DM question 1

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1 Question 1

To model question 1, we introduce $a_{i,j}$, $i \in pros(x,y), j \in cons(x,y)$, a binary variable equal to 1 if the pair (i,j) is used in the (1-1) type explanation, and 0 otherwise.
Let $x_i \in pros(x,y), y_j \in cons(x,y)$,
The problem is therefore as follows:

$$max_{i,j} \sum_{i,j} a_{i,j} \tag{1}$$

s.t
$\forall i, \sum_j a_{i,j} \leq 1$, (each pros is used at most once)
$\forall j, \sum_i a_{i,j} = 1$, (each cons is used one and only once)
$\forall (i,j), x_i + y_j \times a_{i,j} \geq 0$ (if $|y_j| > x_i$, then we can't select this pair so $a_{i,j} = 0$, if this pair is feasible, $a_{i,j}$ may be non null.)
If there is no feasible solution or the maximum is non-positive, there is a certificate of non-existence. We decided not to include inputs in the objective function because we are only looking at the feasibility of a solution, and not necessarily to the optimal solution (i.e the greatest gap)

2 Question 2

Same method :
We model the problem as follows : we keep the same objective functions and we modify the constraints.
The problem is therefore as follows:

$$max_{i,j} \sum_{i,j} a_{i,j} \tag{2}$$

s.t
$\forall j, \sum_i a_{i,j} = 1$, (each cons is used one and only once)
$\forall i, x_i + \sum_j y_j \times a_{i,j} \geq 0$ (if $\sum_j |y_j| > x_i$, then we can't select these elements to be compared with x_i, if x_i compensate the y_j's, the $a_{i,j}$'s).

If there is no feasible solution or the maximum is non-positive, there is a certificate of non-existence.

For the comparison u $\succ$ v, the program returns a certificate a non-existence.

3 Question 3

We model the problem quite the same way as question 2 but this time, the role of the pros and cons are reversed : we are looking for an aggregation of pros to compensate a con.
The problem is therefore as follows :

$$max_{i,j} \sum_{i,j} a_{i,j} \tag{3}$$

s.t
$\forall j, \sum_i a_{i,j} \times x_i + y_j \geq 0$, (aggregation of pros must compensate each con)
$\forall i, \sum_j a_{i,j} \leq 1$ (we have to use each pro at most one time).

The (m-1) type explanation founded by the algorithm is $\{$((G),B),((A),D),((C,F),E)$\}$
The algorithm doesn't find any type (1-m) nor type (m-1) explanation for z$\succ$t.

4 Question 4

To answer this question, we adapt the method and the algorithm presented in the article provided with the project. We introduce new binary variables $t_k^{(x,y)}$, k indexed on $pros(x,y) \cup cons(x,y)$, as well as $e^{(x,y)}$ to validate or invalidate the solution.

4.1 Decision Variables

Let x and y be two alternatives to compare, with $pros(x,y)$ and $cons(x,y)$ denoting the sets of pro and con viewpoints.

- $u_{i,j}^{(x,y)} \in \{0,1\}$: equals 1 if pro viewpoint $i \in pros(x,y)$ is paired with con viewpoint $j \in cons(x,y)$ in a *one vs. q* argument (strong pro).
- $v_{i,j}^{(x,y)} \in \{0,1\}$: equals 1 if pro viewpoint i is paired with con viewpoint j in a *p vs. one* argument (accrual of pros).
- $t_k^{(x,y)} \in \{0,1\}$: equals 1 if viewpoint $k \in pros(x,y) \cup cons(x,y)$ is used alone in an argument.
- $e^{(x,y)} \in \{0,1\}$: equals 1 if an explanation is produced for the comparative statement (x,y).

4.2 Constraints

The following constraints encode the logical structure of the arguments and their alignment with preferences.

