of shaded region $= \iint_R 1 \cdot dR$

$$= \int_0^4 \int_{x^2/4}^{2\sqrt{x}} 1 \cdot dy \cdot dx$$

$$= \int_0^4 [y]_{x^2/4}^{2\sqrt{x}} \cdot dx$$

$$= \int_0^4 \left[2\sqrt{x} - \frac{x^2}{4}\right] dx$$

$$= \int_0^4 \left[2x^{1/2} - \frac{x^2}{4}\right] dx$$

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$$= \left[2 \cdot \frac{x^{3/2}}{3/2} - \frac{x^3}{12}\right]_0^4 \quad ?$$

$$= \left[\frac{4}{3} x^{3/2} - \frac{x^3}{12}\right]^4$$

$$= \left(\frac{4}{3} \cdot 4^{3/2} - \frac{4^3}{12}\right) - 0$$

$$= \frac{4}{3} \times 8 - \frac{\cancel{64}^{16}}{\cancel{12}_{3}}$$

$$= \frac{16}{3} \text{ sq. units}$$

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