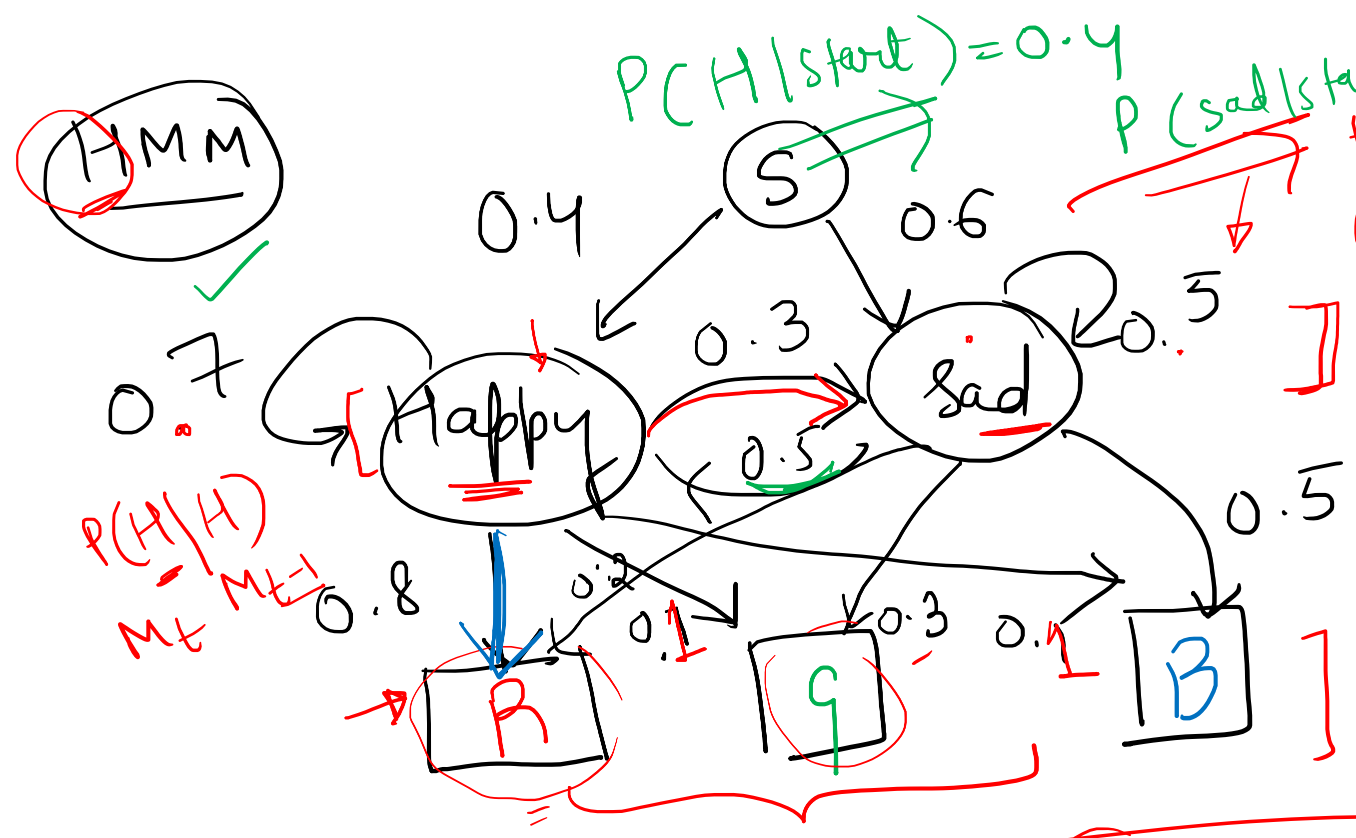
Week 4

1. Hidden Markov Model

Viterbi Algorithm

HMM



Hidden States

States] Mood

Observed States

Transition

$P(M_t | H)$

	H (M_t)	S	
H (M_{t-1})	H|H = 0.7	P(S|H) 0.3	= 1
S	H|S 0.5	0.5	= 1

Emission

	R	G	B
H	R|H 0.8	G|H 0.1	0.1
S	R|S 0.2	G|S 0.3	B|S 0.5

P(S|H) = 0.3
0.5

P(H|S) =

$$P(M_t | M_{t-1})$$

S : start state

t : current time

t-1 : previous time

** imp Rows

transition = 1

Transition prob $\equiv$ Hidden states $\equiv$ Mood

[H → S

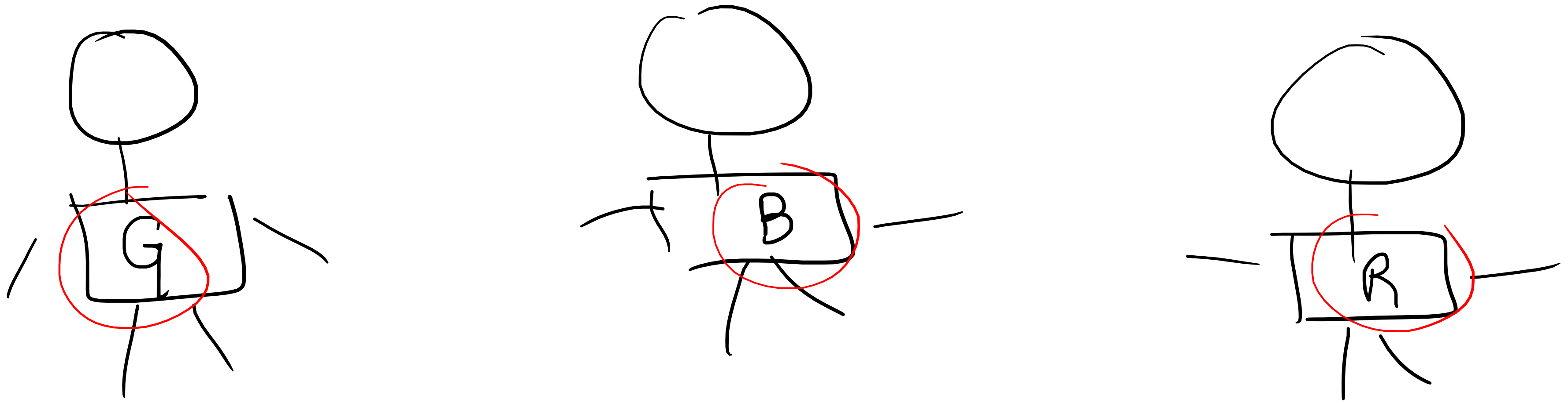
Emission prob $\equiv$ Observed states] R, G, B

Hidden states directly affect the observed states

HMM

Basic Idea of HMM

There are some hidden states which we don't know about ~~those hidden~~ but they directly affect the observed states.



G B B

G R R

Day 1 Day 2 Day 3

$C_1 = G$ $C_2 = B$ $C_3 = R$

$\{M_1, M_2, M_3\} = ?$

?

M: mood (hidden states)

C: obs

$$\text{Max } \mathbb{P}(C_1 = G,\ C_2 = B,\ C_3 = R,\ M_1 = m_1,\ M_2 = m_2,\ M_3 = m_3) \quad \Big| \quad M_t / M_{t-1}$$

obs states

hidden states

realisation of random var ($m_1 \to M_1$, $m_2 \to M_2$, $m_3 \to M_3$)

Ques: what is the most likely sequence** of moods of the prof on these 3 days?

mood $\to$ H

$\quad\ \ \to$ S

D_1	D_2	D_3
H / S	H / S	H / S

How many possible seq. of moods are there for 3 days?

$H = 2$ pos

$2^3 = 8$

__ __ __ $= 8$

$2 * 2 * 2$

$\Rightarrow 2 * 2 * 2$

$M_t \mid M_{t-1}$

$M_t \not\Rightarrow M_{t-1}$

HMM says the prob. of seeing the color seq.

q B R should be maximised the prob. of hidden states + obs. states

Maximize the prob ≡ Maximise the prob. of observed states & hidden states & find which ~~combinat~~ sequence of hidden states ? ✓

Assumptions

→ ~~Transition~~ prob = ?
→ Emission prob = ?

① color shirt worn on any given day only depends on the mood of the professor on that day & nothing else.