Activation of paired variables. Paired variables can only be activated if the corresponding pivot viewpoint is selected:

$$u_{i,j} \leq t_i \qquad \forall i \in Pros,\ \forall j \in Cons \tag{4}$$

$$v_{i,j} \leq t_j \qquad \forall i \in Pros,\ \forall j \in Cons \tag{5}$$

Constraint (4) ensures that a pro i can be involved in a one-vs-m argument only if it is chosen as a pivot. Constraint (5) ensures that a con j can be involved in an m-vs-one argument only if it is chosen as a pivot.

Exclusive use of pro viewpoints. Each pro viewpoint can either serve as the pivot of a one-vs-m argument or be used as a member of an m-vs-one argument, but not both:

$$\sum_{j \in Cons} v_{i,j} + t_i \leq 1 \qquad \forall i \in Pros. \tag{6}$$

Complete coverage of con viewpoints. Each con viewpoint must be used exactly once, either as the pivot of an m-vs-one argument or as part of a one-vs-m argument:

$$\sum_{i \in Pros} u_{i,j} + t_j = 1 \qquad \forall j \in Cons. \tag{7}$$

Preference alignment for one-vs-m arguments. A one-vs-m argument is valid only if its total strength is non-negative when it is activated and when an explanation is produced:

$$\omega_i + \sum_{j \in Cons} \omega_j \, u_{i,j} \;\geq\; -M(1 - t_i) - M(1 - e) \qquad \forall i \in Pros. \tag{8}$$

This constraint is relaxed via a Big-M formulation when the pro i is not selected as a pivot or when no explanation is generated.

Preference alignment for m-vs-one arguments. Similarly, an m-vs-one argument is valid only if its total strength is non-negative:

$$\omega_j + \sum_{i \in Pros} \omega_i \, v_{i,j} \;\geq\; -M(1 - t_j) - M(1 - e) \qquad \forall j \in Cons. \tag{9}$$

This enforces coherence between the selected viewpoints and the user preferences whenever a con j is chosen as a pivot and an explanation is provided.

4.3 Objective Function

The objective is to maximize the explanatory power of the comparison while minimizing the use of standalone viewpoints:

$$\max \; me^{(x,y)} - \sum_{k \in \mathcal{N}^*} t_k^{(x,y)}.$$

Our version of the algorithm is a relaxed version of the (1-q,p-1) algorithm presented in the article, since we allow for as many pros as possible to compensate a single con, and reversely as many cons as possible to be compensated by one pro. We therefore release constraints A-5 and A-6 of the Appendix.

5 Application on Datasets

5.1 RATP

We use a weighted sum to have the order of the 10 stations. Here is the obtained ranking

To explain this rank, we got the "prices" between the best station (i.e. Odéon) and the others for all of the criteria and apply the 4th algorithm ((1-m) type

Rank	Station	Score
1	Odéon	9484
2	Place d'Italie	9457
3	Jussieu	9436
4	Nation	9389
5	La Motte Picquet - Grenelle	9349
6	Porte d'Orléans	9328
7	Reuilly-Diderot	9240
8	Oberkampf	9229
9	Daumenil	9144
10	Vaugirard	9107

Table 1: Ranking obtained with the weighted sum for RAPT dataset

or (m-1) type of trade-off).

The explanations founded are provided in the annex of this document.

We also explain the rank with comparison between two following stations (for example 1 vs 2, 2 vs 3 etc). We got the prices then ran the 4th algorithm on the obtained prices. The explanations are provided in the annex.

5.2 27 criteria

We use a weighted sum to have the order of the 10 stations. Here is the obtained ranking

Rank	Solution	Score
1	Solution 1	19725.3
2	Solution 2	17653.3
3	Solution 4	15711.3
4	Solution 3	-15731.3

Table 2: Ranking obtained with the weighted sum for 27 criteria dataset

5.3 Breast Cancer diagnonis

In this use case, we aim at explaining why one would be more or less likely to have a benign breast cancer, based on a set of 9 quantitative criterions and the outcome (Benign or not Benign).
Firstly, we perform a simple logistic regression on the dataset. We obtain good results with a f1-score of 0.95 for the Benign prediction, showing clearly the statistical association between the observed criterions and the severity of the

