

GRAPH THEORY - SURPRISE TEST

Q-3

no. of vertices = n
and no. of edges = m

Suppose that u and v are any two vertices of G that are not adjacent.

We write $\deg(u)$ ~~or~~
so, we assume that

let H be hamiltonian graph with any edges that have v or u as end vertices of n graph

The H has $n-2$ vertices and $m - \deg(u) - \deg(v)$

$$\text{i.e. } (n-2) \geq \text{vertices} \geq m - \deg(u) - \deg(v)$$

The max no. of edges that H can have is

$${}^{n-2}C_2 = \frac{(n-2)(n-3)}{2}$$

$$\text{or } \frac{1}{2} (n^2 - 5n + 6)$$

$$\text{and when } m - \deg(u) - \deg(v) \leq \frac{1}{2} (n-2)(n-3)$$

So,

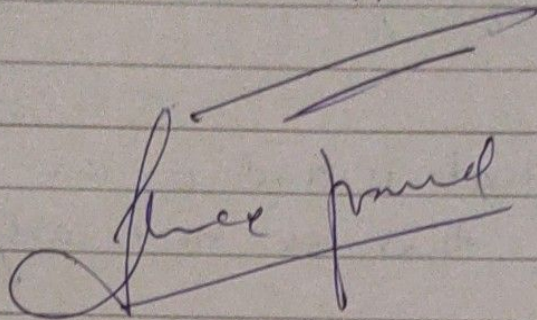
$$\deg(u) + \deg(v) \geq m - \frac{1}{2} (n^2 - 5n + 6)$$

again $\rightarrow \deg(u) + \deg(x) \geq m - \frac{1}{2}(n^2 - 5n + 6)$

By the hypothesis, thus

$$\deg(u) + \deg(v) \geq \frac{1}{2}(n^2 - 3n + 6) - \frac{1}{2}(n^2 - 5n + 6)$$

$$= n$$



Conclusion

Hence we can say that for $m \geq \frac{n^2 - 3n + 6}{2}$

we can infer that the given graph with given condition is a Hamiltonian graph.