RESEARCH STATEMENT

SUSHANT SACHDEVA

1. Introduction

My research interests lie broadly in algorithms – both towards designing better algorithms, and towards understanding the limits of efficient algorithms. Though I am interested in problems from diverse areas, two specific areas of focus in my work include problems related to *graph partitioning* [OSV12, ST11, MS11], and *lower bounds to approximability* [SS11, SS12, MS11, ST11]. I am also interested in giving provable algorithms for problems from more applied areas under justifiable assumptions [AGSS12, AGMS12]. I now sketch my research interests and my related works.

Linear time graph algorithms. I am interested in designing near-linear time algorithms for important graph problems. In [OSV12], we give a near-linear time spectral algorithm for Balanced Separator using techniques from numerical linear algebra and approximation theory. In [SV12], we extend these techniques to show the equivalence of exponentiation and inversion for PSD matrices. I am exploring applications of these techniques to designing fast algorithms for other problems, in particular to obtain a $O(\sqrt{\log n})$ approximation to Balanced Separator.

SDP hierarchies for graph partitioning. I am interested in understanding the limits of SDP Hierarchies for problems like Sparsest Cut, and Balanced Separator. I have worked on understanding the integrality gaps for these problems [MS11, ST11]. I would like to understand the power of the Lasserre hierarchy towards obtaining improved approximations for these problems.

Optimal inapproximability without UGC. For several problems, optimal inapproximability is known only under the Unique Games Conjecture, whose status is uncertain in light of recent results [ABS10]. I am interested in obtaining optimal inapproximability results under NP-hardness. In [SS12], we prove optimal 2-inapproximability for some scheduling problems. In [SS11], we prove (near) optimal inapproximability for Hypergraph Vertex Cover on k-partite k-uniform hypergraphs. I would like to extend these techniques towards proving other optimal inapproximability results.

Provable Algorithms instead of Heuristics. I am interested in applying techniques from TCS to problems in other fields (e.g. Machine Learning, Social Networks) where heuristics are used in practice because of the lack of provable algorithms. An approach we have successfully applied, is to give provable algorithms for restricted input classes. In [AGSS12], we show how to provably and efficiently *explain* a social network as a union of *overlapping communities*. In [AGMS12], we give a provable and efficient algorithm for *Independent Component Analysis* in the presence of noise.

Techniques. My research has utilized tools from numerical linear algebra [OSV12], approximation theory [OSV12, SV12], harmonic analysis [SS12, ST11], and extremal combinatorics [SS11]. I am working on extending and applying these techniques to other problems of interest.

Over the next couple of pages, I give some details about the above themes, my related works, and future directions of research.

2. Graph Partitioning

Some of the most well-studied problems in graph partitioning are Sparsest Cut, where we want to find a cut of minimum conductance[1], and its close variant, Balanced Separator, where additionally the two parts of the cut must be roughly the same size. These two problems have numerous applications in both theory and practice. The exact problems are known to be NP-hard, but from

[1]The ratio of the cost of the edges cut, to the fraction of pairs cut.

an approximation standpoint, our understanding is still incomplete. My research has contributed to improved algorithms and new techniques for these problems, and also an improved understanding of the hard instances.

Near-Linear Time Graph Algorithms. Driven by massive sizes of several interesting graphs (e.g. web graph), the search for near-linear time graph algorithms is an important strand in recent research. Along this direction, in [OSV12] we give a near-linear time algorithm for Balanced Separator with the best possible guarantee for spectral algorithms, *i.e.* it finds a balanced cut of conductance $O(\sqrt{\gamma})$ when the graph has one of conductance γ. Our algorithm runs in time $O(m \cdot \text{polylog}(n))$, without any $\text{poly}(1/\gamma)$ factors, unlike previous works [ST04, AP09, OV11]. This is surprising because our algorithm is based on random walks, and the mixing time of random walks is known to have a $poly(1/\gamma)$ dependence. At a high level, the algorithm simulates $\text{polylog}(n)$ continuous time random walks on the graph, and rounds them to obtain the desired cut. A significant part of the work was to simulate continuous-time random walks (equivalent to approximating $e^{-tL}v$, where L is the graph Laplacian, v is a vector) in near-linear time, independent of the "length" t.