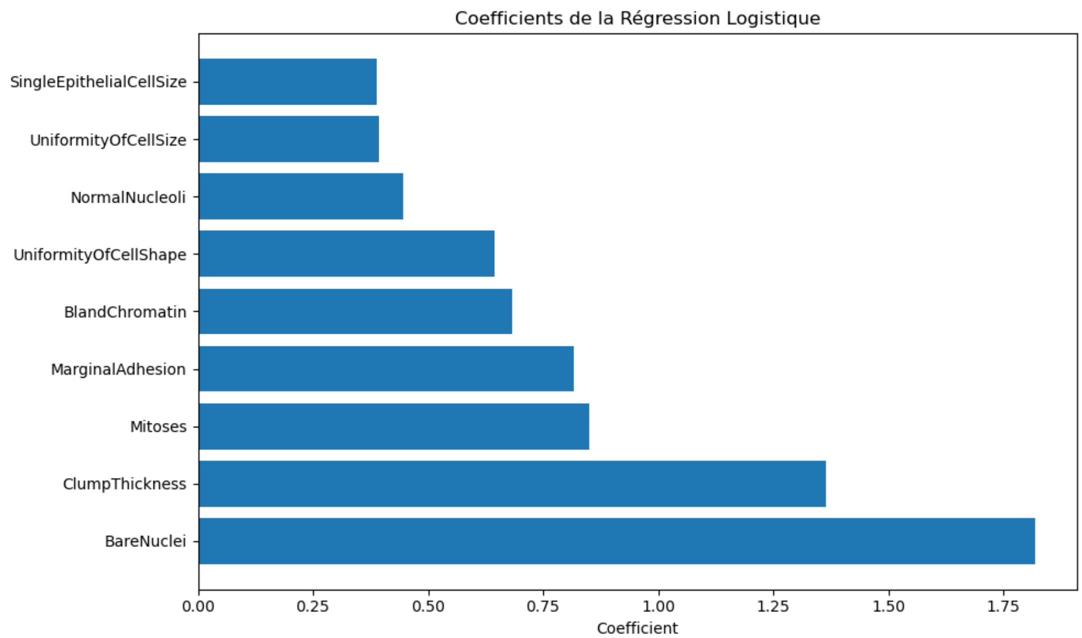

Figure 1: Coefficients of features from the logistic regression

disease.
We then derive the weights for our explanations from the logistic regression algorithm. The values of weight are given in the figure below:

Finally we adapt the data to our explanation algorithm by calculating the scores of each instance (the weighted sum of the criterions and their weights). Thus, instances can be compared with the explanation algorithms. For example, we can explain how the best instance (597) scores better than the fifth best (346):

Comparison	Pros	Cons
(597,346)	ClumpThickness(+2.7), BlandChromatin(+1.4)	SingleEpithelialCellSize (-1.5)

Table 3: Example of a comparison explanation

For this case, our algorithm provides a simple solution:

$$ClumpThickness > SingleEpithelialCellSize$$

A general explanation function is developed, taking as an input 2 instances, and explaining (if possible) why one scores better than the other. Some other random examples are provided in the script.

6 Annex

Station	Explanation
Place d'Italie	*[(('peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'connectivity index [0,100]', 'off-peak-entering-passengers/h'))]*
Jussieu	*[(('peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'off-peak-entering-passengers/h', 'off-peak-passing-passengers/h', 'peak-passing-passengers/h'))]*
Nation	*[(('peak-entering-passengers/h',), ('off-peak-entering-passengers/h', 'off-peak-passing-passengers/h', 'strategic priority [0,10]'))]*
La Motte Picquet-Grenelle	*[(('off-peak-entering-passengers/h',), ('connectivity index [0,100]', 'off-peak-passing-passengers/h', 'strategic priority [0,10]'))]*
Porte d'Orléans	*[(('off-peak-entering-passengers/h',), ('connectivity index [0,100]', 'off-peak-passing-passengers/h', 'strategic priority [0,10]'))]*
Reuilly-Diderot	*[(('peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'off-peak-entering-passengers/h', 'off-peak-passing-passengers/h', 'peak-passing-passengers/h'))]*
Oberkampf	No explanation founded by (1-m) type or (m-1) type trade-off
Daumenil	[(('peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'off-peak-entering-passengers/h', 'off-peak-passing-passengers/h'))]
Vaugirard	*[(('peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'off-peak-entering-passengers/h', 'off-peak-passing-passengers/h', 'strategic priority [0,10]'))]*

