Transmission Line Calculator: Formulas and Calculations

Transmission Line Parameters

The calculator computes the fundamental transmission line parameters—resistance per unit length (R'), inductance per unit length (L'), conductance per unit length (G'), and capacitance per unit length (C')—based on the geometry and material properties. These parameters are then used to determine the characteristic impedance (Z_0).

Constants

- Speed of light: $c = 2.99792458 \times 10^8\,\mathrm{m/s}$
- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$
- Permittivity of free space: $\epsilon_0 = 8.85418782 \times 10^{-12}\,\mathrm{F/m}$
- Angular frequency: $\omega = 2\pi f$, where f is frequency in Hz
- Surface resistance: $R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$, where μ_c is conductor permeability (assumed μ_0) and σ_c is conductor conductivity (S/m)

Material Properties

- Dielectric permeability: $\mu = \mu_r \mu_0$
- Dielectric permittivity: $\epsilon = \epsilon_r \epsilon_0$
- Dielectric conductivity: σ_d (S/m)

0.1 Coaxial Line

- **Resistance per unit length (R'):**

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$

  a: Inner conductor radius (m)
  b: Outer conductor radius (m)

- **Inductance per unit length (L'):**

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$$

- **Conductance per unit length (G'):**

$$G' = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)}$$

- **Capacitance per unit length (C'):**

$$C' = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}$$

- **Characteristic impedance (Z_0):**

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right)$$

0.2 Two-Wire Line

- **Resistance per unit length (R'):**

$$R' = \frac{R_s}{\pi a}$$

  a: Wire radius (m)

  Note: Standard references suggest $R' = \frac{2R_s}{\pi a}$, but the code implements $R' = \frac{R_s}{\pi a}$, potentially an error.

- **Inductance per unit length (L'):**

$$L' = \frac{\mu}{\pi} \ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)$$

  D: Wire spacing (m)

  Approximates to $L' \approx \frac{\mu}{\pi} \ln\left(\frac{2D}{a}\right)$ for large D/a.

- **Conductance per unit length (G'):**

$$G' = \frac{\pi \sigma_d}{\ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)}$$

- **Capacitance per unit length (C'):**

$$C' = \frac{\pi \epsilon}{\ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)}$$

- **Characteristic impedance (Z_0):**

$$Z_0 = \frac{120}{\sqrt{\epsilon_r}} \ln\left(\frac{D}{a} + \sqrt{\left(\frac{D}{a}\right)^2 - 1}\right)$$

0.3 Parallel-Plate Line

- **Resistance per unit length (R'):**

$$R' = \frac{2R_s}{w}$$

  w: Plate width (m)

- **Inductance per unit length (L'):**

$$L' = \frac{\mu h}{w}$$

  h: Plate spacing (m)

- **Conductance per unit length (G'):**

$$G' = \frac{\sigma_d w}{h}$$

- **Capacitance per unit length (C'):**

$$C' = \frac{\epsilon w}{h}$$

- **Characteristic impedance (Z_0):**

$$Z_0 = \frac{377 h}{w\sqrt{\epsilon_r}}$$

1 Propagation Parameters

These parameters describe wave propagation along the transmission line, including attenuation and phase characteristics.

- **Propagation constant (γ):**
$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$
For low-loss lines (approximation used in `PropagationAnalysis`):
$$\alpha \approx \frac{R'}{2Z_0} + \frac{G' Z_0}{2}$$
$$\beta \approx \omega\sqrt{L'C'}$$
α: Attenuation constant (Np/m)
β: Phase constant (rad/m)
- **Phase velocity (v_p):**
$$v_p = \frac{c}{\sqrt{\epsilon_r}}$$
- **Wavelength (λ):**
$$\lambda = \frac{v_p}{f}$$
- **Attenuation in dB/m:**
$$\text{Attenuation (dB/m)} = 8.686 \times \alpha$$
- **Skin depth (δ):**
$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$$

2 Circuit Properties

Circuit properties depend on the load impedance Z_L, line length d, and transmission line parameters.