Techniques and Other Applications. The rounding algorithm uses techniques from SDP rounding and matrix multiplicative weight updates. In order to approximate $e^{-tL}v$, we use techniques from numerical linear algebra and approximation theory to reduce it to a poly-logarithmic number of computations of the form $L^{-1}v$, and combine it with the near-linear time algorithm for inverting laplacians by Spielman-Teng [ST04]. In an ongoing work [SV12], we apply some of these techniques to show that the two problems are in fact equivalent up to logarithmic factors, by approximating $L^{-1}v$ as a sparse weighted sum of matrix exponentials $e^{-tL}v$.

Integrality Gaps for SDP Hierarchies. Another important problem is Non-uniform Sparsest Cut – where we want to minimize the ratio of the cost of the edges cut, to the *weighted* sum of the pairs cut[2] – which has important applications to metric embeddings. As with the Balanced Separator problem (or the Sparsest cut problem), the approximability of this problem is poorly understood. Arora-Lee-Naor [ALN08] showed that the simple SDP relaxation with triangle inequalities achieves an approximation ratio of $\tilde{O}(\sqrt{\log n})$. However, the best known lower bound on the integrality gap of this SDP is $\Omega((\log n)^{\delta})$ for some tiny constant δ, from the work of Cheeger-Kleiner-Naor [CKN09]. Closing this gap remains a very interesting open question.

In [MS11], we prove an improved lower bound of $\tilde{\Omega}((\log n)^{1/3})$ for a slightly weaker SDP, which actually suffices for the upper bound from [ALN08]. We hope that some of these techniques and the understanding gained about the problem could help close the gap.

In [ST11], we asked the following simple question that we think is interesting: Given a particular integrality gap instance for Sparsest Cut (or Balanced Separator) for one of the standard SDP relaxations for the problem[3], can we generate a family of instances of increasing sizes where this gap is at least preserved? Observe that for problems like 3SAT, the question is trivial – by simply combining independent copies of the original instance. Such a simple approach does not work for Sparsest Cut (or Balanced Separator). In [ST11], we prove that if we take the cartesian products of the original instance, the gap is preserved exactly[4].

3. Lower Bounds to Approximability

Hardness of approximation has been a very successful area of research in the last two decades, giving optimal inapproximability results for several problems. Nevertheless, for some important problems like Sparsest Cut and Vertex Cover, such optimal results still seem out of reach.

[2] If one assigns uniform weights to all the pairs, we recover the (Uniform) Sparsest Cut problem defined above

[3] The SDP could derived from any level of any of the known hierarchies – Sherali-Adams SDP/Lasserre hierarchy.

[4] In [ST11], we also prove statements analogous to the KKL theorem and the Friedgut Junta theorem for graphs obtained by applying the cartesian product.

For Sparsest Cut, the situation is dire. Though the best approximation known is $O(\sqrt{\log n})$ by Arora-Rao-Vazirani [ARV09], we can only rule out a PTAS under standard complexity assumptions [Kho06, AMS11]. Research efforts have thus focused on proving lower bounds for classes of SDP algorithms that include [ARV09]. I have discussed my work in this area in the last section.

For Vertex Cover, where a 2 approximation is classic but the best inapproximability factor known is 1.36 by Dinur-Safra [DS04], proving an optimal inapproximability result is a major open problem. Optimal inapproximability is however known under the Unique Games Conjecture (UGC) of Khot [Kho02] by the work of Khot-Regev [KR08]. Though the UGC implies a lot of new optimal inapproximability results, in light of some recent results [ABS10, BRS11], its status is uncertain and leaves something to be desired. I am interested in proving some of these inapproximability results based on standard assumptions, e.g. $\mathbf{P} \neq \mathbf{NP}$, without assuming the UGC. Below, I discuss some of my work in this direction around variants of Vertex Cover, and some scheduling problems.

Near-optimal Hardness for Vertex Cover on k-uniform k-partite Hypergraphs. For variants of the Vertex Cover problem, the situation is similar – simple approximations are known, but proving optimal hardness remains a challenge. For Hypergraph Vertex Cover on k-uniform hypergraphs, Dinur et al. [DGKR05] proved a $k-1$ factor hardness (as compared to a well known k-approximation, and optimal inapproximability assuming UGC [KR08]). In [SS11], we show that even for Hypergraph Vertex Cover on k-uniform k-partite hypergraphs, where a $\frac{k}{2}$-approximation, and optimal inapproximability assuming the UGC (Guruswami-Saket [GS10]) are known, we can prove **NP**-hardness of approximating within $\frac{k}{2} - 1$, improving on factor of $\frac{k}{4}$ by [GS10].

