## Data Science for Engineers

## Week 2

- 1. Are the vectors  $\begin{bmatrix} -2\\4 \end{bmatrix}$ ,  $\begin{bmatrix} 7\\-2 \end{bmatrix}$  and  $\begin{bmatrix} 3\\-6 \end{bmatrix}$  linearly independent?
  - (a) Yes
  - (b) No

Answer: (b)

- 2. Does the set,  $S = \{(1,1), (1,2)\}$  spans  $\mathbb{R}^2$ ?
  - (a) Yes
  - (b) No

Answer: (a)

3. Consider the following system of linear equations of the form Ax = b:

$$2x - 3y + 6z = 14$$

$$x + y - 2z = -3$$

Which among the following are correct?

- (a)  $\begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$  is a solution to Ax = b
- (b)  $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$  is a solution to Ax = b
- (c)  $\begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$  is a solution to Ax = 0
- (d)  $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$  is a solution to Ax = 0

Answer: (a, d)

4. Consider the following system of linear equation:

$$x + y + z = -2$$

$$x + 2y - z = 1$$

$$2x + ay + bz = 2$$

- (i) Find the conditions on a and b for which the above system has no solution.
  - (a) 2a + b 6 = 0
  - (b)  $a \neq 4, 2a + b 6 = 0$
  - (c) a = 4, b = -2
  - (d)  $2a + b 6 \neq 0$

Answer: (b)

- (ii) Find the conditions on a and b for which the above system has a unique solution.
  - (a) 2a + b 6 = 0
  - (b)  $a \neq 4, 2a + b 6 = 0$
  - (c) a = 4, b = -2
  - (d)  $2a + b 6 \neq 0$

Answer: (d)

- (iii) Find the conditions on a and b for which the above system has infite number of solutions.
  - (a) 2a + b 6 = 0
  - (b)  $a \neq 4, 2a + b 6 = 0$
  - (c) a = 4, b = -2
  - (d)  $2a + b 6 \neq 0$

Answer: (c)

5. In solving the system Ax = b in the variables  $x_1, x_2, x_3$  and  $x_4$ , Gaussian elimination on the Augmented matrix  $[A \mid b]$  led to the following row echelon form

$$\begin{pmatrix}
1 & 0 & 0 & 3 & 2 \\
0 & 1 & 1 & 2 & 3 \\
0 & 0 & 0 & 1 & 1/3 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

- (i) Identify the number of free variable from the above rwo echelon matrix.
  - (a) 0
  - (b) 1
  - (c) 2

(d) 3

Answer: (b)

- (ii) Which among the following is correct for the above system Ax = b?
  - (a) It has infinite number of solutions.
  - (b) It has a unique solution.
  - (c) It has no solution.

Answer: (a)

- 6. For what values of a are matrix  $A = \begin{bmatrix} a & 1 \\ -2 & a+3 \end{bmatrix}$  not invertible?
  - (a) a = 1
  - (b) a = -2
  - (c) a = -1
  - (d) a = 2

Answer: (b, c)

- 7. Which among the following is true for the determinant of a matrix?
  - (a) The determinant of a diagonal matrix is the product of its diagonal entries.
  - (b) If one row of a matrix is a scalar multiple of another, the determinant is 1.
  - (c) If one row of a matrix is a scalar multiple of another, the determinant is 0.
  - (d) The determinant of a permutation matrix can only be 1.

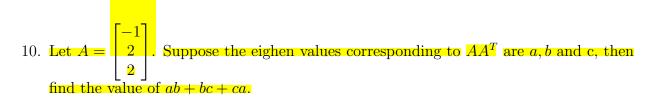
Answer: (a, c)

- 8. Which among the following are the eigenvalues of matrix  $A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$ ?
  - (a) 1, 3, -3
  - (b) 1, 3, 3
  - (c) -1, 3, 3
  - (d) 1, -3, -3

Answer: (d)

9. Find the nullity of  $A = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 1 & -3 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ .

Answer: 2



- (a) 9
- (b) 0
- (c) 81
- (d) 18

Answer: (b)