MST-based Dependency Parsing

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Maximum Spanning Tree Based

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Basic Idea

Starting from all possible connections, find the maximum spanning tree.

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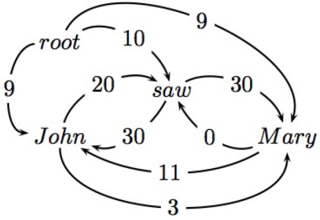
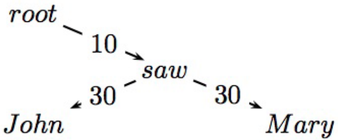
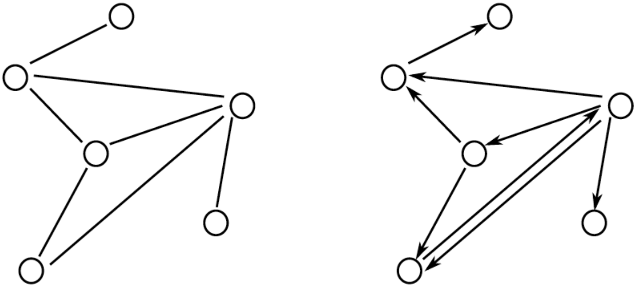




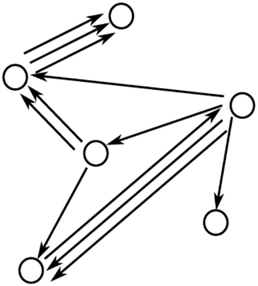
Some Graph Theory Reminders

- A graph $G = (V, A)$ is a set of vertices V and arcs $(i,j) \in A$ where $i,j \in V$.
- Undirected graphs: $(i,j) \in A \Leftrightarrow (j,i) \in A$
- Directed graphs (digraphs) : $(i,j) \in A \nRightarrow (j,i) \in A$



Multi-Digraphs

- A multi-digraph is a digraph where multiple arcs between vertices are possible
- $(i,j,k) \in A$ represents the k^{th} arc from vertex i to vertex j.



Directed Spanning Trees

- A directed spanning tree of a (multi-)digraph $G = (V,A)$ is a subgraph $G' = (V',A')$ such that:
  - $V' = V$
  - $A' \subseteq A$, and $|A'| = |V'| - 1$
  - G' is a tree (acyclic)

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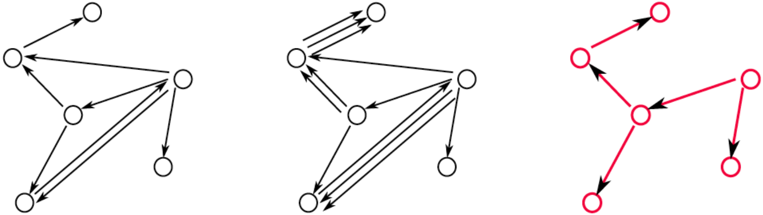
- A spanning tree of the following (multi-)digraphs



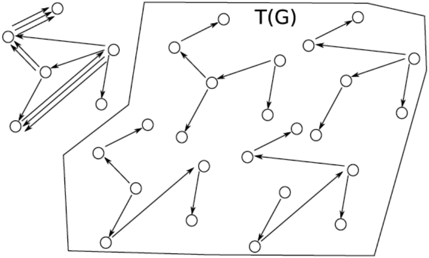
Weighted Directed Spanning Trees

- Assume we have a weight function for each arc in a multi-digraph $G = (V, A)$.
- Define ${w_{ij}}^k \geq 0$ to be the weight of $(i, j, k) \in A$ for a multi-digraph
- Define the weight of directed spanning tree G' of graph G as

$$w(G') = \sum_{(i,j,k) \in G'} {w_{ij}}^k$$

Maximum Spanning Trees (MST)

Let $T(G)$ be the set of all spanning trees for graph G



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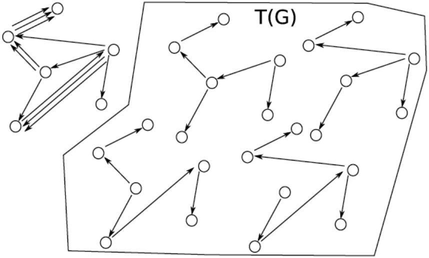

The MST problem

Find the spanning tree G' of the graph G that has the highest weight

$$G' = \arg\max_{G' \in T(G)} w(G') = \arg\max_{G' \in T(G)} \sum_{(i,j,k) \in G'} w_{ij}{}^{k}$$

