IP Experiment No. 10: Discrete Cosine Transform and Image Compression

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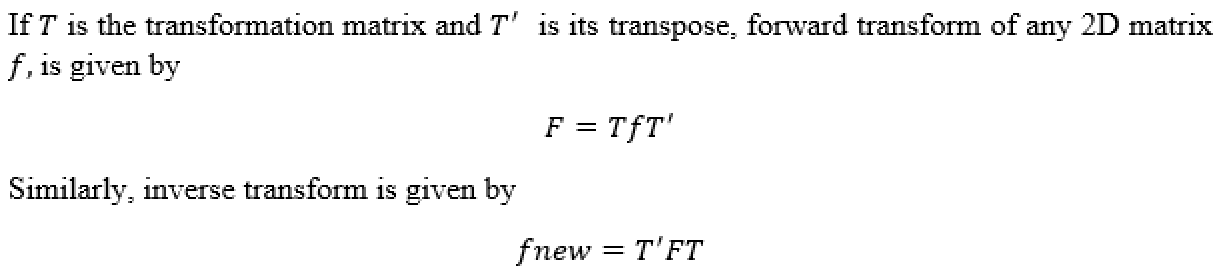
Aim:

1. Apply 2D-Discrete Cosine Transform on the given test image.
2. Obtain compression ratio between the test image and reconstructed image using selected number of highest energy coefficients of the transformed image are retained. Comment on the subjective quality of the reconstructed image.

Theory:



If T is the transformation matrix and T' is its transpose, forward transform of any 2D matrix f, is given by

$$F = TfT'$$

Similarly, inverse transform is given by

$$fnew = T'FT$$

DCT Transform

1. Real valued and unitary
2. Basis vectors are sampled form of cosine signal
3. Widely used for image compression

```
import cv2
import numpy as np
import scipy
import matplotlib.pyplot as plt
from scipy.fftpack import dct
from scipy.fftpack import idct
```

To generate the DCT transform matrix

$$F(u,v)=\alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}f(x,y)\cos\left(\frac{(2x+1)\pi u}{2N}\right)\cos\left(\frac{(2y+1)\pi v}{2N}\right)$$

$$\alpha(u)=\begin{cases}\sqrt{\frac{1}{N}} if u=0\\ \sqrt{\frac{2}{N}} if u\neq 0\end{cases},\alpha(v)=\begin{cases}\sqrt{\frac{1}{N}} if v=0\\ \sqrt{\frac{2}{N}} if v\neq 0\end{cases}$$

A 4 x 4 DCT cosine transform

$$A=1/\sqrt{2}\begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}\\ \cos(\pi/8) & \cos(3\pi/8) & \cos(5\pi/8) & \cos(7\pi/8)\\ \cos(2\pi/8) & \cos(6\pi/8) & \cos(10\pi/8) & \cos(14\pi/8)\\ \cos(3\pi/8) & \cos(9\pi/8) & \cos(15\pi/8) & \cos(21\pi/8)\end{bmatrix}=\begin{bmatrix}0.5 & 0.5 & 0.5 & 0.5\\ 0.653 & 0.271 & -0.271 & -0.653\\ 0.5 & -0.5 & -0.5 & 0.5\\ 0.271 & -0.653 & 0.653 & -0.271\end{bmatrix}$$

To compute DFT of 1D matrix

$$F=S\,.\,f^T$$

$$=\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ \frac{3}{\sqrt{5}} & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & -\frac{3}{\sqrt{5}}\\ 1 & -1 & -1 & 1\\ \frac{1}{\sqrt{5}} & -\frac{3}{\sqrt{5}} & \frac{3}{\sqrt{5}} & -\frac{1}{\sqrt{5}}\end{bmatrix}\begin{bmatrix}1\\2\\2\\1\end{bmatrix} \qquad F=\begin{bmatrix}3\\0\\-1\\0\end{bmatrix}$$

Example:

```
# five input array f from above example, compute the 1D DCT and display the value
x = [1, 2, 2, 1]
X = dct(x, norm='ortho')
print('DCT Transform: ', X)

