Data Science for Engineers

Week 5 assignment

1. Which of the following statements is/are not TRUE with respect to the multi variate optimization?

   I - The gradient of a function at a point is parallel to the contours
   II - Gradient points in the direction of greatest increase of the function
   III - Negative gradients points in the direction of the greatest decrease of the function
   IV - Hessian is a non-symmetric matrix

   (a) I
   (b) II and III
   (c) I and IV
   (d) III and IV

   Answer: (c)

2. The solution to an unconstrained optimization problem is always the same as the solution to the constrained one.

   (a) True
   (b) False

   Answer: (b)

3. Gradient based algorithm methods compute

   (a) only step length at each iteration
   (b) both direction and step length at each iteration
   (c) only direction at each iteration
   (d) none of the above

   Answer: (b)

4. For an unconstrained multivariate optimization given $f(\overline{x})$, the necessary second order condition for $\overline{x}^*$ to be the minimizer of $f(\overline{x})$ is

   (a) $\nabla^2 f(\overline{x}^*)$ must be negative definite.

(b) $\nabla^2 f(\overline{x}^*)$ must be positive definite.

(c) $\nabla f(\overline{x}^*) = 0$

(d) $f''(x^*) > 0$

Answer: (b)

5. Consider an optimization problem $\min_{x_1, x_2 \in \mathbb{R}} f(x_1, x_2) = x_1^2 + 4x_2^2 - 2x_1 + 8x_2$.

(i) Which among the following is the stationary point for $f(x_1, x_2)$?

(a) $(0, 0)$

(b) $(1, -1)$

(c) $(-1, -1)$

(d) $(-1, 1)$

Answer: (b)

(ii) Find the eighen values corresponding to Hessian matrix of f.

(a) $1, -1$

(b) $1, 1$

(c) $2, 8$

(d) $0, 2$

Answer: (c)

(iii) Find the minimum value of f.

(a) 0

(b) -5

(c) -1

(d) 1

Answer: (b)

(iv) Now, in order to find the minimum value of f subject to the constraint

$$x_1 + 2x_2 = 7,$$

what should be the first order condition for $\overline{\mathbf{x}^*}$ to be a minimizer of $f(x_1, x_2)$?

(a)

$$2x_1^* + 2 = \lambda$$
$$-8x_2^* - 8 = 2\lambda$$
$$x_1^* + 2x_2^* = 7$$

(b)

$$-2x_1^* + 2 = \lambda$$
$$-8x_2^* - 8 = 2\lambda$$
$$x_1^* + 2x_2^* = 7$$

(c)

$$2x_1^* - 2 = -\lambda$$
$$8x_2^* + 8 = -2\lambda$$
$$x_1^* + 2x_2^* = 7$$

(d)

$$-2x_1^* + 2 = -\lambda$$
$$-8x_2^* - 8 = -2\lambda$$
$$x_1^* + 2x_2^* = 7$$

Answer: (b)

(v) What is the minimum value of $f(x_1, x_2)$ subject to the above mentioned constrained?

(a) -5
(b) -1
(c) 27
(d) 0

Answer: (c)

6. Find the maximum value of $f(x, y) = 49 - x^2 - y^2$ subject to the constraints $x + 3y = 10$.

(a) 49
(b) 46
(c) 59
(d) 39

Answer: (d)

7. Consider an optimization problem $\min\limits_{x_1, x_2} x^2 - xy + y^2$ subject to the constraints

$$2x + y \leq 1$$
$$x + 2y \geq 2$$
$$x \geq -1$$

Find the lagrangian function for the above optimization problem.

(a) $L(x, y, \mu_1, \mu_2, \mu_3) = x^2 - xy + y^2 + \mu_1(2x + y - 1) + \mu_2(2 - x - 2y) + \mu_3(-x - 1)$

(b) $L(x, y, \mu_1, \mu_2, \mu_3) = x^2 - xy + y^2 + \mu_1(2x + y - 1) + \mu_2(x + 2y - 2) + \mu_3(-x - 1)$

(c) $L(x, y, \mu_1, \mu_2, \mu_3) = x^2 - xy + y^2 + \mu_1(2x + y - 1) + \mu_2(x + 2y - 2) + \mu_3(x + 1)$

(d) $L(x, y, \mu_1, \mu_2, \mu_3) = x^2 - xy + y^2 + \mu_1(1 - 2x - y) + \mu_2(2 - x - 2y) + \mu_3(-x - 1)$

Answer: (a)