Finding MST

Directed Graph

For each sentence x, define the directed graph $G_x = (V_x, E_x)$ given by

$$V_x = \{x_0 = root, x_1, \ldots, x_n\}$$

$$E_x = \{(i,j) : i \neq j, (i,j) \in [0:n] \times [1:n]\}$$

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G_x is a graph with

- the sentence words and the dummy root symbol as vertices and
- a directed edge between every pair of distinct words and
- a directed edge from the root symbol to every word

Chu-Liu-Edmonds Algorithm

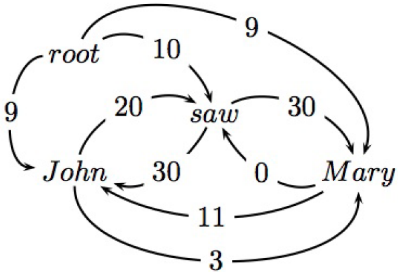
Chu-Liu-Edmonds Algorithm

- Each vertex in the graph greedily selects the incoming edge with the highest weight.
- If a tree results, it must be a maximum spanning tree.
- If not, there must be a cycle.
  - Identify the cycle and contract it into a single vertex.
  - Recalculate edge weights going into and out of the cycle.

Chu-Liu-Edmonds Algorithm

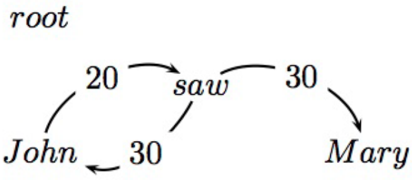
x = John saw Mary

- Build the directed graph



Chu-Liu-Edmonds Algorithm

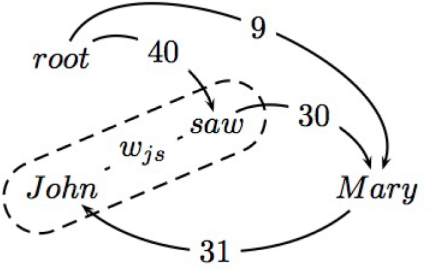
- Find the highest scoring incoming arc for each vertex



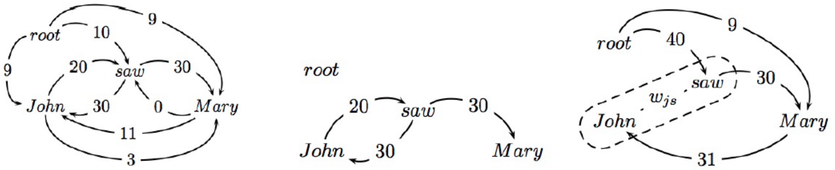
- If this is a tree, then we have found MST.

Chu-Liu-Edmonds Algorithm

- If not a tree, identify cycle and contract
- Recalculate arc weights into and out-of cycle



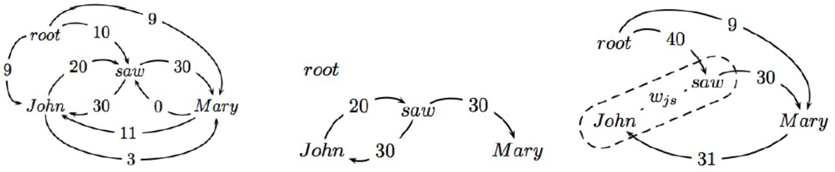
Chu-Liu-Edmonds Algorithm



Outgoing arc weights

- Equal to the max of outgoing arc over all vertices in cycle
- e.g., John → Mary is 3 and saw → Mary is 30.

Chu-Liu-Edmonds Algorithm

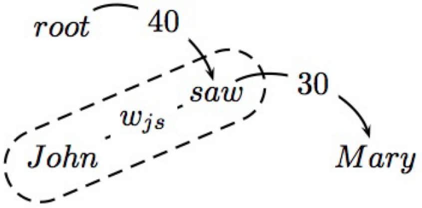


Incoming arc weights

- Equal to the weight of best spanning tree that includes head of incoming arc and all nodes in cycle
- root → saw → John is 40
- root → John → saw is 29

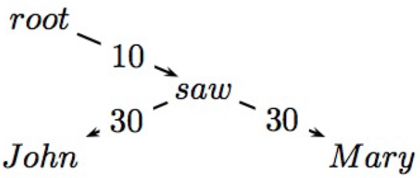
Chu-Liu-Edmonds Algorithm

Calling the algorithm again on the contracted graph:



- This is a tree and the MST for the contracted graph
- Go back up the recursive call and reconstruct final graph

Chu-Liu-Edmonds Algorithm



- The edge from w_{js} to *Mary* was from *saw*
- The edge from *root* to w_{js} represented a tree from *root* to *saw* to *John*.