University of Exeter

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Contents

1 Project 1

1.1 Part A

In operations, inputs are the resources required for a process to generate a certain output. Typically, inputs are independent variables whereas outputs are dependent variables. I believe "person-days" and "CPU time (hours)" should be the model's inputs, while "Profit (million pounds)" should be the model's output. Person-days and CPU time are two independent variables that measure time and Profit, in million of pounds, is proportionate to person-days and CPU time. This indicates that any variation in person-days and CPU time will have an effect on profit.

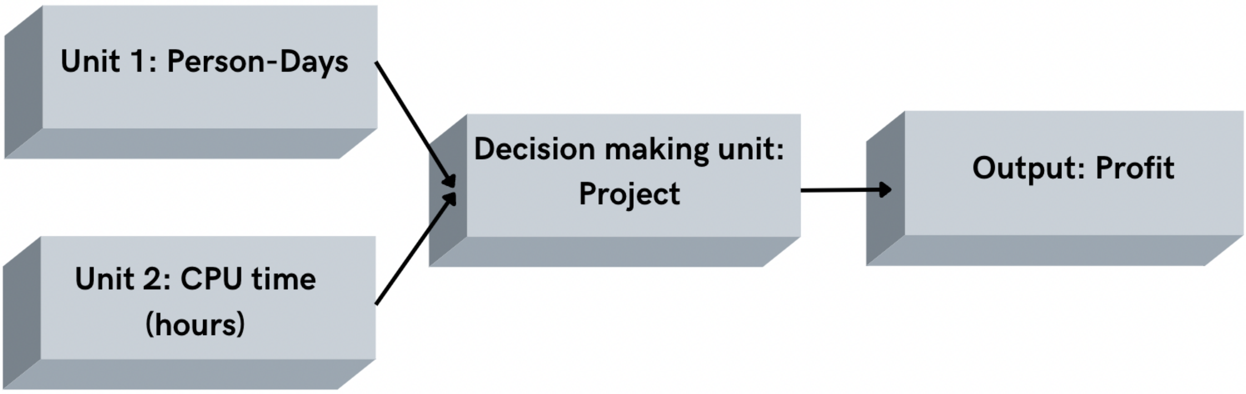


Figure 1: Data Envelopment Analysis Model

1.2 Part B

DEA is a method to compare homogeneous operating units on efficiency. DEA makes it possible to compare the operating units by comparing the level of output they secure to their input levels. Output efficiency is measured by the extent to which actual output levels may be raised to the theoretical maximum for a given set of input levels. Input efficiency is calculated as a percentage of the number of inputs needed to produce a certain amount of output. I have used a 2-input, 1-output model to assess the efficacy of the various projects. The results can be seen in figure 2 and 3.

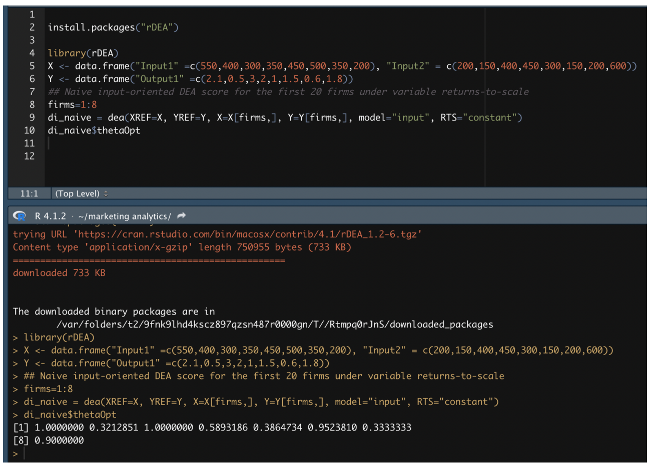


Figure 2: part B

After putting the model into place, it was clear that projects 1 and 3 have the greatest efficiency and should get the most attention. When projects 1 and 3 are done, the focus may shift to projects 6 and 8,

