Large-scale Shell Model: application to studies of nuclear spectrosopy and gamma transitions

Mohamed-Rami Ahmane, Sacha Rejai

Faculté de Physique et Ingénierie
Université de Strasbourg

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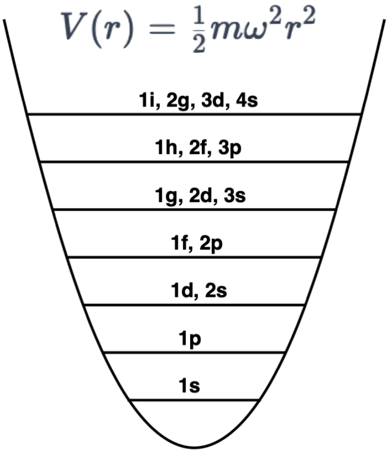
Faculté de physique et ingénierie Université de Strasbourg

Outline

- Introduction
- Properties of the Shell model
- Applications of the Shell model
- Large-scale shell model
- Code Antoine
- Nuclear deformation

Properties of the Shell model

- The independent particle model (IPM)
- Harmonic oscillator potential



Properties of the Shell model

$$V_{SO} = \alpha \overrightarrow{L} . \overrightarrow{S}$$

The model can be improved by adding Spin-Orbit Coupling.

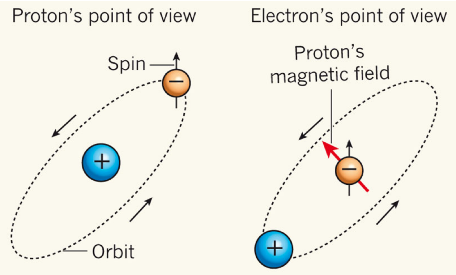
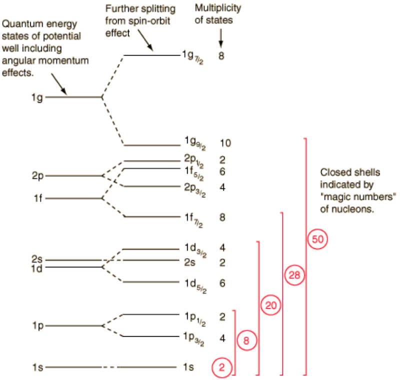


Figure: Spin-Orbit Coupling

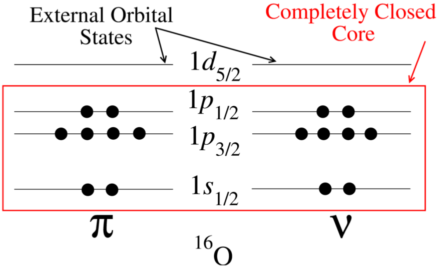
Properties of the Shell model

- Based on experimental observations of nuclear binding energies and the properties of excited nuclear states.
- Magic numbers



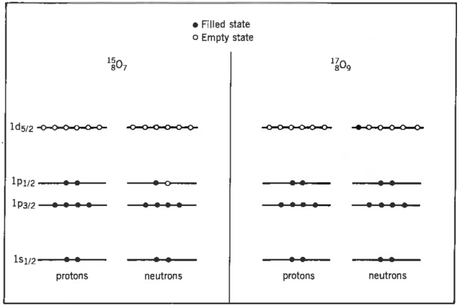
Properties of the Shell model

Nucleons are positioned in the lowest shells and that allow us to determine the angular momentum and the parity of the state J^{π}.



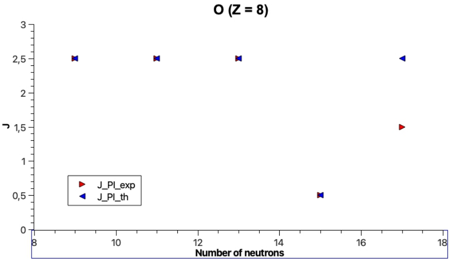
Properties of the Shell model

1. ${}^{15}_{8}\mathrm{O}_7$: The last neutron $\rightarrow 1p_{1/2}$
   - $j = \frac{1}{2}$
   - $\ell = 1$ (from p orbital)
     $\implies$ parity: $\pi = (-1)^{\ell} = -$
   - $\therefore \quad J^{\pi}\mathrm{gs}({}^{15}\mathrm{O}) = \frac{1}{2}^{-}$
2. ${}^{17}_{8}\mathrm{O}_9$: The last neutron $\rightarrow 1d_{5/2}$
   - $j = \frac{5}{2}$
   - $\ell = 2$ (from d orbital)
     $\implies$ parity: $\pi = (-1)^{\ell} = +$
   - $\therefore \quad J^{\pi}\mathrm{gs}({}^{17}\mathrm{O}) = \frac{5}{2}^{+}$

