

Probability

Probability is a measure of the likelihood of occurrence of an event. The measure probability ranges between 0 to 1, zero means an impossible event and 1 means a certain event.

Terminology

1. Trial or Experiment: An action whose result is uncertain. For example, throwing a dice, tossing a coin etc.
2. Event: A single result of an experiment.
3. Sample space: It is the total number of possible outcomes for an experiment.
4. Sample point: One of the possible number of outcomes.

Formula

The Probability of an event = Number of favourable outcomes / Total number of possible outcomes

Few common experiments and important points to remember:

- Tossing a coin
Tossing a coin can lead to 2 possible outcomes:
 - Heads
 - Tails
 - The probability of a coin landing as heads or tails is equal and is 0.5
- Throwing a dice
Throwing a dice can lead to 6 outcomes, you can get any value between 1 to 6, both included.
 - The probability of occurrence of any number is $\frac{1}{6}$
- Picking a card from a deck of cards
Picking a card from a deck of cards can lead to 52 outcomes. In a deck, there are:
 - 13 cards of Hearts
 - 13 cards of Spades
 - 13 cards of Clubs
 - 13 cards of Diamonds
- Balls of different colour in bag

Steps to find the probability:

Step 1: List the outcomes of the experiment.

Step 2: Count the number of possible outcomes of the experiment.

Step 3: Count the number of favorable outcomes.

Step 4: Use the probability formula.

Demonstrations: Basic Probability

1. If 2 coins are tossed simultaneously, what is the probability of getting 2 heads?

Solution:

The total number of possible outcomes are 4 i.e, (H,H), (H,T), (T,H), (T,T)

Total number of favourable outcomes = 1 i.e, (H,H)

Thus $P = 1/4$

2. If 2 dice are thrown simultaneously, what is the probability of getting same number on both the dice?

Solution:

Total number of possible outcomes = $6 \times 6 = 36$

Total number of favourable outcome = 6 , i.e, (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

Thus probability of getting same number on both dices = $6/36 = 1/6$

3. What is the probability of getting an Ace from a deck of cards?

Solution:

Total number of possible outcomes = 4 (Ace of each type of card)

Total number of favourable outcome = 52

Thus $P = 4/52 = 1/13$

Joint Probability

Joint probability is the probability of 2 events taking place simultaneously, simply put it is the probability that event x occurs at same time as event y. The basic assumption while calculating joint probability of 2 events is that the 2 events are independent of each other.

The formula for joint probability is:

$$P(X \cap Y) = P(X) \times P(Y)$$

Demonstrations: Joint Probability

1. What is the probability of getting a 8 and a red while drawing a card from a deck.

Solution:

$$\begin{aligned}\text{Using the formula, } P(8 \cap \text{red}) &= P(8) * P(\text{red}) \\ &= (4/52) * (13/52) \\ &= 1/26\end{aligned}$$

2. A bag contains 3 red balls, 5 green balls and 10 black balls, what is the probability of getting either a red or a green ball, when a ball is randomly drawn from the bag?

Solution:

The probability of getting a red ball = $3/18$

The probability of getting a green ball = $5/18$

The probability of either getting a red or green ball = $15/18$

Conditional Probability

Conditional probability is probability of an event given that some other event has already occurred.

Notation: $P(B|A)$, this notation means probability of event 'B' given that event 'A' has already occurred. In conditional probability the 2 events are dependent.

$$\text{Formula: } P(B|A) = P(B \cap A) / P(A)$$

Demonstrations: Conditional Probability

1. In a game of cards, a player wins if he draws 2 cards of same unit. If the first card is a spade, what is the probability that the player will win?

Solution: If the first card was spade, now you the player is left with 12 spades and 51 cards, thus the probability that second card being spade and player winning the game is $12/51$.

2. A student has applied into an university and has 50% chance of getting an admission, as per university rules, 50% of the admitted students will get hostels for accommodation. What is the probability of student getting admitted and getting a hostel is?

$$\begin{aligned}\text{Solution: } P(\text{Hostel} | \text{Admission}) &= P(\text{Hostel} \cap \text{Admission}) / P(\text{Admission}) \\ P(\text{Hostel} | \text{Admission}) &= 0.5 * 0.5 \\ P(\text{Hostel} | \text{Admission}) &= 0.25\end{aligned}$$

3. A bag contains 4 red balls and 5 green balls, you draw a ball from the bag without replacement, what is the probability of drawing a green ball in second draw provided that the it was a red ball in first draw?