Optimal Inapproximability for Scheduling Problems. For some classic scheduling problems too viz. Concurrent Open Shop and the Assembly Line problem, optimal 2-inapproximability is known assuming UGC [BK10], whereas the best NP-hardness factors known are $\frac{6}{5}$ [MQS$^+$10] and $1 + \epsilon$ (for a small $\varepsilon > 0$) respectively. In a recent work [SS12], we prove an optimal NP-hardness factor of 2, without assuming the UGC.

4. Provable Algorithms instead of Heuristics

I am interested in applying the techniques from TCS to important problems in other fields, e.g. machine learning, where the algorithms used in practice often have no proven guarantees though they seem to work very well. For these problems, the worst-case paradigm is often too stringent, and the attempts at average-case paradigm are unsatisfactory. An approach we have successfully applied to the problems below, is to make justifiable assumptions about the input, motivated by the application, and give algorithms with provable guarantees for the restricted input class.

Independent Component Analysis. A classic problem in Signal Analysis is the *Cocktail Party* problem: There are n microphones at a party, picking up instantaneous combinations of n conversations. The goal is to recover the individual conversations. A particular formalization and approach to this question is called Independent Component Analysis (ICA). In ICA, we are given several samples of the form $y = Ax + \eta$, where x is a random variable with independent components, η is an independent random noise vector, and the goal is to recover A. ICA has a rich literature and several empirical approaches are known. However, to the best of our knowledge, the only provable algorithm with a finite sample size is from Frieze-Jerrum-Kannan [FJK96], in the case $\eta = 0$.

In [AGMS12], we give a polynomial time algorithm that provably recovers A when η is an independent gaussian with an unknown covariance matrix. Our contribution is two fold: (1) We deal with noise that is an unknown gaussian. (2) We give the first rigorous analysis for recovering all of A, avoiding a blow-up in the error. The work of [FJK96] recovers just one column of A, and the naive way of recovering all of them incurs exponential blow-up in the error.

Identifying Communities. With the rise of online social networks, the challenge of extracting useful information from these graphs has gained importance. One natural way to achieve this goal is to *explain* the structure of the graph as a union of *communities*. The question is old, but several works in the literature have tried to identify a community as a cut of low conductance, with the assumption that all the underlying communities are disjoint. Unfortunately, this modeling choice often leads to problems that are computationally hard, and no objective measure of whether the low conductance cuts found (by heuristics or approximation) represent meaningful communities.

In [AGSS12], we give a different approach to the problem. We allow our graph to be described by overlapping communities with certain natural assumptions on the local structure of the communities. Under these assumptions, we provably and efficiently recover the underlying communities. Our assumptions about the structure of the communities accord with the sociology literature and our current understanding of social networks.

5. Future Directions

Near-Linear Time Graph Algorithms. I am interested in extending the techniques from [OSV12] to obtain near-linear time graph algorithms. Our work in [OSV12] falls in what has been called the *Laplacian Paradigm*, that uses the near-linear time Laplacian system solver by Spielman and Teng [ST11] to design fast algorithms. In [OSV12, SV12], we show that one could as well use the matrix exponential, which can be directly applied to obtain fast SDP algorithms using the Arora-Kale[AK07] matrix multiplicative weight framework. I would like to employ these techniques to other interesting problems. There are several problems that I find exciting here:

(1) Currently [OSV12] just finds one balanced cut in near-linear time. Can we in fact find a decomposition of the graph into *expanders* in near-linear time? Such a result might be of practical significance.
(2) Can we give a near-linear time algorithm that achieves the best known $\Theta(\sqrt{\log n})$ approximation to Balanced Separator?
(3) One of the biggest challenges in this area is to obtain (approximate) max-flow in near-linear time. This would also answer the previous question (using the work of Sherman [She09]).