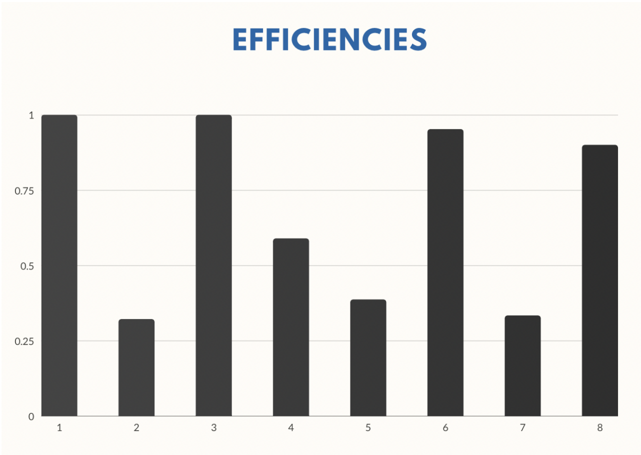


Figure 3: part B graph

which are right behind 1 and 3 in terms of how efficiently they work. The model says that projects 2, 4, and 7 will not be efficient and that the input metrics for these projects must be improved before they can be done.

1.3 Part C

As I see it, the director and the three partners at the engineering consulting firm should make Project 3 a top priority. As can be observed from the output, Project 3 has an efficiency score of 100 per cent. Project 3 is projected to provide the biggest profit at GBP 3 million, surpassing even Project 1's forecast of GBP 2 million. Since it takes fewer people (300) and more computers (400) to complete the job, it strikes the optimal balance between the two. With the usage of technology and the daily cost of hiring the personnel, this combination is certainly ideal.

1.4 Part D

I think it is possible to create more input and output variables, in addition to those already included in the data-set, that may be analysed to strengthen my recommendation. These are the items;
Input:

1. **Labor Force:** The entire number of qualified individuals required to do the work at hand.

2. **Cost**: The project's expenditures

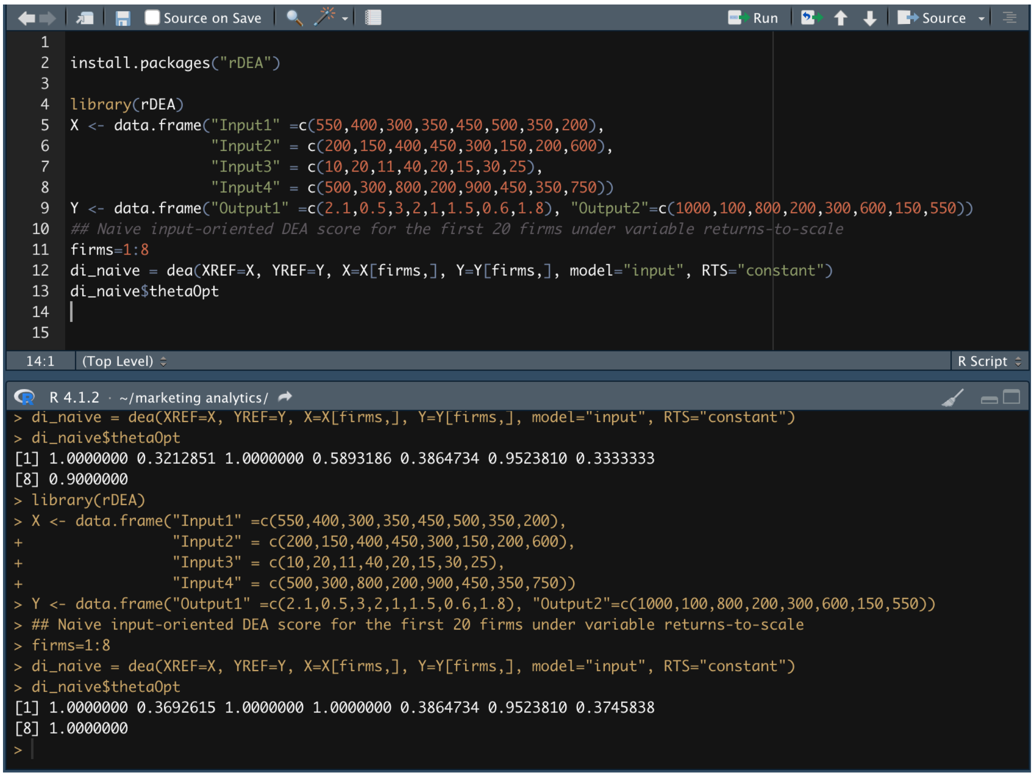
Output:

1. **Unit Sales:** Sales may indicate the project's success by demonstrating how many individuals were exposed to it. Profits may increase as a consequence of increased brand recognition.