```

```
DCT Transform:  [ 3.  0. -1.  0.]
```

To recover the original matrix using inverse DCT

$$f = S^T . F$$

$$= \frac{1}{2}\begin{bmatrix} 1 & \frac{3}{\sqrt{5}} & 1 & \frac{1}{\sqrt{5}} \\ 1 & \frac{1}{\sqrt{5}} & -1 & -\frac{3}{\sqrt{5}} \\ 1 & -\frac{1}{\sqrt{5}} & -1 & \frac{3}{\sqrt{5}} \\ 1 & -\frac{3}{\sqrt{5}} & 1 & -\frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} \qquad f = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

```
# compute the Inverse DCT of the above transformed image F
x_rec = idct(X, norm='ortho')
print('Recovered x: ', x_rec)
```

```
Recovered x:  [1. 2. 2. 1.]
```

Generate 2D DCT of given matrix

$$f = \begin{bmatrix} 2 & 3 & 3 & 3 \\ 2 & 3 & 3 & 2 \\ 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 3 \end{bmatrix}$$

$$AfA^T = F = \begin{bmatrix} 10 & -0.19 & -0.5 & -0.46 \\ 0.46 & -0.35 & -0.73 & 0.35 \\ 0.5 & -0.84 & 0.0 & -0.73 \\ -0.19 & 0.35 & 0.84 & 0.35 \end{bmatrix}$$

```
# Compute the 2D DCT for the input f given in above example
x = [
    [2, 3, 3, 3],
    [2, 3, 3, 2],
    [3, 2, 2, 2],
    [2, 3, 2, 3]
]

X = dct(dct(x, axis=0, norm='ortho'), axis=1, norm='ortho')
print('DCT Transform: ', X)

```

```
DCT Transform:  [[10.         -0.19134172 -0.5        -0.46193977]
 [ 0.46193977 -0.35355339 -0.73253782  0.35355339]
 [ 0.5        -0.8446232   0.         -0.73253782]
 [-0.19134172  0.35355339  0.8446232   0.35355339]]
```

Inverse DCT

```
# conpute the inverse DCT of F
x_rec = idct(idct(X, axis=0, norm='ortho'), axis=1, norm='ortho')
print('Recovered x: ', x_rec)

```

```
Recovered x:  [[2. 3. 3. 3.]
 [2. 3. 3. 2.]
 [3. 2. 2. 2.]
 [2. 3. 2. 3.]]
```

To find the DCT of an image

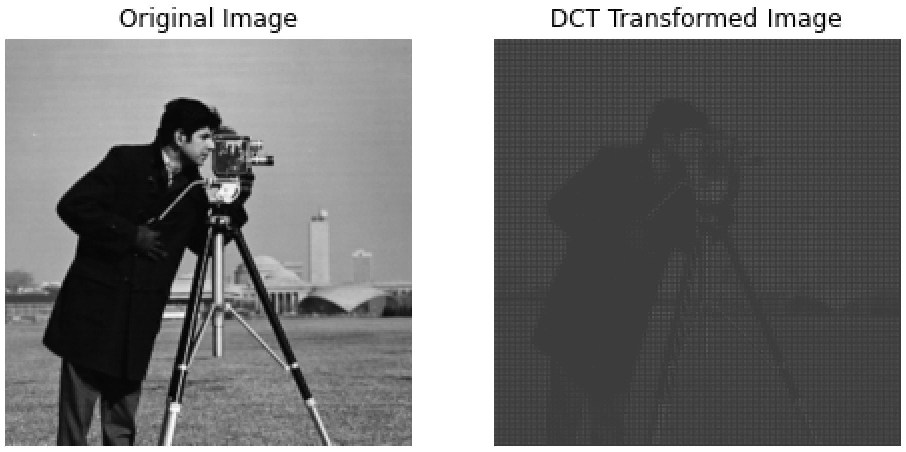
```
#Function to implement 2D DCT and IDCT
def dct2(a):
  return dct(dct(a, axis=0, norm= "ortho"), axis= 1, norm="ortho")
```

```
def idct2(a):
  return idct(idct(a, axis=0, norm="ortho"), axis=1, norm="ortho")
```

```
# Read the input image
img = cv2.imread('cameraman.tif', 0)

# Compute the rows and columns of the image
r, c = img.shape

# Define a imgdct matrix of zeros of size of the image which is to be filled with the
img_dct = np.zeros((r, c), dtype=int)

# Since the DCT matrix is a 4x4 matrix , we have to take 4x4 matrix of image and then
n = 4
```

```
for i in range(0, r//n):
  for j in range(0, c//n):
      block = img[i*n:(i+1)*n, j*n:(j+1)*n]
      img_dct[i * n:(i + 1) * n, j * n:(j + 1) * n] = dct2(block)

#plot the DCT transformed image
plt.figure(figsize=(8, 8))
plt.subplot(1, 2, 1)
plt.imshow(img, cmap='gray')
plt.title('Original Image')
plt.axis('off')
plt.subplot(1, 2, 2)
plt.imshow(img_dct, cmap='gray')
plt.title('DCT Transformed Image')
plt.axis('off')
```

```
(-0.5, 511.5, 511.5, -0.5)
```