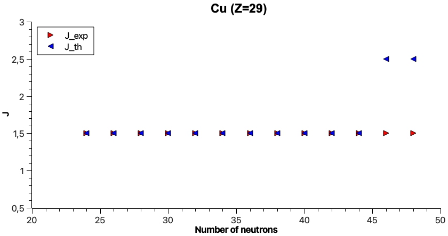


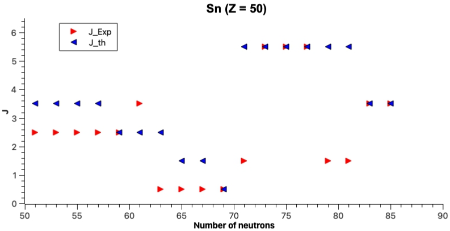
Applications of the Shell model

comparison of the angular momentum predicted by our model with experimental results for different isotopes of O, Cu and Sn



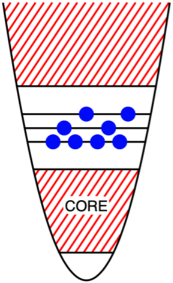
Applications of the Shell model





Large-scale shell model

- Inert core: orbits that are always full.
- Valence space: orbits that contain the physical degrees of freedom relevants to a given property. The distribution of the valence particles among these orbitals is governed by the interaction.
- External space: all the remaining orbits that are always empty (limit of calculation)



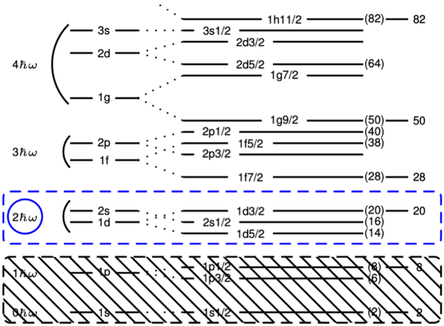
- Define a valence space
- Derive an effective interaction $H\psi = E\psi \longrightarrow H_{eff}\psi = E_{eff}\psi$
- Build and diagonalize the Hamiltonian matrix.

$$\Psi = \frac{1}{\sqrt{n!}} \sum_P (-1)^P P \begin{pmatrix} \phi_1(1) & \cdots & \phi_n(1) \\ \vdots & \ddots & \vdots \\ \phi_1(n) & \cdots & \phi_n(n) \end{pmatrix}$$

Large-scale shell model

Valence space :

- 4 A 16 : Cohen-Kurath interaction
- 16 A 40 : sd shell USD interaction
- 40 A 80 : KB3 interactions



Antoine Code

- Computer program used to perform shell model calculations in nuclear physics.
- Based on the large-scale shell model and allows for the calculation of nuclear properties, such as excitation energies, magnetic moments, and decay rates.
- Diagonalisation of large Hamiltonian matrices for solving the nuclear many-body problem.

Antoine Code

- option 1 : Calculate Dimensions of the basis
- option 2 : Compute number of non-zero terms in the matrix
- option 4 : Diagonalization from random initial pivot
- option 12 : Wave-function Quadrupole moment
- option 31 : Change truncation basis

Option 1: Calculate Dimensions of the basis
Option 2: Compute number of non-zero terms in the matrix
1 0 0 ***** Option 1
4 3 205 203 1001 0 0 0 10
6 3 205 203 1001 0 0 0 10
0 0 0

Basis definition:
for each fluid (proton or neutrons),
- number of particles
- number of shells
- denomination of of shells
- shell class c_i
- t_{max} in each fluid
- total J_z value
- total parity
- total t_{max}

