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GRAPH THEORY

ASSIGNMENT - 3

Ques 1 let α be the edge connectivity of Graph G,
$\therefore$ $\exists$ a cutset S in G with α edges
let S partition the vertices of G into V_1 & V_2 Now,
by removing at most α vertices from V_1 (or V_2)
which the edges in S are incident we can
effect the removal S from G.

$\therefore$ Vertex connectivity $\leq$ edge connectivity
we known that the Graph will have 2e degree
which is divided among the n vertices, so there
must be among the n or vertex in G whose degree
$\leq \frac{2e}{n}$ and since edge connectivity $\leq$ smallest degree in G

$\rightarrow$ edge connectivity $\leq 2e/n$

vertex connectivity $\leq$ edge connectivity $\leq 2e/n$

Ques 2 ~~A~~ A connected graph is said to be seperable if
its vertex connectivity is one.

let G be a non-seperable graph, let V be same vertex
in G, let edge $\{e_1, e_2, \ldots e_n\}$ are

are incident, the graph will be disconnected as there will be 2 component $G - \{V_1\}$ & $\{V_2\}$

let $\{e_j, e_{j+1}, \ldots e_i\}$ be a subset of $\{e_1, e_2 \ldots e_k\}$

now let $\{e_j, e_{j+1}, \ldots e_i\}$ also be a cut set of G.

The block of having vertex V has vertex $\{v_1, v_2 \ldots v_i\}$ such that an edge e_i from v to n exists

→ removal of v from G should disconnect Graph. but since, G is non-separable, hence it is a contraction

The block having vertex V from v to n exists.

→ removal of v from G should disconnect Graph. but since is a non-separable, hence its is a contraction.

→ $\{e_1, e_2, \ldots e_i\}$ can't have a subset which is a cut set of G.

→ $\{e_1 \ldots e_i\}$ is a cut set

→ set of edges incident on each vertex of G is a cut set.

Q: 3 Capacity of a cut set is defined as sum of the capacity of each edge in the cut set

②

let Gf (residual graph obtained) defined 2 subset of vertices,

1: $A \rightarrow$ set of vertices reachable from a in Gf

2: $A^c \rightarrow V - A$

<u>claim</u> :- value (f) $\Rightarrow$ (A, A^c) when c is the capacity

we know, $val(f) = c(A, A^c) \Rightarrow f_{out}(A) - f_{in}(A)$

$\therefore$ for $val(f) \Rightarrow c(A, A^c)$ we need

(i) outgoing edges $\rightarrow$ fully saturated.

(ii) incoming edges must have 0 flow.

The prove claim, we consider 2 cases,

(i) in G $\exists$ outgoing edge (x, y) $x \in A$, $y \in A^c$ not saturated

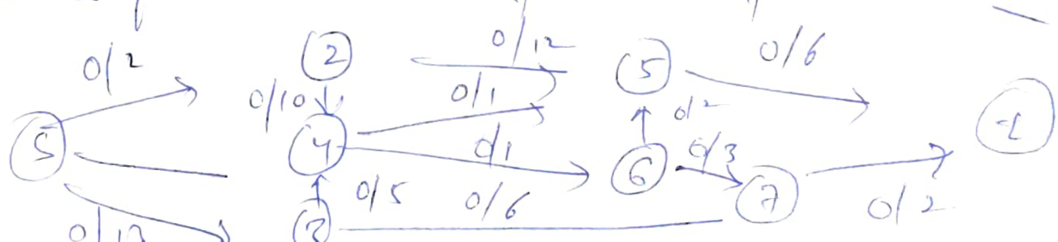
i.e. $f(x, y) < x, y \Rightarrow \exists$ forward edge $x \rightarrow y$ in Gf

$\rightarrow$ $\exists$ a path from $a \rightarrow y$ Gf which is a contradiction

(ii) in G $\exists$ an incoming edge (y, x) $x \in A$ $y \in A^c$ s.t. has non-zero flow of contradiction

$\therefore$ flow $\leq$ capacity

④ The ford fulkerson algorithm is a greedy algorithm that compute the max flow in flow networks



augmented path	capacity
$S \to 2 \to 5 \to t$	2
$S \to 3 \to 7 \to t$	2
$S \to 3 \to 4 \to 6 \to 5 \to t$	1
$S \to 3 \to 4 \to 5 \to t$	1