Power of Lasserre Hierarchy for Graph Partitioning. Another important direction that interests me is the applications of strong SDP hierarchies (Lasserre hierarchy in particular) towards improved approximation algorithms for problems like Sparsest Cut and Small Set Expansion. There have recently been a few positive results [BRS11, GS11], but we essentially have no real lower bounds beyond a few constant rounds of the hierarchy. These are very challenging questions, but there is a lot to be understood about these problems.

Constant-quality Vertex Sparsifiers. A graph $H = (W \cup T, F)$ is called a Vertex Sparsifier for $G = (V \cup T, E)$, if for every bi-partition of T, the costs of min-cuts separating this bi-partition in G and H are same up to a factor of Q, called the quality. After a sequence of works [Moi09, LM10, EGK$^+$10, CLLM10], we know how to efficiently construct sparsifiers with $Q = \tilde{O}(\log n)$ and $W = \emptyset$. A recent work of Chuzhoy [Chu12] showed that in $2^{|T|}\mathrm{poly}(n)$ time, we could construct vertex sparsifiers with $Q = O(1)$, and $|W| = \mathrm{poly}(|T|)$. There are several interesting open questions here that I am working on with Julia Chuzhoy and Yury Makarychev:

(1) Can we efficiently construct *small* sparsifiers, $|W| = \mathrm{poly}(|T|)$, with constant quality?
(2) What is the optimal size-quality tradeoff? Can it be achieved efficiently?

Hardness of Hypergraph Vertex Cover. In [SS12], we extended the powerful techniques from Fourier analysis that were used in the work of Dinur-Safra [DS04]. I believe these techniques can be applied for other structural hardness results for Hypergraph Vertex Cover and their applications, and in particular to a previous work of Bansal and Khot [BK09].

Independent Component Analysis. Our work for ICA in [AGMS12] has two disadvantages: (1) It is not robust to the presence of a tiny fraction of outliers. (2) The time bounds are impractical. I am interested in designing a robust and efficient algorithm with provable guarantees, which could have significant practical applications.

References

[ABS10] Sanjeev Arora, Boaz Barak, and David Steurer. Subexponential algorithms for unique games and related problems. FOCS, pages 563–572, Washington, DC, USA, 2010. IEEE Computer Society.

[AGMS12] Sanjeev Arora, Rong Ge, Ankur Moitra, and Sushant Sachdeva. Provable ICA with unknown gaussian noise, with implications for gaussian mixtures and autoencoders. To appear at NIPS, 2012.

[AGSS12] Sanjeev Arora, Rong Ge, Sushant Sachdeva, and Grant Schoenebeck. Finding overlapping communities in social networks: toward a rigorous approach. EC, pages 37–54, 2012.

[AK07] Sanjeev Arora and Satyen Kale. A combinatorial, primal-dual approach to semidefinite programs. STOC, pages 227–236, 2007.

[ALN08] Sanjeev Arora, James R. Lee, and Assaf Naor. Euclidean distortion and the sparsest cut. *J. Amer. Math. Soc.*, 21:1–21, 2008.

[AMS11] Christoph Ambühl, Monaldo Mastrolilli, and Ola Svensson. Inapproximability results for maximum edge biclique, minimum linear arrangement, and sparsest cut. *SIAM J. Comput.*, 40(2):567–596, April 2011.

[AP09] Reid Andersen and Yuval Peres. Finding sparse cuts locally using evolving sets. STOC, pages 235–244, 2009.

[ARV09] Sanjeev Arora, Satish Rao, and Umesh Vazirani. Expander flows, geometric embeddings and graph partitioning. *J. ACM*, 56(2):5:1–5:37, April 2009.

[BK09] N. Bansal and S. Khot. Optimal long code test with one free bit. In *FOCS*, pages 453–462, 2009.

[BK10] N. Bansal and S. Khot. Inapproximability of hypergraph vertex cover and applications to scheduling problems. In *ICALP*, pages 250–261, 2010.

[BRS11] Boaz Barak, Prasad Raghavendra, and David Steurer. Rounding semidefinite programming hierarchies via global correlation. FOCS, pages 472–481, 2011.

[Chu12] Julia Chuzhoy. On vertex sparsifiers with steiner nodes. STOC, pages 673–688, 2012.

[CKN09] Jeff Cheeger, Bruce Kleiner, and Assaf Naor. A $(\log n)^{\Omega(1)}$ integrality gap for the sparsest cut sdp. FOCS, pages 555–564, 2009.