Project	Profit (in million GBP)	Person - Days	CPU Time (hours)	Unit Sales (in thousands)	Labour Force	Cost (in millions GBP)
1	2.1	550	200	1000	10	0.5
2	0.5	400	150	100	20	0.3
3	3	300	400	800	11	0.8
4	2	350	450	200	40	0.2
5	1	450	300	300	20	0.9
6	1.5	500	150	600	15	0.45
7	0.6	350	200	150	30	0.35
8	1.8	200	600	550	25	0.75

Figure 4: Part D New Data Table

On this new data table, I ran the DEA analysis once again.



```
install.packages("rDEA")

library(rDEA)
X <- data.frame("Input1" =c(550,400,300,350,450,500,350,200),
                "Input2" = c(200,150,400,450,300,150,200,600),
                "Input3" = c(10,20,11,40,20,15,30,25),
                "Input4" = c(500,300,800,200,900,450,350,750))
Y <- data.frame("Output1" =c(2.1,0.5,3,2,1,1.5,0.6,1.8), "Output2"=c(1000,100,800,200,300,600,150,550))
## Naive input-oriented DEA score for the first 20 firms under variable returns-to-scale
firms=1:8
di_naive = dea(XREF=X, YREF=Y, X=X[firms,], Y=Y[firms,], model="input", RTS="constant")
di_naive$thetaOpt
```

```
> di_naive = dea(XREF=X, YREF=Y, X=X[firms,], Y=Y[firms,], model="input", RTS="constant")
> di_naive$thetaOpt
[1] 1.0000000 0.3212851 1.0000000 0.5893186 0.3864734 0.9523810 0.3333333
[8] 0.9000000
> library(rDEA)
> X <- data.frame("Input1" =c(550,400,300,350,450,500,350,200),
+                 "Input2" = c(200,150,400,450,300,150,200,600),
+                 "Input3" = c(10,20,11,40,20,15,30,25),
+                 "Input4" = c(500,300,800,200,900,450,350,750))
> Y <- data.frame("Output1" =c(2.1,0.5,3,2,1,1.5,0.6,1.8), "Output2"=c(1000,100,800,200,300,600,150,550))
> ## Naive input-oriented DEA score for the first 20 firms under variable returns-to-scale
> firms=1:8
> di_naive = dea(XREF=X, YREF=Y, X=X[firms,], Y=Y[firms,], model="input", RTS="constant")
> di_naive$thetaOpt
[1] 1.0000000 0.3692615 1.0000000 1.0000000 0.3864734 0.9523810 0.3745838
[8] 1.0000000
>
```

Figure 5: R code for DEA analysis of Part D

This new DEA analysis tells us that projects 1,3,4 and 8 are all running on maximum efficiency while project 6 is close to maximum while projects 2, 5 and 7 are not well efficient.