$P(C_i \mid M_i)$

② Markov assumption

$(M_t \mid M_{t-1})$

Mood on any given day depends directly on the mood on the very previous day

$$P(C_3 \mid C_2, C_1, M_3, M_2, M_1)$$

$$\times \; P(C_2 \mid C_1, M_3, M_2, M_1)$$

$$\times \; P(C_1 \mid M_3, M_2, M_1)$$

$$\times \; P(M_3 \mid M_2, M_1)$$

$$\times \; P(M_2 \mid M_1) \times$$

$$P(M_1)$$

$$P(C_3 \mid M_3)$$

$$\times \; P(C_2 \mid M_2)$$

$$\times \; P(C_1 \mid M_1)$$

$$\times \; P(M_3 \mid M_2)$$

$$\times \; P(M_2 \mid M_1)$$

$$\to \; P(M_1 \mid S)$$

SSH

SHS

HHH

HSH

SSS

SHH

HHS

HSS

$C = \{R, G, B\}$; $M = \{H, S\}$

Day 1

Day 2

Day 3

C_1

C_2

C_3

M_1

M_2

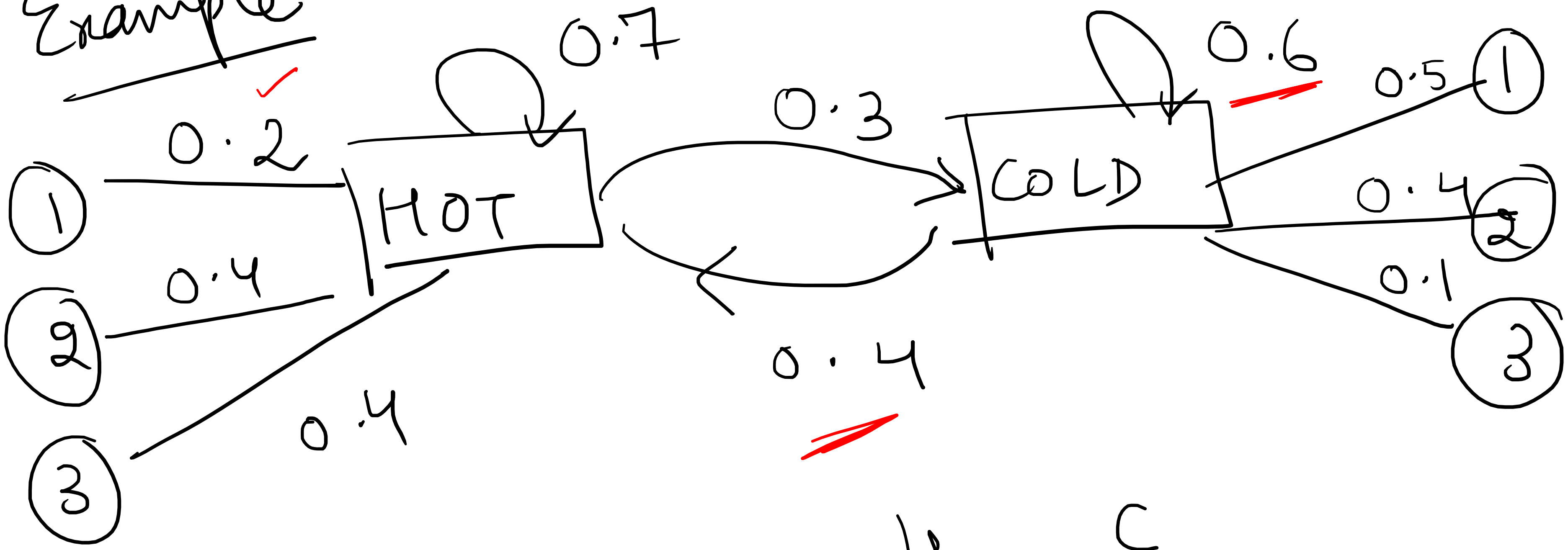
M_3 → S → H

RGBR G B

M_4

[mood of prof on day 3 ?