Antoine Code

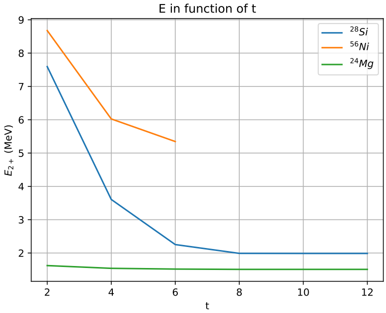
```
INITIAL STATE   P= 0     2*J= 4    N= 1
    Q(L=2)=     -16.339347
       P= 0      2*J= 0    N=  1 DE=  1.509  BE(L)=     69.47740369

INITIAL STATE   P= 0     2*J= 4    N= 2
    Q(L=2)=      16.578464
       P= 0      2*J= 4    N=  1 DE=  2.613  BE(L)=     15.15699610
       P= 0      2*J= 0    N=  1 DE=  4.122  BE(L)=      6.20638656

INITIAL STATE   P= 0     2*J= 8    N= 1
    Q(L=2)=     -19.857931
       P= 0      2*J= 4    N=  2 DE=  0.256  BE(L)=      0.46933800
       P= 0      2*J= 4    N=  1 DE=  2.869  BE(L)=     92.35718046

INITIAL STATE   P= 0     2*J= 8    N= 2
    Q(L=2)=      -9.085790
       P= 0      2*J= 8    N=  1 DE=  1.556  BE(L)=     12.53539846
       P= 0      2*J= 4    N=  2 DE=  1.812  BE(L)=     35.30633272
       P= 0      2*J= 4    N=  1 DE=  4.425  BE(L)=      3.11890756

INITIAL STATE   P= 0     2*J=12    N= 1
    Q(L=2)=     -16.733946
       P= 0      2*J= 8    N=  2 DE=  2.328  BE(L)=      7.26175402
       P= 0      2*J= 8    N=  1 DE=  3.885  BE(L)=     87.48414172

INITIAL STATE   P= 0     2*J=12    N= 2
    Q(L=2)=     -20.484379
       P= 0      2*J=12    N=  1 DE=  1.322  BE(L)=      9.25116535
       P= 0      2*J= 8    N=  2 DE=  3.650  BE(L)=     45.96907524
       P= 0      2*J= 8    N=  1 DE=  5.207  BE(L)=      0.15884931

INITIAL STATE   P= 0     2*J=16    N= 1
    Q(L=2)=      15.147648
       P= 0      2*J=12    N=  2 DE=  2.503  BE(L)=      0.65399637
       P= 0      2*J=12    N=  1 DE=  3.825  BE(L)=     10.94581083
```

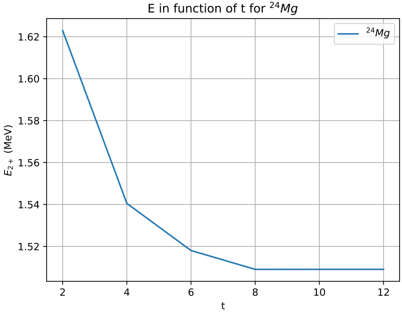
Figure: Data from Antoine code

```
 GROUND-STATE (AMONG THE READ STATES) ENERGY=   -87.08958

2*J=  0 T-TZ= 0 COUL=0 N=  1 P=0 2*M= 0 C= 0  EXC=   0.00000 E=   -87.08958
2*J=  4 T-TZ= 0 COUL=0 N=  1 P=0 2*M= 0 C= 0  EXC=   1.50907 E=   -85.58051
2*J=  4 T-TZ= 0 COUL=0 N=  2 P=0 2*M= 0 C= 0  EXC=   4.12220 E=   -82.96738
2*J=  8 T-TZ= 0 COUL=0 N=  1 P=0 2*M= 0 C= 0  EXC=   4.37828 E=   -82.71130
2*J=  6 T-TZ= 0 COUL=0 N=  1 P=0 2*M= 0 C= 0  EXC=   5.09658 E=   -81.99300
2*J=  8 T-TZ= 0 COUL=0 N=  2 P=0 2*M= 0 C= 0  EXC=   5.93446 E=   -81.15512
2*J= 10 T-TZ= 0 COUL=0 N=  1 P=0 2*M= 0 C= 0  EXC=   7.88347 E=   -79.20611
2*J= 12 T-TZ= 0 COUL=0 N=  1 P=0 2*M= 0 C= 0  EXC=   8.26295 E=   -78.82663
2*J= 12 T-TZ= 0 COUL=0 N=  2 P=0 2*M= 0 C= 0  EXC=   9.58485 E=   -77.50473
2*J= 16 T-TZ= 0 COUL=0 N=  1 P=0 2*M= 0 C= 0  EXC=  12.08806 E=   -75.00152
2*J= 16 T-TZ= 0 COUL=0 N=  2 P=0 2*M= 0 C= 0  EXC=  13.16017 E=   -73.92941
   REWIND FILE           50

 TIME FOR THIS OPTION=         0.180
 *****************************************************************
NORMAL END OF THE JOB
```