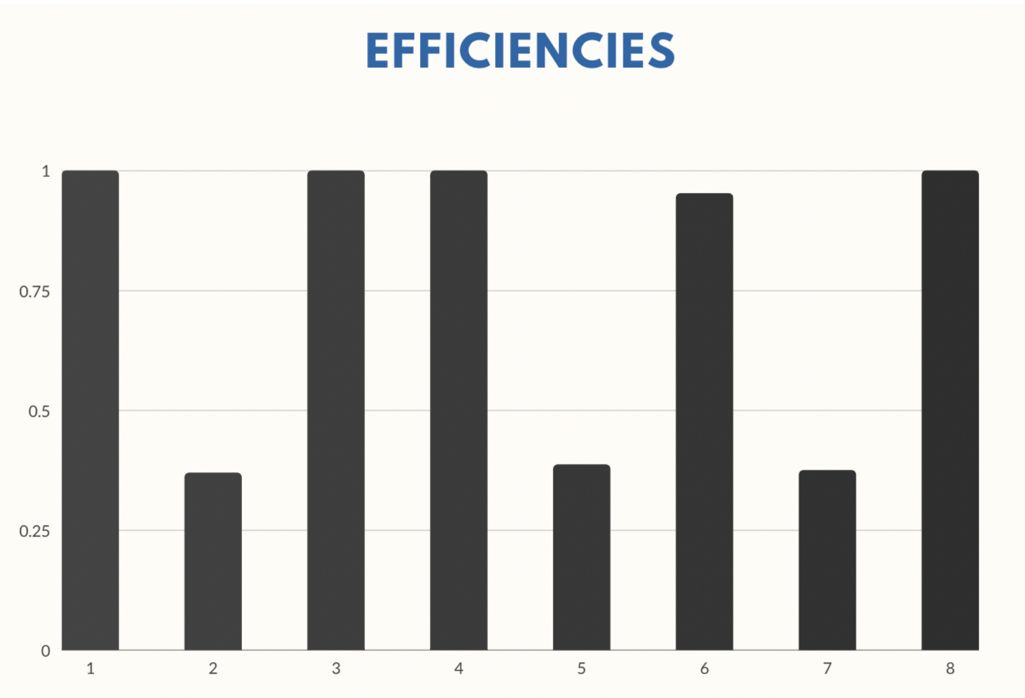


Figure 6: Part D Graph

1.5 Part E

To adopt a more general approach, I developed a set of inputs and outputs applicable to almost all projects. That is;
Inputs:

1. **Time:** The number of days a project takes to complete.

2. **Labour Forces:** This includes the number of skilled workers needed to complete the project.

3. **Production costs:** The cost of raw materials, labour, and other resources used in production.

Outputs:

1. **Sales:** The quantity of sales generated by the project.

2. **Profit:** The difference between the net value of all sales and the cost of production; it is the entire benefit from a project.

3. **Units:** The total number of items sold

While collecting these data points, we must consider the following:

1. Identify the many inputs and outputs, as well as the Decision Making Units, of the project.

2. Establish the model's input and output counts.

3. If the inputs' scaling differs from the outputs', they are normalised.

4. Model orientation is identified by identifying whether the model is input-oriented or output-oriented.

5. The project's goals should inform the selection of the appropriate return to scale model.

6. Making sure their efficiency scale is consistent throughout.

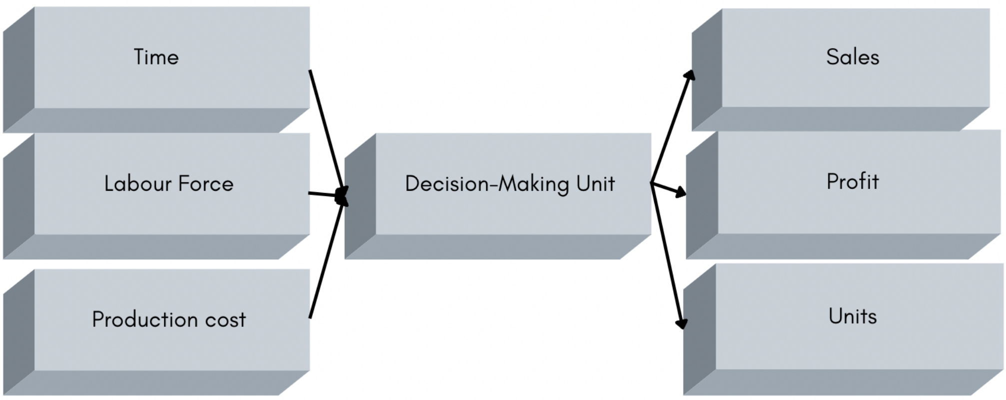


Figure 7: DEA Matrix

2 Project 2

2.1 Part A

According to the table we can see that shipping costs and the ability of each manufacturer to hold the goods are factors that influence distribution. In this instance, we could conclude that demand exceeds supply. Availability surpasses demand by 1,800 tonnes. Therefore, we add a column with the value 0 for each man-

Factory	Steel	Iron	Capacity
1	200	500	2000
2	800	400	1500
3	500	1000	2500
Demand	3200	1000	

