Data Science for Engineers

Week 3 assignment

1. Sumit wants to contact one of his friends, but he remembers only the first 9 of the 10 digits of the contact number. He is sure that the last digit of the contact number is an odd number. He selects an odd number randomly. If the random variable X denotes the last digit of the contact number, then calculate Var(X). [1 Mark]

   (a) 5

   (b) 8

   (c) 33

   (d) None of the above

   Answer: b

2. Suppose $X \sim$ Normal(μ, 4). For $n = 20$ iid samples of X, the observed sample mean is 5.2. What conclusion would a z-test reach if the null hypothesis assumes $\mu = 5$ (against an alternative hypothesis $\mu \neq 5$) at a significance level of $\alpha = 0.05$?
   Use $F_Z^{-1}(0.025) = -1.9599$

   a. Accept H_0

   b. Reject H_0

   Answer: a

3. A box contains 8 items out of which 2 are defective. A sample of 5 items is to be selected randomly (without replacement) from the box. If the random variable X represents the number of defective items in a selection of 5 items, then find $E(X)$.(Enter the answer correct to 2 decimal places) [1 Mark]

   (a) 1.25

   (b) 5

   (c) 0.25

   (d) 1

Answer: a

4. Suppose $X \sim$ Normal(μ, 9). For n = 100 iid samples of X, the observed sample mean is 11.8. What conclusion would a z-test reach if the null hypothesis assumes μ = 10.5 (against an alternative hypothesis μ 10.5)? [1 mark]

   (a) Accept H_0 at a significance level of 0.10.

   (b) Reject H_0 at a significance level of 0.10.

   (c) Accept H_0 at a significance level of 0.05.

   (d) Reject H_0 at a significance level of 0.05.

   Answer: b, d

5. Let X and Y be two independent random variables with Var(X) = 9 and Var(Y) = 3, find Var($4X - 2Y + 6$). [1 mark]

   (a) 100

   (b) 140

   (c) 156

   (d) None of the above

   Answer: c

6. The correlation coefficient of two random variable X and Y is 14, their variance is given by 3 and 5. Compute Cov(X, Y). [1 mark]

   (a) -0.854

   (b) 0.561

   (c) -0.968

   (d) None of the above

   Answer: c

7. When will you reject the Null hypothesis? [1 mark]

   (a) p value greater than α

   (b) p value less than α

   (c) p value equal to α

   (d) None of the above

   Answer: b

8. A sample of N observations are independently drawn from a normal distribution. The sample variance follows

(a) Normal distribution

(b) Chi-square with N degrees of freedom

(c) Chi-square with $N - 1$ degrees of freedom

(d) t-distribution with $N - 1$ degrees of freedom

Answer: c

9. A car manufacturer purchases car batteries from two different suppliers. Supplier X provides 55% of the batteries and supplier Y provides the rest. 5% of all batteries from supplier X are defective and 4% of all batteries from supplier Y are defective. You select a battery from the bulk and you found it to be defective. What is the probability that it is from Supplier X?

(a) 0.0455

(b) 0.455

(c) 0.0275

(d) 0.018

Answer: c

10. Which one of the following is best measure of central tendency for categorical data?

(a) Mean

(b) Median

(c) Mode

(d) None of the above

Answer: c