Transmission Line Calculator: Formulas and Calculations

Transmission Line Parameters

The calculator computes the fundamental transmission line parameters—resistance per unit length (R'), inductance per unit length (L'), conductance per unit length (G'), and capacitance per unit length (C')—based on the geometry and material properties. These parameters are then used to determine the characteristic impedance (Z_0).

Constants

- Speed of light: $c = 2.99792458 \times 10^8\,\mathrm{m/s}$
- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$
- Permittivity of free space: $\epsilon_0 = 8.85418782 \times 10^{-12}\,\mathrm{F/m}$
- Angular frequency: $\omega = 2\pi f$, where f is frequency in Hz
- Surface resistance: $R_s = \sqrt{\dfrac{\pi f \mu_c}{\sigma_c}}$, where μ_c is conductor permeability (assumed μ_0) and σ_c is conductor conductivity (S/m)

Material Properties

- Dielectric permeability: $\mu = \mu_r \mu_0$
- Dielectric permittivity: $\epsilon = \epsilon_r \epsilon_0$
- Dielectric conductivity: σ_d (S/m)

0.1 Coaxial Line

- **Resistance per unit length (R'):**

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$

  a: Inner conductor radius (m)
  b: Outer conductor radius (m)

- **Inductance per unit length (L'):**

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$$

- **Conductance per unit length (G'):**

$$G' = \frac{2\pi\sigma_d}{\ln\left(\dfrac{b}{a}\right)}$$

- **Capacitance per unit length (C'):**

$$C' = \frac{2\pi\epsilon}{\ln\left(\dfrac{b}{a}\right)}$$

- **Characteristic impedance (Z_0):**

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right)$$

0.2 Two-Wire Line

- **Resistance per unit length (R'):**

$$R' = \frac{R_s}{\pi a}$$

  a: Wire radius (m)

- **Inductance per unit length (L'):**

$$L' = \frac{\mu}{\pi} \ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)$$

  D: Wire spacing (m)
  Approximates to $L' \approx \frac{\mu}{\pi} \ln\left(\frac{2D}{a}\right)$ for large D/a.

- **Conductance per unit length (G'):**

$$G' = \frac{\pi \sigma_d}{\ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)}$$

- **Capacitance per unit length (C'):**

$$C' = \frac{\pi \epsilon}{\ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)}$$

- **Characteristic impedance (Z_0):**

$$Z_0 = \frac{120}{\sqrt{\epsilon_r}} \ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)$$

0.3 Parallel-Plate Line

- **Resistance per unit length (R'):**

$$R' = \frac{2R_s}{w}$$

  w: Plate width (m)

- **Inductance per unit length (L'):**

$$L' = \frac{\mu h}{w}$$

  h: Plate spacing (m)

- **Conductance per unit length (G'):**

$$G' = \frac{\sigma_d w}{h}$$

- **Capacitance per unit length (C'):**

$$C' = \frac{\epsilon w}{h}$$

- **Characteristic impedance (Z_0):**

$$Z_0 = \frac{377h}{w\sqrt{\epsilon_r}}$$

0.4 Microstrip Line

- **Effective relative permittivity (ϵ_{eff}):**

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left(1 + \frac{10}{s}\right)^{-xy}$$

  where:

$$x = 0.56\left(\frac{\epsilon_r - 0.9}{\epsilon_r + 3}\right)^{0.05}$$

$$y = 1 + 0.02\ln\left(\frac{s^4 + 3.7\times 10^{-4}s^2}{s^4 + 0.43}\right) + 0.05\ln(1 + 1.7\times 10^{-4}s^3)$$

$$s = \frac{w}{h}$$

  w: Strip width (m)
  h: Substrate height (m)

- **Characteristic impedance (Z_0):**

$$Z_0 = \frac{60}{\sqrt{\epsilon_{eff}}}\ln\left(\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right)$$

  where:

$$t = \left(\frac{30.67}{s}\right)^{0.75}$$

- **Phase velocity (v_p):**

$$v_p = \frac{c}{\sqrt{\epsilon_{eff}}}$$

- **Wavelength (λ):**

$$\lambda = \frac{v_p}{f}$$

- **Capacitance per unit length (C'):**

$$C' = \frac{\sqrt{\epsilon_{eff}}}{Z_0 c}$$

- **Inductance per unit length (L'):**

$$L' = Z_0^2 C'$$

- **Resistance and Conductance:** For ideal conductors and ideal dielectrics:

$$R' \approx 0, \quad G' \approx 0$$

1 Propagation Parameters

These parameters describe wave propagation along the transmission line, including attenuation and phase characteristics.