Total Demand = 4200

Total Supply = 6000

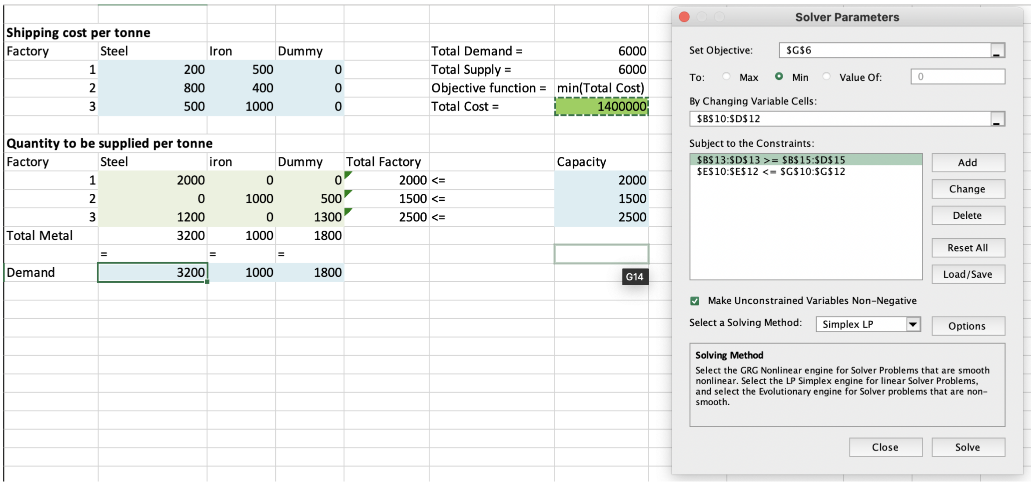
ufacturer and the value 1800 for demand to achieve equilibrium. Since the shipments until this point are fictitious, the shipping cost is set to '0' in all fields. I used an Excel solver with the restrictions that total de-

Factory	Steel	Iron	Dummy	Capacity
1	200	500	0	2000
2	800	400	0	1500
3	500	1000	0	2500
Demand	3200	1000	1800	

Total Demand = 6000

Total Supply = 6000

mand must equal total supply and total supply must be less than or equal to the storage capacity of each firm.



Factory 1 must ship 2,000 tonnes of steel, Factory 2 must ship 1,000 tonnes of iron, and Factory 3 must ship 1,200 tonnes of steel for optimum transportation. This idea would cost a total of £1.4 million.

2.2 Part B

For this part I deleted Factory 3 from the table and examined the changes, since it will not be available for the next three months.

Factory	Steel	Iron	Dummy	Capacity
1	200	500	0	2000
2	800	400	0	1500
Demand	3200	1000	1800	

Total Demand = 4200

Total Supply = 3500

The situation is infeasible because it cannot be satisfied by the current supply given the current demand. Therefore, in this case, the corporation will incur a penalty for each product sent to the retailer. Currently, the corporation is concerned not just with reducing transportation costs, but also with avoiding these fines.

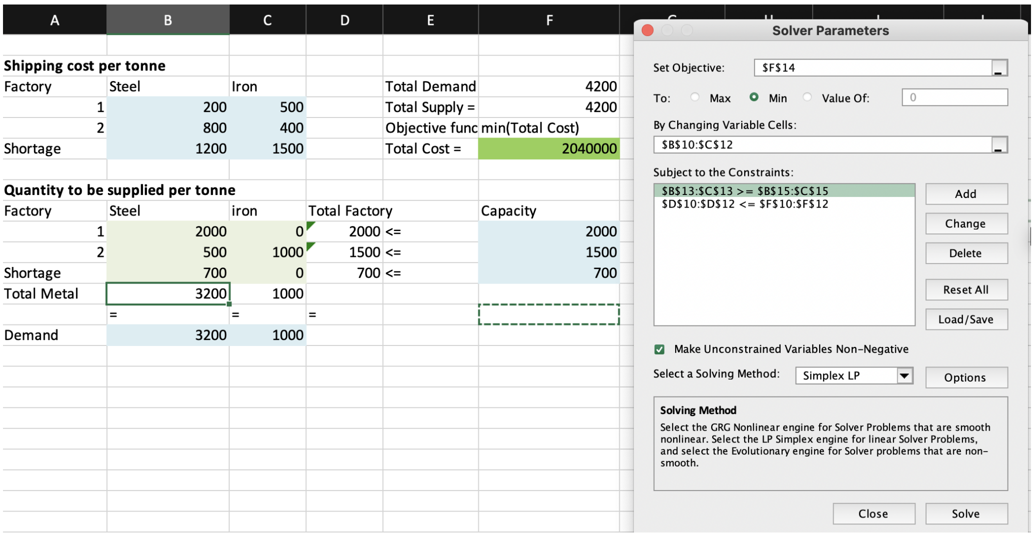
Factory	Steel	Iron	Capacity
1	200	500	2000
2	800	400	1500
Shortage	1200	1500	700
Demand	3200	1000	