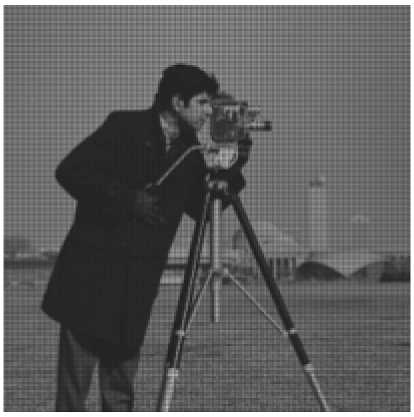


```
# Read the input image
img = cv2.imread('cameraman.tif', 0)

# Compute the rows and columns of the image
r, c = img.shape

# Define a imgdct matrix of zeros of size of the image which is to be filled with the D
rimg_dct = np.zeros((r, c), dtype=int)

# Since the DCT matrix is a 4x4 matrix , we have to take 4x4 matrix of image and then a
n = 4

for i in range(0, r//n):
  for j in range(0, c//n):
      block = img_dct[i*n:(i+1)*n, j*n:(j+1)*n]
      rimg_dct[i * n:(i + 1) * n, j * n:(j + 1) * n] = dct2(block)

#plot the DCT transformed image
plt.figure(figsize=(8, 8))
plt.subplot(1, 2, 1)
plt.imshow(img, cmap='gray')
plt.title('Original Image')
plt.axis('off')
plt.subplot(1, 2, 2)
plt.imshow(rimg_dct, cmap='gray')
plt.title('DCT Transformed Image')
plt.axis('off')
```

```
(-0.5, 511.5, 511.5, -0.5)
```

Original Image

DCT Transformed Image



For compression we retain the first few coefficients having 1% of total energy

Find the total energy of the original image
$TotalEnergy = sum|F(x,y)|^2$

```
# compute the total energy of the image using the formula given above


#Total energy of image
E = np.sum(img**2)

```

Compute P = 1% of the total energy

```
# compute 1% of the total energy
#we want 1% of TE of image as we wish to retain only those coeffiecients in img dct whi
P = 0.01*E

```

Select those coefficients in the Transformed image which have more than 1% energy from Transformed coefficients

```
# Retain only those coefficients for imagdct which are > 1% of total energy
compressed_blurred = np.where(img_dct**2>P, img_dct, 0)     #Blurs the image
```

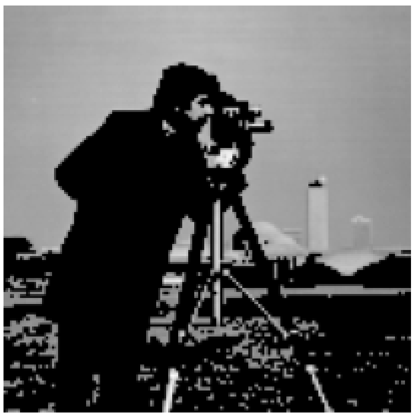
```python
#Find IDCT of the imgdct matrix of the retained coefficeints
r,c = img.shape
img_rec = np.zeros((r,c),dtype=int)
for i in range(0,r//n):
    for j in range(0,c//n):
        temp = compressed_blurred[i*n:(i+1)*n,j*n:(j+1)*n]
        img_rec[i*n:(i+1)*n,j*n:(j+1)*n] = idct2(temp)
```

```python
#Plott the original and the compressed image
plt.figure(figsize=(8, 8))

plt.subplot(1, 2, 1)
plt.imshow(img, cmap='gray')
plt.title('Original Image')
plt.axis('off')

plt.subplot(1, 2, 2)
plt.imshow(img_rec, cmap='gray',vmin=0,vmax=255)
plt.title('Recovered Image from Compressed Data Image')
plt.axis('off')
```

```
(-0.5, 511.5, 511.5, -0.5)
```