- **Propagation constant (γ):**

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

  For low-loss lines (approximation used in `PropagationAnalysis`):

$$\alpha \approx \frac{R'}{2Z_0} + \frac{G' Z_0}{2}$$

$$\beta \approx \omega\sqrt{L'C'}$$

  α: Attenuation constant (Np/m)
  β: Phase constant (rad/m)

- **Phase velocity (v_p):**

$$v_p = \frac{c}{\sqrt{\epsilon_r}}$$

For microstrip:

$$v_p = \frac{c}{\sqrt{\epsilon_{eff}}}$$

- **Wavelength (λ):**

$$\lambda = \frac{v_p}{f}$$

- **Attenuation in dB/m:**

$$\text{Attenuation (dB/m)} = 8.686 \times \alpha$$

- **Skin depth (δ):**

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$$

2 Circuit Properties

Circuit properties depend on the load impedance Z_L, line length d, and transmission line parameters.

- **Reflection coefficient (Γ):**

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$Z_L = Z_{L\text{real}} + jZ_{L\text{imag}}$ (complex load impedance in ohms)
Magnitude: $|\Gamma| = \sqrt{\Gamma_{\text{real}}^2 + \Gamma_{\text{imag}}^2}$
Angle: $\theta_\Gamma = \tan^{-1}\left(\frac{\Gamma_{\text{imag}}}{\Gamma_{\text{real}}}\right) \times \frac{180}{\pi}$ (degrees)

- **Voltage Standing Wave Ratio (VSWR):**

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Protected against division by zero with a small epsilon ($\epsilon = 0.000001$).

- **Wave impedance ($Z(d)$):** General case:

$$Z(d) = Z_0 \frac{1 + \Gamma e^{-2\gamma d}}{1 - \Gamma e^{-2\gamma d}}$$

$e^{-2\gamma d} = e^{-2\alpha d}(\cos(-2\beta d) + j\sin(-2\beta d))$
Short circuit ($Z_L = 0$):

$$Z(d) = Z_0 \tanh(\gamma d)$$

$$\tanh(\gamma d) = \frac{\sinh(\alpha d)\cos(\beta d) + j\cosh(\alpha d)\sin(\beta d)}{\cosh(\alpha d)\cos(\beta d) + j\sinh(\alpha d)\sin(\beta d)}$$

Open circuit ($Z_L \approx \infty$):

$$Z(d) = Z_0 \coth(\gamma d)$$

$$\coth(\gamma d) = \frac{\cosh(\alpha d)\cos(\beta d) - j\sinh(\alpha d)\sin(\beta d)}{\sinh(\alpha d)\cos(\beta d) + j\cosh(\alpha d)\sin(\beta d)}$$

3 Visualizations

The calculator provides multiple visualizations to help understand transmission line behavior.

3.1 Standing Wave Pattern

The standing wave pattern is visualized by plotting the voltage magnitude along the line.

- **Voltage magnitude ($|V(d)|$):**

$$|V(d)| = |V_0^+|\sqrt{1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_\Gamma)}$$

  $|V_0^+|$: Incident voltage magnitude (assumed 1 for plotting)
  θ_Γ: Reflection coefficient angle (radians)
  Maxima and minima are found numerically by sampling and identifying peaks/troughs.

- **Instantaneous voltage:**

$$V(d,t) = \cos(\omega t - \beta d) + |\Gamma|\cos(\omega t + \beta d - \theta_\Gamma)$$

  Animated over time t from 0 to 2π.

3.2 Propagating Wave Visualization

This visualization shows the incident and reflected waves propagating on the transmission line, and their superposition.

- **Incident wave:**

$$V_{inc}(d,t) = \cos(\omega t - \beta d)$$

- **Reflected wave:**

$$V_{ref}(d,t) = |\Gamma|\cos(\omega t + \beta d - \theta_\Gamma)$$

- **Total wave:**

$$V_{total}(d,t) = V_{inc}(d,t) + V_{ref}(d,t)$$

The animation shows the waves traveling in opposite directions and their interference pattern, which creates the standing wave.