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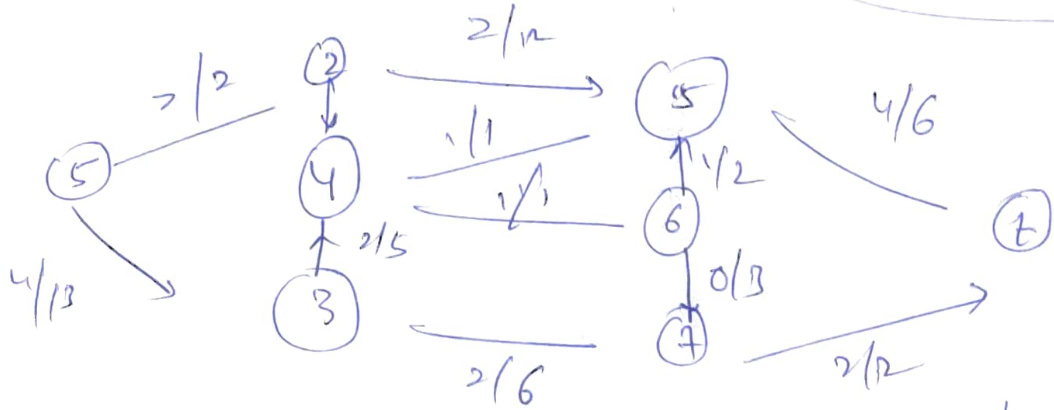
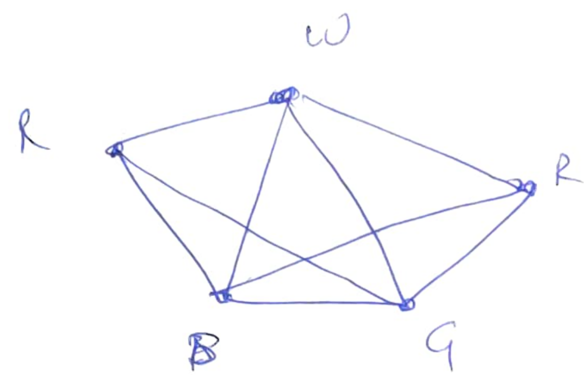
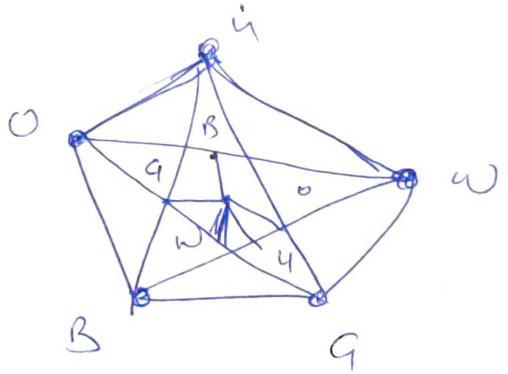


total flow = 4+2 = 6

Qn 5



total colors needed = 4 | chromatic number = 4



chromatic number = 6

(6) let G be a bipartite graph

→ Vertex set of a G can be partitioned into two set V_1 & V_2 such that $V_1 \cup V_2 = V$

→ Since in V_1 for any u, v edge $u \to v$ doesn't exist

∴ We can (exist) (assign) color 1 to V_1 & 2 to V_2

Conversely, let G be bicolorable,

let V_1 ⇒ set of vertices having color 1

V_2 ⇒ ——— " ——— — " 2

Since, no two vertex u, v, in V_1 are adjacent same can be said ~~abs~~ about V_2

So, any edge $\{u, v\}$ in G will be set $u \in V$ & $v_0 \in V_2$

G is bipartite.

(7) Complete matching in a graph: → matching in graph G is said to be perfect if every vertex is connected to exactly one edge.

Know → complete, bipartite graph with n vertices each subset, let V_1 & V_2 the subsets.

let $\{v_1, v_2, v_2 \ldots v_2\}$ be vertex in V_1

& $\{v_1, v_2, v_3 \ldots v_n\}$ be vertex in V_2.

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now for vertex v_1, we have n options.

—— v_2 —— $(n-1)$ options

then total no. of match $\Rightarrow n \times (n-1) \ldots 1$

$\Rightarrow n!$

Qn 8 A matching M is said to be perfect if vertex of Graph is incident to an edge in matching.

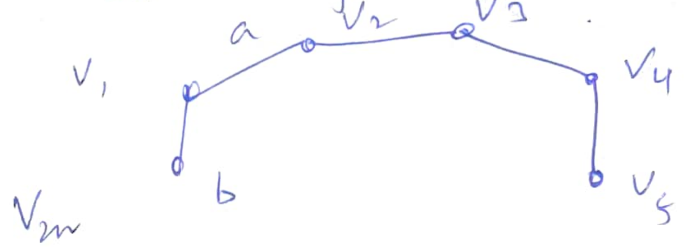
a) K_{2n} → complete graph with $2n$ vertices for 1^{st} vertex, we have $2n-1$ choices

2^{nd} —— " $2n-3$ choices

3^{rd} —— " $2n-5$ choices

no. of perfect matching $= \dfrac{2n!}{2^n \times n!}$

b) C_{2n} → cycle with $2n$ vertices,



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Starting from V_1, we have 2 options edge a & b, (i) after choosing either of these we can only above the alternate edge.

⇒ no. of perfect matching = 2