Table 4: RATP dataset: Explanations for comparison between Odeon and the other stations

Station	Explanation
Odéon (Ligne 4) vs Place d'Italie (Ligne 6)	*[(('peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'connectivity index [0,100]', 'off-peak-entering-passengers/h'))]*
Place d'Italie (Ligne 6) vs Jussieu (Ligne 7)	*[(('peak-entering-passengers/h',), ('off-peak-passing-passengers/h', 'peak-passing-passengers/h', 'strategic priority [0,10]'))]*
Jussieu (Ligne 7) vs Nation (Ligne 9)	*No explanation founded by (1-m) type or (m-1) type trade-off*
Nation (Ligne 9) vs La Motte Picquet-Grenelle (Ligne 10)	*[(('peak-entering-passengers/h',), ('connectivity index [0,100]', 'peak-passing-passengers/h', 'strategic priority [0,10]'))]*
La Motte Picquet-Grenelle (Ligne 10) vs Porte d'Orléans (Ligne 4)	*[(('off-peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'off-peak-passing-passengers/h'))]*
Porte d'Orléans (Ligne 4) vs Reuilly-Diderot (Ligne 1)	*No explanation founded by (1-m) type or (m-1) type trade-off*
Reuilly-Diderot (Ligne 1) vs Oberkampf (Ligne 9)	*No explanation founded by (1-m) type or (m-1) type trade-off*
Oberkampf (Ligne 9) vs Daumenil (Ligne 6)	*[(('peak-passing-passengers/h',), ('Station degradation level ([0,20] scale)', 'off-peak-passing-passengers/h')), (('peak-entering-passengers/h',), 'off-peak-entering-passengers/h')]*
Daumenil (Ligne 6) vs Vaugirard (Ligne 12)	*[(('peak-entering-passengers/h',), ('Station degradation level ([0,20] scale)', 'off-peak-entering-passengers/h', 'peak-passing-passengers/h', 'strategic priority [0,10]'))]*

Table 5: RATP dataset: Explanations for comparison between two following stations

Solution	Explanation
Solution 2	*[(('a',), ('p', 'r', 'w')), (('b',), ('e', 'f', 'i', 'k', 'q', 'v', 'x')), (('m',), ('g', 'l', 't')), (('s',), 'A')]*
Solution 3	*[(('h',), ('f', 'p')), (('m',), 'c'), (('s',), 'n'), (('u',), ('A', 'q')), (('w',), 'o'), (('z',), ('a', 't', 'x')), (('d', 'e', 'g'), 'b'), (('j', 'k', 'v'), 'l'), (('i', 'y'), 'r')]*
Solution 4	*[(('c',), 'l'), (('m',), ('b', 'i', 'k', 'p', 'v', 'y'))]*

Table 6: 27 criteria dataset: Comparaison between Solution 1 (best solution) and others

Solution	Explanation
Solution 3	*[(('c',), ('a', 'd', 'h', 'i', 'p', 'v')), (('m',), ('b', 'k', 'l', 'y'))]*
Solution 4	*No explanation founded by (1-m) type or (m-1) type trade-off*

Table 7: 27 criteria dataset: Comparaison between Solution 2 (second best solution) and others (except the best solution)

Solution	Explanation
Solution 3	*[(('m',), ('g', 'i', 's', 'w')), (('o',), ('h', 'k', 'y')), (('A',), 'v')]*

Table 8: 27 criteria dataset: Comparison between solution 4 (third best solution) and the others (excluding the best and second best solutions)