3.3 Frequency Response

The frequency response visualization shows how the characteristic impedance (Z_0) varies with frequency.

- For each geometry type, the calculator plots Z_0 over a frequency range from $0.5f$ to $2f$, where f is the current operating frequency.
- For microstrip lines, the effective permittivity ϵ_{eff} varies with frequency, causing the characteristic impedance to change as well.
- The plot helps in identifying the optimal operating frequency range for a given line configuration.

Overall Analysis

Functionality

- **Calculator Component**: Computes R', L', G', C', Z_0, γ, v_p, and λ based on user inputs (geometry, frequency, material properties). Handles input validation and updates parameters dynamically.
- **CircuitProperties Component**: Calculates Γ, VSWR, and $Z(d)$ using Z_0 and γ from the calculator, incorporating user-defined Z_L and d. Supports complex loads, short, and open circuits.
- **PropagationAnalysis Component**: Provides detailed propagation metrics (α, β, attenuation in dB/m, skin depth) using a low-loss approximation for α.
- **StandingWave Component**: Visualizes the standing wave pattern, identifying voltage maxima and minima numerically, and animates the instantaneous voltage.
- **PropagatingWaveVisualizer Component**: Shows the traveling incident and reflected waves and their combination in real-time, helping to understand how standing waves form.
- **FrequencyResponse Component**: Visualizes how the characteristic impedance changes with frequency for different transmission line geometries.

Microstrip Line Analysis

The microstrip line implementation presents unique challenges due to its quasi-TEM nature:

- **Effective Permittivity**: Unlike other geometries, microstrip lines use an effective permittivity (ϵ_{eff}) to account for the mixed air/dielectric environment.
- **Frequency Dependence**: The effective permittivity and hence the characteristic impedance have stronger frequency dependence compared to other transmission line types.
- **Approximations**: The calculator uses curve-fit approximations (Wheeler's equations with modifications) for the effective permittivity and characteristic impedance calculations, accurate within approximately 2% for most practical microstrip configurations.
- **Limitations**: The calculator assumes a thin conductor (thickness much less than strip width) and ignores dispersion effects at very high frequencies.

Assumptions and Approximations

- **Lossless Approximation**: Z_0 formulas assume negligible R' and G', valid for low-loss lines.
- **Low-Loss Approximation**: α and β are approximated in `PropagationAnalysis`, suitable for lines where losses are small.
- **Numerical Methods**: Voltage maxima/minima are determined by sampling rather than analytical solutions, ensuring accuracy for lossy lines but introducing computational overhead.
- **Microstrip Assumptions**: For microstrip lines, $R' \approx 0$ and $G' \approx 0$ are assumed, which is valid for good conductors and good dielectrics at moderate frequencies.

Potential Issues

- **Units**: Geometric inputs are in millimeters, converted to meters (multiplied by 10^{-3}) for consistency with SI units.
- **Error Handling**: Division by zero and invalid inputs are managed with checks (e.g., $a > 0$, $b > a$), setting parameters to ∞ or NaN as needed.
- **Microstrip Width/Height Ratio**: The microstrip formulas become less accurate for extreme width-to-height ratios (s ¡ 0.1 or s ¿ 10).
- **Animation Performance**: The wave animations in StandingWave and PropagatingWaveVisualizer components may impact performance on older devices.

Practical Applications

This calculator is a versatile tool for engineers designing RF and microwave circuits, enabling analysis of line behavior under various conditions. It supports educational purposes by illustrating wave propagation and impedance concepts visually and numerically. The addition of microstrip line calculations makes it especially useful for PCB-based RF design.

Notes

- Verify the two-wire R' formula against design requirements, as the discrepancy may affect loss calculations.
- For high-loss lines, use the exact γ formula instead of approximations to ensure accuracy.
- All calculations assume SI units after conversion from user inputs.
- For microstrip lines, more advanced models may be needed above 10 GHz to account for dispersion effects.
- The visualizations provide qualitative insights but may not capture all subtleties of actual transmission line behavior, especially in high-frequency applications.

This document provides a complete reference for the formulas and their implementation, offering insights into the calculator's capabilities and limitations.