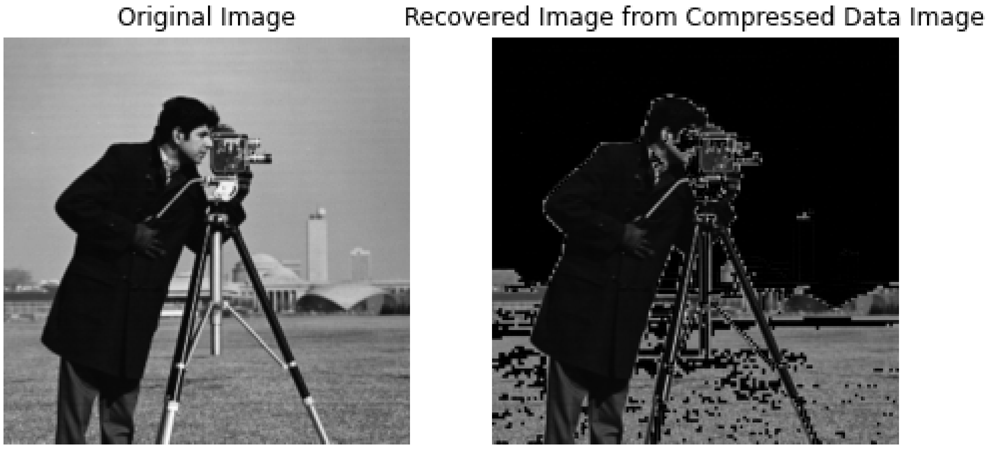
Original Image

Recovered Image from Compressed Data Image



```python
# Retain only those coefficients for imagdct which are < 1% of total energy
compressed_edges = np.where(img_dct**2<P, img_dct, 0)    # Gives edges

#Find IDCT of the imgdct matrix of the retained coefficeints
img_rec2 = np.zeros((r,c),dtype=int)
for i in range(0,r//n):
    for j in range(0,c//n):
        temp2 = compressed_edges[i*n:(i+1)*n,j*n:(j+1)*n]
        img_rec2[i*n:(i+1)*n,j*n:(j+1)*n] = idct2(temp2)

#Plott the original and the compressed image
plt.figure(figsize=(8, 8))

plt.subplot(1, 2, 1)
plt.imshow(img, cmap='gray')
plt.title('Original Image')
plt.axis('off')

plt.subplot(1, 2, 2)
plt.imshow(img_rec2, cmap='gray',vmin=0,vmax=255)
plt.title('Recovered Image from Compressed Data Image')
plt.axis('off')
```

```
(-0.5, 511.5, 511.5, -0.5)
```



Compression Ratio = size after compression/size before compression

```
#Finding the nonzero values in the Inew matrix
non_zero = np.count_nonzero(compressed_blurred)
```

```
#Total coefficients in the original image
total_coeff = r*c
```

```
# compute the compression ratio by calculating CR=number of non-zero coefficients/total
CR = (non_zero/total_coeff)*100
print("Compression Ratio = ",CR,"%")
```

```
Compression Ratio =  3.28826904296875 %
```

Conclusion

1. We implemented the code for Discrete Cosine Transform(DCT) for 1D Array, 2D Array and Image.
2. We implemented the code for Image Compression using DCT and observed that by retaining low frequency coefficients of the DCT Transformed image results in blurring of image while retaining the high frequency coefficients results in egdes.
3. Compression Ratio of the compressed image was calculated as the ratio of number of non zero coefficients to the total coefficients.
4. We achieved compression ratio of 3.3% by retaining the coefficients having energy greater than 0.01% of total image energy.