- **Reflection coefficient (Γ):**
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$Z_L = Z_{L\text{real}} + jZ_{L\text{imag}}$ (complex load impedance in ohms)
Magnitude: $|\Gamma| = \sqrt{\Gamma_{\text{real}}^2 + \Gamma_{\text{imag}}^2}$
Angle: $\theta_\Gamma = \tan^{-1}\left(\frac{\Gamma_{\text{imag}}}{\Gamma_{\text{real}}}\right) \times \frac{180}{\pi}$ (degrees)
- **Voltage Standing Wave Ratio (VSWR):**
$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$
Protected against division by zero with a small epsilon ($\epsilon = 0.000001$).
- **Wave impedance ($Z(d)$):** General case:
$$Z(d) = Z_0 \frac{1 + \Gamma e^{-2\gamma d}}{1 - \Gamma e^{-2\gamma d}}$$
$e^{-2\gamma d} = e^{-2\alpha d}(\cos(-2\beta d) + j\sin(-2\beta d))$
Short circuit ($Z_L = 0$):
$$Z(d) = Z_0 \tanh(\gamma d)$$
$$\tanh(\gamma d) = \frac{\sinh(\alpha d)\cos(\beta d) + j\cosh(\alpha d)\sin(\beta d)}{\cosh(\alpha d)\cos(\beta d) + j\sinh(\alpha d)\sin(\beta d)}$$
Open circuit ($Z_L \approx \infty$):
$$Z(d) = Z_0 \coth(\gamma d)$$
$$\coth(\gamma d) = \frac{\cosh(\alpha d)\cos(\beta d) - j\sinh(\alpha d)\sin(\beta d)}{\sinh(\alpha d)\cos(\beta d) + j\cosh(\alpha d)\sin(\beta d)}$$

3 Standing Wave Pattern

The standing wave pattern is visualized by plotting the voltage magnitude along the line.

- **Voltage magnitude ($|V(d)|$):**

$$|V(d)| = |V_0^+|\sqrt{1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_\Gamma)}$$

  $|V_0^+|$: Incident voltage magnitude (assumed 1 for plotting)
  θ_Γ: Reflection coefficient angle (radians)
  Maxima and minima are found numerically by sampling and identifying peaks/troughs.

- **Instantaneous voltage:**

$$V(d,t) = \cos(\omega t - \beta d) + |\Gamma|\cos(\omega t + \beta d - \theta_\Gamma)$$

  Animated over time t from 0 to 2π.

Overall Analysis

Functionality

- **Calculator Component**: Computes R', L', G', C', Z_0, γ, v_p, and λ based on user inputs (geometry, frequency, material properties). Handles input validation and updates parameters dynamically.
- **CircuitProperties Component**: Calculates Γ, VSWR, and $Z(d)$ using Z_0 and γ from the calculator, incorporating user-defined Z_L and d. Supports complex loads, short, and open circuits.
- **PropagationAnalysis Component**: Provides detailed propagation metrics (α, β, attenuation in dB/m, skin depth) using a low-loss approximation for α.
- **StandingWave Component**: Visualizes the standing wave pattern, identifying voltage maxima and minima numerically, and animates the instantaneous voltage.

Assumptions and Approximations

- **Lossless Approximation**: Z_0 formulas assume negligible R' and G', valid for low-loss lines.
- **Low-Loss Approximation**: α and β are approximated in `PropagationAnalysis`, suitable for lines where losses are small.
- **Numerical Methods**: Voltage maxima/minima are determined by sampling rather than analytical solutions, ensuring accuracy for lossy lines but introducing computational overhead.

Potential Issues

- **Two-Wire R'**: The code uses $R' = \frac{R_s}{\pi a}$, whereas standard theory suggests $R' = \frac{2R_s}{\pi a}$, potentially underestimating resistance by a factor of 2.
- **Units**: Geometric inputs are in millimeters, converted to meters (multiplied by 10^{-3}) for consistency with SI units.
- **Error Handling**: Division by zero and invalid inputs are managed with checks (e.g., $a > 0$, $b > a$), setting parameters to ∞ or NaN as needed.

Practical Applications

This calculator is a versatile tool for engineers designing RF and microwave circuits, enabling analysis of line behavior under various conditions. It supports educational purposes by illustrating wave propagation and impedance concepts visually and numerically.

Notes

- Verify the two-wire R' formula against design requirements, as the discrepancy may affect loss calculations.
- For high-loss lines, use the exact γ formula instead of approximations to ensure accuracy.
- All calculations assume SI units after conversion from user inputs.

This document provides a complete reference for the formulas and their implementation, offering insights into the calculator's capabilities and limitations.