Mathematical formulation

$\dot{x} = f(x)$, $x = x_*$ is equi pt

Linear Analysis

$x(t) = x_* + \underbrace{y(t)}_{\text{perturbation}}$ $|y(t)| << 1$

$\Rightarrow \dot{x}(t) = \dot{y}(t)$

$\Rightarrow \dot{y}(t) = f(x) = f(x_* + y(t))$ [$\because f(x_* + y(t))$ is smooth fn

$= f(x_*) + f'(x_*)\,y(t) + \cdots$

$f(x_*) = 0$ & neglected higher order terms

$\Rightarrow \dot{y}(t) = f'(x_*)\,y(t) \equiv \alpha\, y_t$

$|y(t)| \to 0$ if $\alpha < 0$

$\Rightarrow x(t) \to x_*$ as $t \to \infty$ if $f'(x_*) < 0$ Stable

$|y(t)| \to \infty$ if $\alpha > 0$

$x(t) \not\to x_*$ as $t \to \infty$ if $f'(x_*) > 0$ Unstable

If $f'(x_*) = 0 \Rightarrow$ Nothing can be said by linear stability analysis.

x_* is locally asymptotically stable if

$f'(x_*) < 0$ &

unstable $f'(x_*) > 0$

We will check the previous example with this condition :

$$\dot{x}(t) = x^2 - 1 \quad , \quad x_{1*} = 1, \quad x_{2*} = -1$$

$$f(x) = x^2 - 1$$

$$f'(x) = 2x$$

$f'(x_{1*}) = 2 > 0 \rightarrow$ unstable $\Rightarrow$ x_{1*} is unstable

$f'(x_{2*}) = -2 < 0 \rightarrow$ stable $\Rightarrow$ x_{2*} is stable

28 April Friday

2017
Week 17th • Day 118th
Sunrise : 05:29 am • Sunset : 06:31 pm

Important

$\frac{dy}{dx} < 0$	\
$\frac{dy}{dx} = 0$	—
$\frac{dy}{dx} > 0$	/

for phase diagram

8.00

9.00

10.00

11.00

12.00

1.00

2.00

3.00

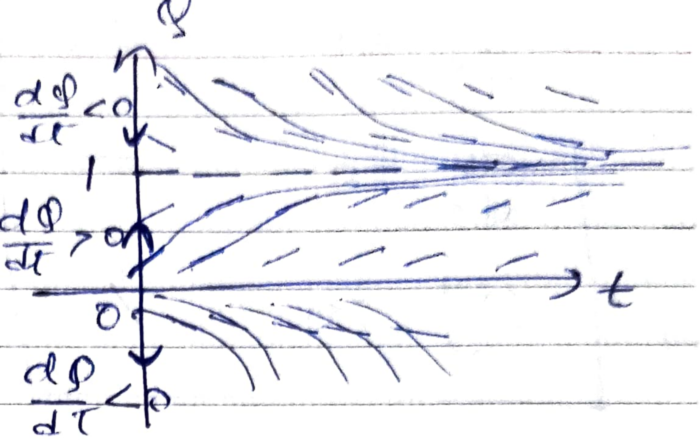
4.00

5.00

Phase diagram

Autonomous as t is not in RHS

$\underline{Q}$ $\quad \frac{dQ}{dt} = Q(1-Q)$



When

$0 < Q < 1$

$\Rightarrow \frac{dQ}{dt} > 0$

$Q > 1, \frac{dQ}{dt} < 0$

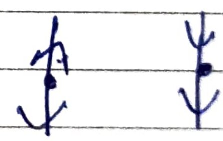
$Q < 0, \quad \frac{dQ}{dt} < 0$

$Q = 0$
$Q = 1$] Equilibrium : sol^n doesn't change

Autonomous: slope doesnot change left to right b'coz t is not present on RHS

$\underline{Q=1}$ → stable equilibrium point

$\underline{Q=0}$ → unstable



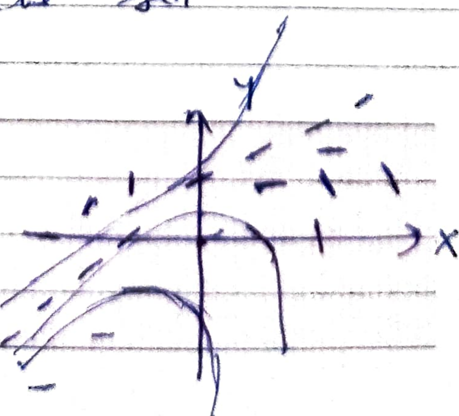
<u>Slope field</u>: picture of sol^n $\frac{dy}{dx} = f(x,y)$ without knowing the sol^n

<u>Ex</u> $\frac{dy}{dx} = y - x$

$\frac{dy}{dx} = 0$ at $y = x$

$\frac{dy}{dx} < 0$ at $y < x$

$\frac{dy}{dx} > 0$ at $y > x$



MAY 2017					
Monday	1	8	15	22	29
Tuesday	2	9	16	23	30
Wednesday	3	10	17	24	31
Thursday	4	11	18	25	
Friday	5	12	19	26	
Saturday	6	13	20	27	
Sunday	7	14	21	28	