Example



$$\pi_{start} = \{ \overset{H}{0.8}, \overset{C}{0.2} \}$$

Transition matrix

	H	C
H	H/H 0.7	C/H 0.3 ✓
C	H/C 0.4	0.6 ✓

$\omega_t | \omega_{t-1}$

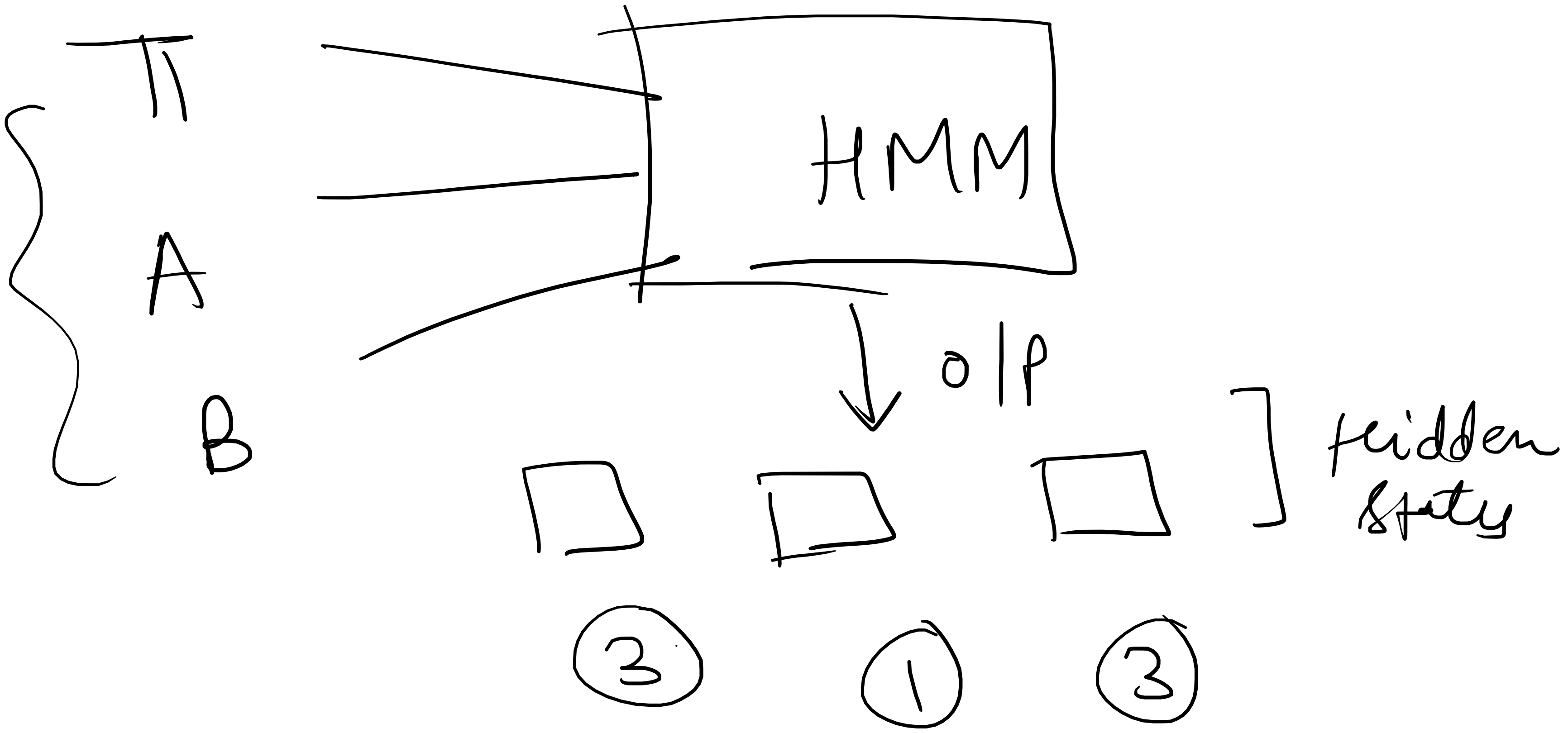
emission matrix

	1	2	3
H	0.2	2/H 0.4	3/H 0.4
C	1/C 0.5	0.4	0.1

Observable states
(# ice-creams a person eats)

3 1 3 } i/p

Model data



weather : hidden states ≡ Transition

$w = \{H, C\}$

ice-creams : observed states ≡ emission

$\{1, 2, 3\}$

3 1 3

T.C

$N_H = 2$; $T_0 = 3$

weather sequence?

