

Data Science for Engineers

Week 4 assignment

1. Let $f(x) = x^3 + 3x^2 - 24x + 7$. Select the correct options from the following: [3 marks]

- (a) $x = 2$ will give the maximum for $f(x)$.
- (b) $x = 2$ will give the minimum for $f(x)$.
- (c) Maximum value of $f(x)$ is 87.
- (d) The stationary points for $f(x)$ are 2 and 4.

Answer: (b), (c)

2. Find the gradient of $f(x, y) = x^2y$ at $(x, y) = (1, 3)$. [2 marks]

- (a) $\nabla f = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$
- (b) $\nabla f = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$
- (c) $\nabla f = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$
- (d) $\nabla f = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

Answer: (b)

3. Find the Hessian matrix for $f(x, y) = x^2y$ at $(x, y) = (1, 3)$. [2 marks]

- (a) $\nabla^2 f = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$
- (b) $\nabla^2 f = \begin{bmatrix} 3 & 3 \\ 3 & 0 \end{bmatrix}$
- (c) $\nabla^2 f = \begin{bmatrix} 6 & 2 \\ 2 & 0 \end{bmatrix}$
- (d) $\nabla^2 f = \begin{bmatrix} 6 & 3 \\ 3 & 0 \end{bmatrix}$

Answer: c

4. Let $f(x, y) = -3x^2 - 6xy - 6y^2$. The point $(0, 0)$ is a

- (a) saddle point
- (b) maxima
- (c) minima

Answer: (b)

5. For which numbers b is the matrix $A = \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix}$ positive definite?

- (a) $-3 < b < 3$
- (b) $b = 3$
- (c) $b = -3$
- (d) $-3 \leq b \leq 3$

Answer: (a)

6. Consider $f(x) = x^3 - 12x - 5$. Which among the following statements are true?

- (a) $f(x)$ is increasing in the interval $(-2, 2)$.
- (b) $f(x)$ is increasing in the interval $(2, \infty)$.
- (c) $f(x)$ is decreasing in the interval $(-\infty, -2)$.
- (d) $f(x)$ is decreasing in the interval $(-2, 2)$.

Answer: (b, d)

7. Consider the following optimization problem:

$$\max_{x \in \mathbb{R}} f(x)$$

, where

$$f(x) = x^4 + 7x^3 + 5x^2 - 17x + 3$$

Let x^* be the maximizer of $f(x)$. What is the second order sufficient condition for x^* to be the maximizer of the function $f(x)$?

- (a) $4x^3 + 21x^2 + 10x - 17 = 0$
- (b) $12x^2 + 42x + 10 = 0$
- (c) $12x^2 + 42x + 10 > 0$
- (d) $12x^2 + 42x + 10 < 0$

Answer: (d)

8. In optimization problem, the function that we want to optimize is called

- (a) Decision function
- (b) Constraints function
- (c) Optimal function
- (d) Objective function

Answer: (d)

9. The optimization problem $\min_x f(x)$ can also be written as $\max_x f(x)$.

- (a) True
- (b) False

Answer: b

10. Gradient descent algorithm converges to the local minimum.

- (a) True
- (b) False

Answer: a