Solution: The probability of drawing a red ball in first draw is $4/9$ and probability of drawing a green ball in second draw is $5/8$, so probability of drawing a green ball in second draw is $(4/9)*(5/8) = 5/18$

Bayes Theorem

The Bayes theorem describes the probability of an event based on the prior knowledge of the conditions that might be related to the event. If we know the conditional probability $P(A|B)$, we can use the bayes rule to find out the reverse probabilities $P(B|A)$.

Formula:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

We will understand the application of bayes theorem using a demonstration:

2 Bags contain red and green balls, the first bag contains 2 red and 3 green balls, the second bag contains 5 red and 7 green balls, if a green ball is drawn out from one of the bag , what is the probability that it came from the first bag.

Solution:

$P(1)$ - Probability of choosing bag 1 = $1/2$

$P(2)$ - Probability of choosing bag 2 = $1/2$

$P(G1)$ - Probability of getting a green ball from bag 1 = $3/5$

$P(G2)$ - Probability of getting a green ball from bag 2 = $7/12$

Applying bayes theorem to determine the probability of green ball coming from bag 1

$$\begin{aligned} &= (P(1) * P(G1)) / (P(1) * P(G1)) * (P(2) * P(G2)) \\ &= ((1/2) * (3/5)) / ((1/2) * (3/5)) * ((1/2) * (7/12)) \\ &= 12/27 \end{aligned}$$

You can learn more about conditional probability and bayes theorem from here - [Link](#)

Assessments

1. If a card is chosen from a standard deck of cards, what is the probability of getting a five or a seven?

- a. $4/52$
- b. $1/28$
- c. $8/52$
- d. $1/169$

Ans: C

2. If a card is chosen from a standard deck of cards, what is the probability of getting a diamond or a club?

- a. $1/2$
- b. $13/52$
- c. $20/52$
- d. $12/52$

Ans: A

3. If 3 coins are tossed simultaneously, what is the probability of getting all heads?

- a. $1/3$
- b. $1/216$
- c. $1/36$
- d. $3/36$

Ans: B

4. A bag contains 7 red balls and 11 green balls, if a ball is drawn out at random from bag, what is the probability that the ball is red?

- a. $11/18$
- b. $7/11$
- c. $7/18$
- d. $11/18$

Ans: C

5. A card is drawn at random from a deck of cards, what is the probability it is a face card?

- a. $3/52$

- b. $\frac{4}{52}$
- c. $\frac{3}{13}$
- d. $\frac{4}{13}$

Ans: C

6. In a throw of dice what is the probability of getting number greater than 5

- a. $\frac{1}{6}$
- b. $\frac{2}{6}$
- c. $\frac{3}{6}$
- d. $\frac{4}{6}$

Ans: A

7. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even ?

- a. $\frac{3}{4}$
- b. $\frac{1}{4}$
- c. $\frac{7}{4}$
- d. $\frac{1}{2}$

Ans: A

8. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 4 and 9?

- a. $\frac{1}{4}$
- b. $\frac{3}{10}$
- c. $\frac{1}{5}$
- d. $\frac{7}{20}$

Ans: D

9. A number is selected at random from first thirty natural numbers. What is the chance that it is a multiple of either 3 or 13?

- a. $\frac{17}{30}$
- b. $\frac{2}{5}$
- c. $\frac{11}{30}$
- d. $\frac{4}{15}$

Ans: B

10. Mr Lee has 2 kids, the first one is a boy, what is the probability that second kid is a girl?

- a. $\frac{1}{2}$
- b. $\frac{1}{3}$
- a. $\frac{1}{4}$
- b. $\frac{1}{5}$

Ans: B

11. A jar contains 4 marbles. 3 Red & 1 white. Two marbles are drawn with replacement after each draw. What is the probability that the same color marble is drawn twice?

- a. $\frac{1}{2}$
- b. $\frac{1}{3}$
- c. $\frac{5}{8}$
- d. $\frac{1}{2}$

Ans: C