Evaluation of Language Models, Basic Smoothing

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Week 2: Lecture 5

Evaluating Language Model

Does it prefer good sentences to bad sentences?

Assign higher probability to real (or frequently observed) sentences than ungrammatical (or rarely observed) ones

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Training and Test Corpora

- Parameters of the model are trained on a large corpus of text, called **training set**.
- Performance is tested on a disjoint (held-out) **test data** using an **evaluation metric**

Extrinsic evaluation of N-grams models

Comparison of two models, A and B

- Use each model for one or more tasks: *spelling corrector, speech recognizer, machine translation*
- Get accuracy values for A and B
- Compare accuracy for A and B

Intrinsic evaluation: Perplexity

Intuition: The Shannon Game

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- The president of India is ...
- I wrote a ...

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A better model of text

is one which assigns a higher probability to the actual word

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Applying chain Rule

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For bigrams

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&= \left(\left(\frac{1}{10}\right)^N\right)^{-\frac{1}{N}}
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&= \left(\left(\frac{1}{10}\right)^N\right)^{-\frac{1}{N}} \\
&= \left(\frac{1}{10}\right)^{-1} \\
&= 10
\end{aligned}
$$

Lower perplexity = better model

WSJ Corpus

Training: 38 million words

Test: 1.5 million words

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N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

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Unigram perplexity: 962?

The model is as confused on test data as if it had to choose uniformly and independently among 962 possibilities for each word.

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Use the language model to generate word sequences

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```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                          food  </s>
I want to eat Chinese food
```

Shakespeare as Corpus

- N = 884,647 tokens, V = 29,066
- Shakespeare produced 300,000 bigram types out of $V^2 = 844$ million possible bigrams.

Approximating Shakespeare

Unigram

To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have
Every enter now severally so, let
Hill he late speaks; or! a more to leg less first you enter
Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like

Bigram

What means, sir. I confess she? then all sorts, he is trim, captain.
Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.
What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman?

Trigram

Sweet prince, Falstaff shall die. Harry of Monmouth's grave.
This shall forbid it should be branded, if renown made it empty.
Indeed the duke; and had a very good friend.
Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.

Quadrigram

King Henry.What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
Will you not tell me who I am?
It cannot be but so.
Indeed the short and the long. Marry, 'tis a noble Lepidus.

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Training set

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- ... denied the reports
- ... denied the claims
- ... denied the request

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Zero probability n-grams

- P(offer | denied the) = 0

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Zero probability n-grams

- P(offer | denied the) = 0
- The test set will be assigned a probability 0
- And the perplexity can't be computed

Language Modeling: Smoothing

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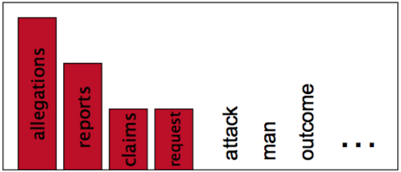
With sparse statistics

P(w | denied the)
3 allegations
2 reports
1 claims
1 request

7 total



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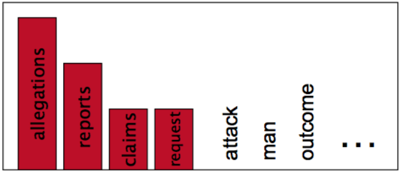
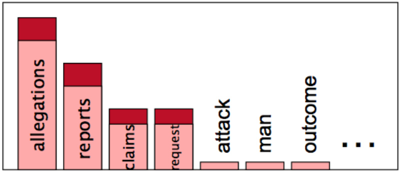


Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total



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- Just add one to all the counts!
- MLE estimate for bigram: $P_{MLE}(w_i|w_{i-1}) = \frac{c(w_{i-1},w_i)}{c(w_{i-1})}$
- Add-1 estimate: $P_{Add-1}(w_i|w_{i-1}) = \frac{c(w_{i-1},w_i)+1}{c(w_{i-1})+V}$

Reconstituted counts as effect of smoothing

Effective bigram count ($c^*(w_{n-1}w_n)$)

$$\frac{c^*(w_{n-1}w_n)}{c(w_{n-1})} = \frac{c(w_{n-1}w_n)+1}{c(w_{n-1})+V}$$

Comparing with bigrams: Restaurant corpus

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

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i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

More general formulations: Add-k

$$P_{Add-k}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c(w_{i-1}) + kV}$$

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Unigram prior smoothing:

$$P_{UnigramPrior}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i) + mP(w_i)}{c(w_{i-1}) + m}$$

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A good value of k or m?

Can be optimized on held-out set