Total Demand = 4200

Total Supply = 4200

I have assumed the fines as £1200 for steel and £1500 for iron. Since the supply is 700 tonnes less than the demand, a supply shortage row must be added to the model to account for the resulting penalties.

Now that Total supply is equal to total demand we can derive optimal supply and total shipping cost.



2.3 Part C

In this section, the table will remain same except for the addition of a raw material column to the data-set, which, like the shipping cost, will be multiplied by the number of units and added to the main cost.

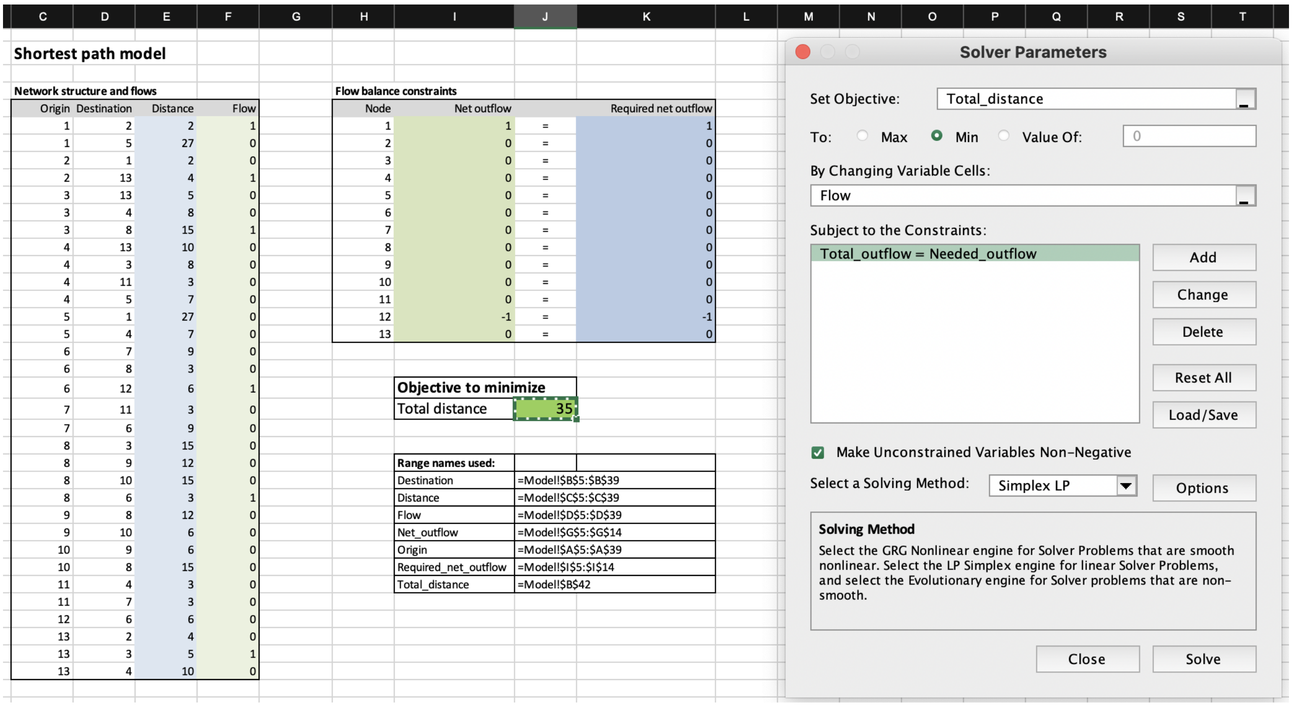
Shipping cost per tonne						
Factory	Steel	Iron	Dummy		Total Demand =	6000
1	200	500	0		Total Supply =	6000
2	800	400	0		Objective function =	min(Total Cost)
3	500	1000	0		Total Cost =	1400000
					Raw Material Cost =	274000
Quantity to be supplied per tonne					Final Cost =	1674000
Factory	Steel	iron		Total Factory		Capacity
1	2000	0	0	2000	<=	2000
2	0	1000	500	1000	<=	1500
3	1200	0	1300	2500	<=	2500
Total Metal	3200	1000	1800			
	=	=		=		
Demand	3200	1000				
Factory	Steel Raw Material Cost	Iron Raw Material Cost				
1	50	100				
2	70	120				
3	45	130				