[CLLM10] Moses Charikar, Tom Leighton, Shi Li, and Ankur Moitra. Vertex sparsifiers and abstract rounding algorithms. FOCS, pages 265–274, 2010.

[DGKR05] I. Dinur, V. Guruswami, S. Khot, and O. Regev. A new multilayered pcp and the hardness of hypergraph vertex cover. *SIAM J. Comput.*, 34:1129–1146, May 2005.

[DS04] I. Dinur and S. Safra. On the hardness of approximating minimum vertex cover. *Annals of Mathematics*, 162:439–485, 2004.

[EGK$^+$10] Matthias Englert, Anupam Gupta, Robert Krauthgamer, Harald Räcke, Inbal Talgam-Cohen, and Kunal Talwar. Vertex sparsifiers: new results from old techniques. APPROX/RANDOM, pages 152–165, 2010.

[FJK96] Alan M. Frieze, Mark Jerrum, and Ravi Kannan. Learning linear transformations. In *FOCS*, pages 359–368, 1996.

[GS10] V. Guruswami and R. Saket. On the inapproximability of vertex cover on k-partite k-uniform hypergraphs. In *ICALP*, pages 360–371, 2010.

[GS11] Venkatesan Guruswami and Ali Kemal Sinop. Lasserre hierarchy, higher eigenvalues, and approximation schemes for graph partitioning and quadratic integer programming with psd objectives. FOCS, pages 482–491, 2011.

[Kho02] Subhash Khot. On the power of unique 2-prover 1-round games. STOC, pages 767–775, 2002.

[Kho06] Subhash Khot. Ruling out ptas for graph min-bisection, dense k-subgraph, and bipartite clique. *SIAM J. Comput.*, 36(4):1025–1071, December 2006.

[KR08] S. Khot and O. Regev. Vertex cover might be hard to approximate to within 2-epsilon. *J. Comput. Syst. Sci.*, 74(3):335–349, 2008.

[LM10] F. Thomson Leighton and Ankur Moitra. Extensions and limits to vertex sparsification. STOC, pages 47–56, 2010.

[Moi09] Ankur Moitra. Approximation algorithms for multicommodity-type problems with guarantees independent of the graph size. FOCS, pages 3–12, 2009.

[MQS$^+$10] M. Mastrolilli, M. Queyranne, A. S. Schulz, O. Svensson, and N. A. Uhan. Minimizing the sum of weighted completion times in a concurrent open shop. *Oper. Res. Lett.*, 38(5):390–395, September 2010.

[MS11] Mohammad Moharrami and Sushant Sachdeva. Power of weak v/s strong triangle inequalities. *Manuscript*, 2011.

[OSV12] Lorenzo Orecchia, Sushant Sachdeva, and Nisheeth K. Vishnoi. Approximating the exponential, the lanczos method and an $\tilde{O}(m)$-time spectral algorithm for balanced separator. STOC, pages 1141–1160, 2012.

[OV11] Lorenzo Orecchia and Nisheeth K. Vishnoi. Towards an sdp-based approach to spectral methods: A nearly-linear-time algorithm for graph partitioning and decomposition. In *SODA*, pages 532–545, 2011.

[She09] Jonah Sherman. Breaking the multicommodity flow barrier for $O(\sqrt{\log n})$-approximations to sparsest cut. FOCS, 2009.

[SS11] Sushant Sachdeva and Rishi Saket. Nearly optimal np-hardness of vertex cover on k-uniform k-partite hypergraphs. In *APPROX-RANDOM*, volume 6845 of *Lecture Notes in Computer Science*, pages 327–338, 2011.

[SS12] Sushant Sachdeva and Rishi Saket. Optimal inapproximability for scheduling problems via structural hardness for hypergraph vertex cover. *In preparation*, 2012.

[ST04] Daniel A. Spielman and Shang-Hua Teng. Nearly-linear time algorithms for graph partitioning, graph sparsification, and solving linear systems. STOC, pages 81–90, 2004.

[ST11] Sushant Sachdeva and Madhur Tulsiani. Cuts in cartesian products of graphs. *CoRR*, abs/1105.3383, 2011.

[SV12] Sushant Sachdeva and Nisheeth K. Vishnoi. Inversion is as easy as exponentiation. *In preparation*, 2012.