

## Data Science for Engineers

### Week 2

1. Are the vectors  $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ ,  $\begin{bmatrix} 7 \\ -2 \end{bmatrix}$  and  $\begin{bmatrix} 3 \\ -6 \end{bmatrix}$  linearly independent?

(a) Yes

(b) No

Answer: (b)

2. Does the set,  $S = \{(1, 1), (1, 2)\}$  spans  $\mathbb{R}^2$ ?

(a) Yes

(b) No

Answer: (a)

3. Consider the following system of linear equations of the form  $Ax = b$ :

$$2x - 3y + 6z = 14$$

$$x + y - 2z = -3$$

Which among the following are correct?

(a)  $\begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$  is a solution to  $Ax = b$

(b)  $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$  is a solution to  $Ax = b$

(c)  $\begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$  is a solution to  $Ax = 0$

(d)  $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$  is a solution to  $Ax = 0$

Answer: (a, d)

4. Consider the following system of linear equation:

$$x + y + z = -2$$

$$x + 2y - z = 1$$

$$2x + ay + bz = 2$$

(i) Find the conditions on  $a$  and  $b$  for which the above system has no solution.

(a)  $2a + b - 6 = 0$

(b)  $a \neq 4, 2a + b - 6 = 0$

(c)  $a = 4, b = -2$

(d)  $2a + b - 6 \neq 0$

Answer: (b)

(ii) Find the conditions on  $a$  and  $b$  for which the above system has a unique solution.

(a)  $2a + b - 6 = 0$

(b)  $a \neq 4, 2a + b - 6 = 0$

(c)  $a = 4, b = -2$

(d)  $2a + b - 6 \neq 0$

Answer: (d)

(iii) Find the conditions on  $a$  and  $b$  for which the above system has infinite number of solutions.

(a)  $2a + b - 6 = 0$

(b)  $a \neq 4, 2a + b - 6 = 0$

(c)  $a = 4, b = -2$

(d)  $2a + b - 6 \neq 0$

Answer: (c)

5. In solving the system  $Ax = b$  in the variables  $x_1, x_2, x_3$  and  $x_4$ , Gaussian elimination on the Augmented matrix  $[A | b]$  led to the following row echelon form

$$\left( \begin{array}{cccc|c} 1 & 0 & 0 & 3 & 2 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

(i) Identify the number of free variable from the above row echelon matrix.

(a) 0

(b) 1

(c) 2

(d) 3

Answer: (b)

(ii) Which among the following is correct for the above system  $Ax = b$ ?

(a) It has infinite number of solutions.

(b) It has a unique solution.

(c) It has no solution.

Answer: (a)

6. For what values of  $a$  are matrix  $A = \begin{bmatrix} a & 1 \\ -2 & a+3 \end{bmatrix}$  not invertible?

(a)  $a = 1$

(b)  $a = -2$

(c)  $a = -1$

(d)  $a = 2$

Answer: (b, c)

7. Which among the following is true for the determinant of a matrix?

(a) The determinant of a diagonal matrix is the product of its diagonal entries.

(b) If one row of a matrix is a scalar multiple of another, the determinant is 1.

(c) If one row of a matrix is a scalar multiple of another, the determinant is 0.

(d) The determinant of a permutation matrix can only be 1.

Answer: (a, c)

8. Which among the following are the eigenvalues of matrix  $A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$ ?

(a) 1, 3, -3

(b) 1, 3, 3

(c) -1, 3, 3

(d) 1, -3, -3

Answer: (d)

9. Find the nullity of  $A = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 1 & -3 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ .

Answer: 2

10. Let  $A = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ . Suppose the eigen values corresponding to  $AA^T$  are  $a, b$  and  $c$ , then find the value of  $ab + bc + ca$ .

(a) 9

(b) 0

(c) 81

(d) 18

Answer: (b)