The assignment strategy will remain unchanged from the main scenario. This is because the raw materials have no distinct demand and supply locations and are not constrained. The entire cost, however, went from £1.4 million to £1.67 million.

3 Project 3

3.1 Part A

I constructed an Excel model by converting the figure data to tabular format. As the company wishes to maintain a connection between influencers 1 and 12, node 1 will serve as the starting point with a necessary net outflow of 1, while node 12 will serve as the ending point with a needed net outflow of -1. All remaining nodes will have a value of 0 following normal guidelines. Since the flow direction is not specified, I presumed that it was bidirectional. The below findings are obtained after running the model in Excel Solver: I would



Shortest path model

Network structure and flows

Origin	Destination	Distance	Flow
1	2	2	1
1	5	27	0
2	1	2	0
2	13	4	1
3	13	5	0
3	4	8	0
3	8	15	1
4	13	10	0
4	3	8	0
4	11	3	0
4	5	7	0
5	1	27	0
5	4	7	0
6	7	9	0
6	8	3	0
6	12	6	1
7	11	3	0
7	6	9	0
8	3	15	0
8	9	12	0
8	10	15	0
8	6	3	1
9	8	12	0
9	10	6	0
10	9	6	0
10	8	15	0
11	4	3	0
11	7	3	0
12	6	6	0
13	2	4	0
13	3	5	1
13	4	10	0

Flow balance constraints

Node	Net outflow		Required net outflow
1	1	=	1
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	0	=	0
8	0	=	0
9	0	=	0
10	0	=	0
11	0	=	0
12	-1	=	-1
13	0	=	0

Objective to minimize	
Total distance	35

Range names used:	
Destination	=Model!$B$5:$B$39
Distance	=Model!$C$5:$C$39
Flow	=Model!$D$5:$D$39
Net_outflow	=Model!$G$5:$G$14
Origin	=Model!$A$5:$A$39
Required_net_outflow	=Model!$I$5:$I$14
Total_distance	=Model!$B$42

Figure 8: Shortest Path Model: Project 3 Part A

propose that the user take the course shown in Figure 9, that is 1-2-13-3-8-6-12. It is the quickest route with the lowest overall investment cost of 35 units for promoting digital linkages between influencer 1 and influencer 12.

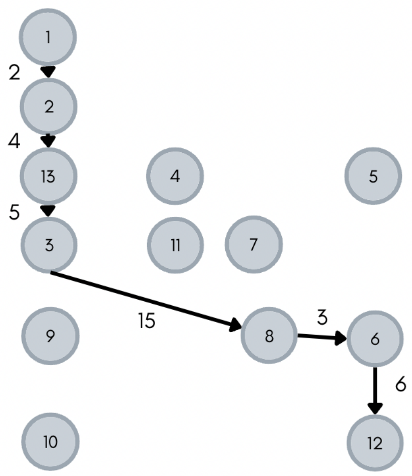


Figure 9: Shortest Path Model

3.2 Part B

Shortest path model

Network structure and flows

Origin	Destination	Distance	Flow
1	2	2	1
1	5	27	0
2	1	2	0
2	13	4	0
3	13	5	1
3	4	8	0
3	8	15	0
4	13	8	0
4	3	7	1
4	11	3	0
4	5	10	0
5	1	27	1
5	4	7	0
6	7	9	0
6	8	3	0
6	12	6	0
7	11	3	0
7	6	9	0
8	3	15	1
8	9	3	0
8	10	12	0
8	6	15	0
9	8	12	0
9	10	6	1
10	9	6	0
10	8	15	1
11	4	3	0
11	7	3	0
12	6	6	0
13	2	4	0
13	3	5	0
13	4	10	1