Figure: Data from Antoine code

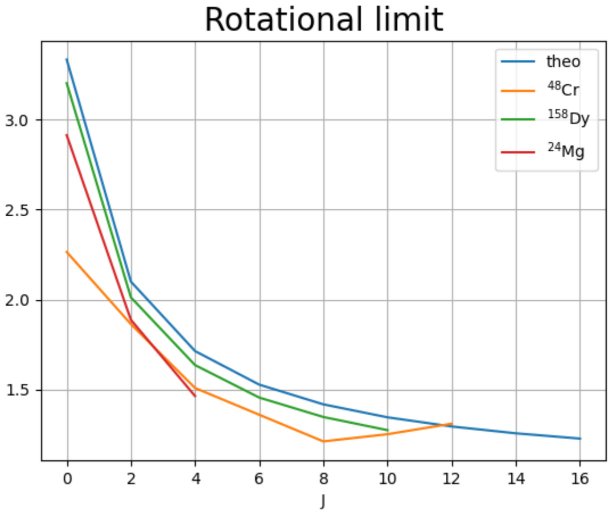
Results of numerical calculation





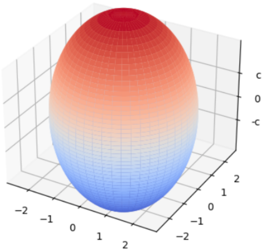
Evolution of the excitation energy of the 2^+ state as a function of the truncation

Results of numerical calculation



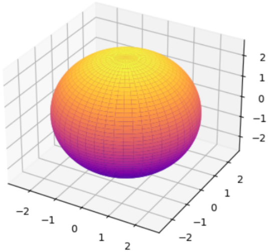
Nuclear deformation

Shape of the prolate nucleus



$\beta > 0$

Shape of the oblate nucleus



$\beta < 0$

$$R(\theta,\phi) = R_0\left(1 + \beta\sqrt{\frac{5}{16\pi}}(cos\gamma(3cos^2\theta - 1) + \sqrt{3}sin\gamma sin^2\theta cos2\phi)\right)$$

Nuclear deformation

- Quadrupole deformation :

$$\beta = \frac{4\pi}{3}\frac{Q}{ZR^2}$$

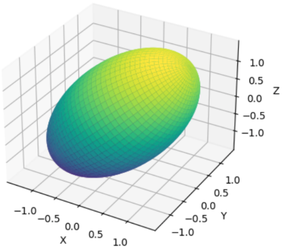
- Quadrupole moment

$$Q = \int \rho(\vec{r})(3Z^2 - r^2)d^3r$$

- Core radius

$$R = r_0 A^{1/3}$$

Shape of the triaxial core



$$\gamma = \frac{2}{3}\frac{\beta}{R}\sqrt{\frac{5\pi}{16\pi - 3}}$$

Nuclear deformation

- Quadrupole moment :

$$Q = Q_0 \frac{3K^2 - I(I+1)}{(2I+3)(I+1)}$$

- Transition probability :

$$B(E2; KI_1 \longrightarrow KI_2) = \frac{5}{16\pi} e^2 Q_0^2 \langle I_1 K 2 0 | I_2 K \rangle^2$$