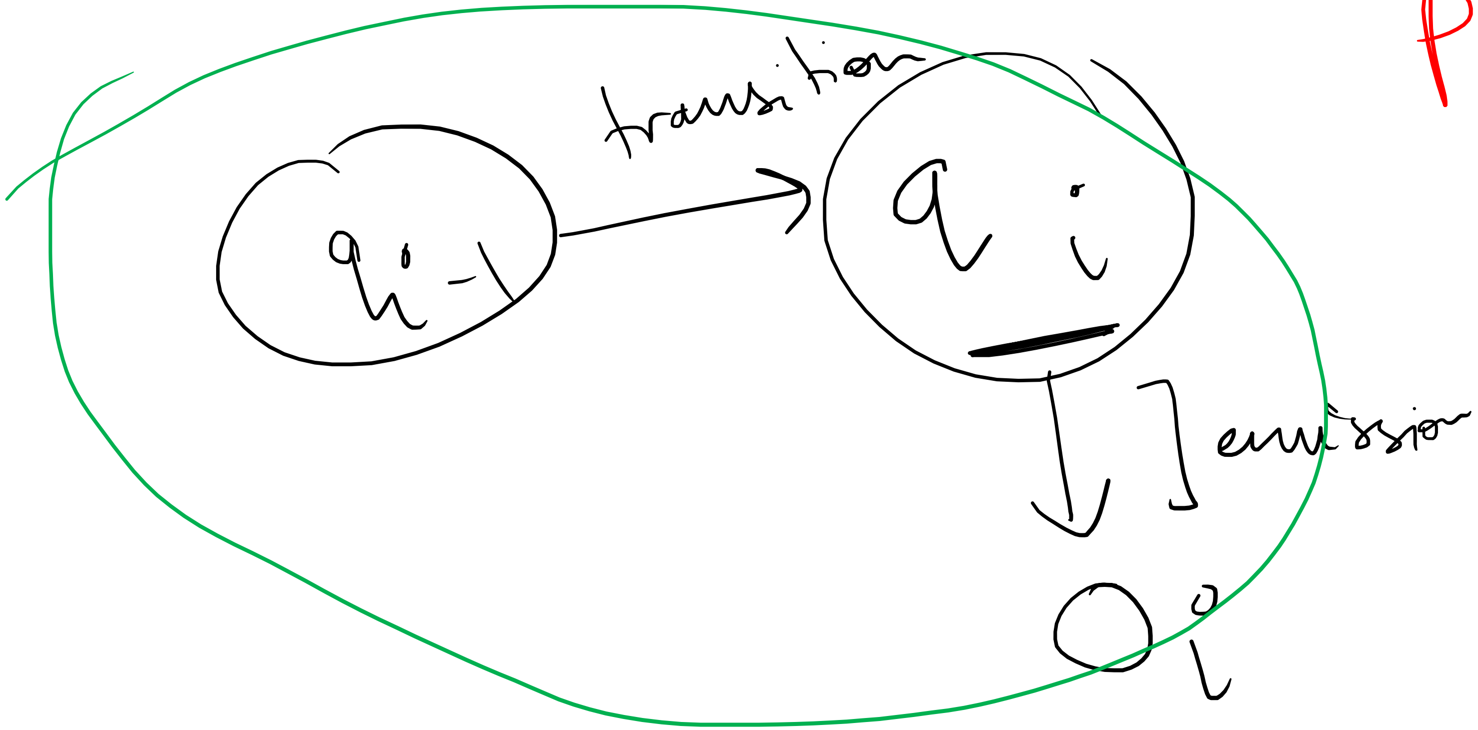
$2 \times 2 \times 2 = 8$

H H H

C C C

2^3

$$P(O \mid Q) = \arg\max \Big[\prod \overbrace{P(O_i \mid q_i)}^{\text{emission}} , \underbrace{P(q_i \mid q_{i-1})}_{\text{transition}} \Big]$$