Flow balance constraints

Node	Net outflow		Required net outflow
1	0	=	1
2	-1	=	0
3	-1	=	0
4	0	=	0
5	1	=	0
6	0	=	0
7	0	=	0
8	0	=	0
9	1	=	0
10	0	=	0
11	0	=	0
12	0	=	-1
13	0	=	0

Objective to minimize	
Total distance	87

Range names used:	
Destination	=Model!$B$5:$B$39
Distance	=Model!$C$5:$C$39
Flow	=Model!$D$5:$D$39
Net_outflow	=Model!$G$5:$G$14
Origin	=Model!$A$5:$A$39
Required_net_outflow	=Model!$I$5:$I$14
Total_distance	=Model!$B$42

Figure 10: Part B

Using the same model, it was discovered via trial and error (by modifying the starting and ending node) that the optimal project sequence could get the corporation up to GBP 87 million by setting the solver aim to maximum. Figure 11 provides a graphic depiction of the project's chronology. To maximise profits, the brand's sequence should be 9-10-8-3-13-4-5-1-2, beginning with project 9.

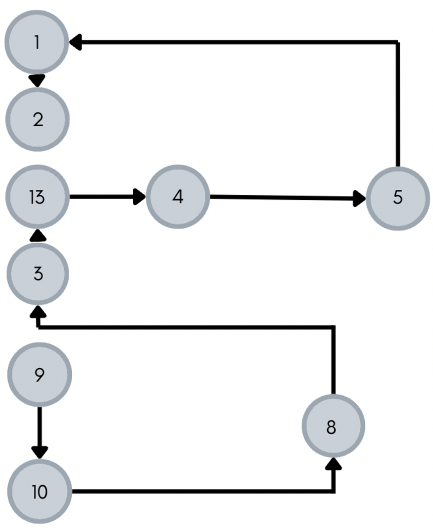


Figure 11: Part B

3.3 Part C

In several contexts, it is useful to be able to solve issues involving shortest paths and maximum flows. That includes:

Short Distance problem

1. Delivery apps such as Amazon's, Deliveroo's, Door- Dash's, and others utilise it to determine the most effective routes for their drivers to travel so that they can accomplish the most deliveries in the shortest period of time.
2. Architects and designers utilise this data to plan warehouse layouts that facilitate the efficient movement of goods. Since warehouse employees are paid on an hourly basis, it is financially prudent to minimise the distances they must travel while carrying items.
3. Proper placement of advertising is essential to the success of any firm selling a product in print or online. Identifying the optimal location is a major issue for several modern businesses. Even newspapers and websites have begun increasing their prices for prime real estate on their papers.

Maximum Flow Problem

1. When designing electrical circuits, it is essential to bear in mind that various kinds and brands of electrical equipment have varying maximum current ratings within which they may work safely. The cable permits the transmission of electricity between electrical devices. The maximum flow model may be used to establish a safe upper limit on the amount of power that may flow into the system in order to prevent overloading of the individual links in the chain.
2. When planning and installing cell towers, network operators heavily rely on this optimization. As a result, the businesses' reception is enhanced, while the maximum possible territory is covered. This not only offers them a competitive edge over other network providers, but also allows them to save money and increase the efficiency of their data infrastructure.
3. In football, attackers are tasked with taking shots and generating multiple scoring opportunities. To maximise their goal-scoring potential, clubs use the players that provide a large number of scoring chances. Left-footed strikers, such as Messi and Mohamed Salah, are favoured to operate on the right side of the goal. When they hold the ball on the right side of the field, they have greater space to cut inside and may pick between near- and far-post to have their chances.