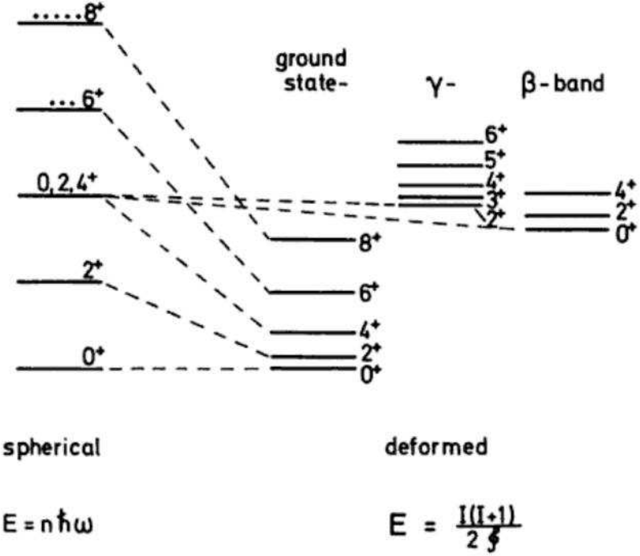
Nuclear deformation

States	$Q_{0\ exp}$	$Q_{0\ from\ BE2}$	$Q_{0\ from\ Q}$	β
2+	121,7	113,2	109,4	0,317
4+	100,2	110,9	114,2	0,330
6+	97,4	105,4	105,1	0,304
8+	87,6	98,6	98,2	0,284
10+	76,2	82,1	55,8	0,161
12+	79,5	68,9	13,1	0,038
14+	64,9	60,7	13,2	0,038
16+	35,9	43,7	15,7	0,046

Table: Values of Quadrupole Moment for ^{48}Cr

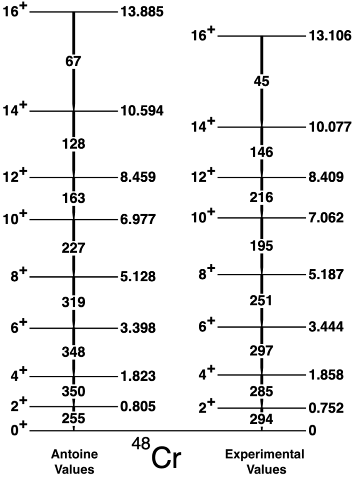
$\beta > 0$ for $^{48}Cr \implies$ prolate

Nuclear deformation



Nuclear deformation

We observe that values obtain with the code for energy levels and probability of transition are very close to experimental values.



Nuclear deformation

- $Q(2_1+) = -Q(2_\gamma+)$ and $Q(3+)$ is near to 0 : That's the proof of the γ band exist for Mg24
- We can determine $\gamma = 15$ non-zero and deduce that the Nuclei is triaxial

States	$Q_{spectro}$	Q_0 from code	β
2+	-16,34	57,19	0,525
4+	-19,86	54,61	0,502
6+	-16,73	41,83	0,384
8+	15,15	-35,98	-0,331

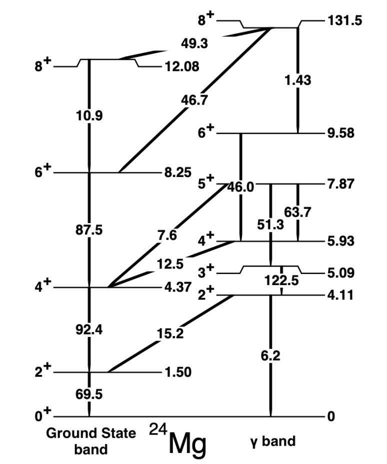
Table: Values from code for ^{24}Mg for ground state band

States	$Q_{spectro}$	Q_0 from code	β
2+	16,58	-58,02	-0,53
3+	-0,05	0,16	0,001
4+	-9,08	62,46	0,573
5+	-13,70	59,35	0,545
6+	-20,48	71,69	0,659

Table: Values from code for ^{24}Mg for γ band

Nuclear deformation

Then we have built the levels scheme of Mg24 and we can see the γ band next to the ground state band.



Summary

- Basis of shell model, IPM, adding spin orbit coupling
- Angular momentum coupling for two and three particles
- Fortran code compilation
- We learn how to use the code with exercices ; some examples :
  - Verify with the code, the basis dimensions of ^{16}O and ^{18}F
  - Calculate the basis dimensions of all even-even N=Z nuclei from ^{20}Ne to ^{36}Ar
  - Calculate the number of non-zero matrix element of all even-even N=Z nuclei from ^{20}Ne to ^{36}Ar
  - Compute the energy spectrum of ^{28}Si for J^{π} =0+;2+;4+;6+
  - Compute the energy spectrum of ^{22}O ; Compute E2 transitions and moments.

Summary

- Computing of energy levels and probability transition and Creation of levels scheme with another fortran code.

- Study of general properties of electromagnetic transitions in nuclei, Bohr-Mottelson model . . .

- Study of notion about nuclear deformation and apply to ^{24}Mg and ^{48}Cr

- Python script to plot in 3D the nuclei form in function of quadrupole moment.