⑧ ✓

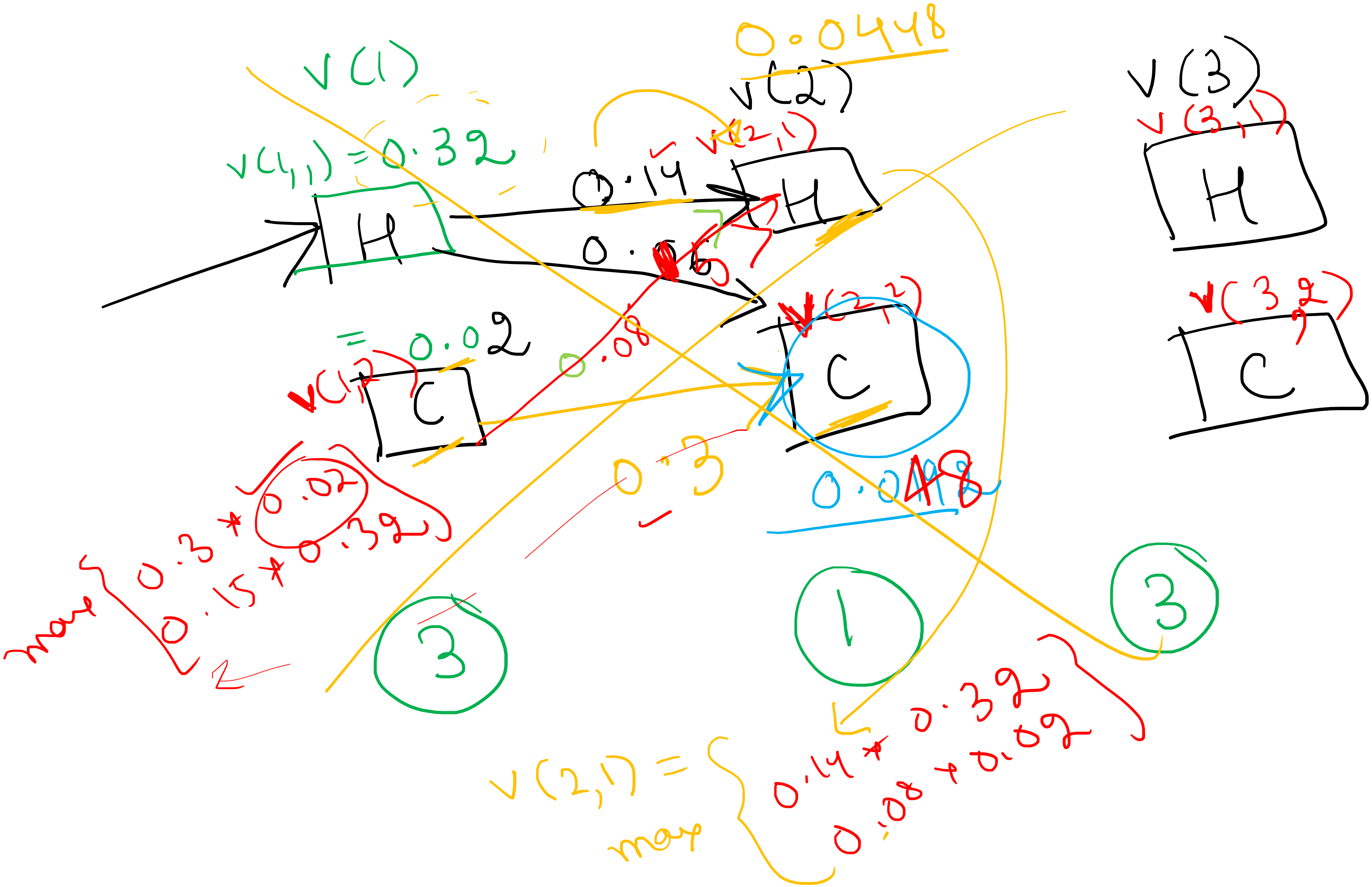
Hidden states = 50

obs states = 10

~~Brute force~~ 50^{10} ≈ million/billion

⇒ Dynamic prog

break down data into smaller chunks & store o/p of intermediate steps to be used in later steps.


0.0448
V(1)
V(2)
V(3)
V(1,1) = 0.32
H
0.14
V(2,1)
H
V(3,1)
H
= 0.02
V(1,2)
C
0.08
V(2,2)
C
V(3,2)
C
0.3
0.048
max{0.3 * 0.02, 0.15 * 0.32}
3
1
3
V(2,1) = max{0.14 * 0.32, 0.08 * 0.02}

$$P(A, B) = P(A|B) \cdot P(B)$$

Step 1 : V1

$v(1,1)$ $P(3, H) = P(3|H) \cdot P(H)$ $\pi P(H) =$

$= 0.4 \times 0.8 = 0.32$

$v(1,2)$ $P(3, C) = P(3|C) \cdot P(C) =$

$0.1 * 0.2 = 0.02$

Step 2 (i) $P(1, H) = P(1|H) \cdot P(H|H)$

$= 0.2 * 0.7 = 0.14$

(ii) $P(1, C) = P(1|C) \cdot \boxed{P(C|H)}$

$= 0.5 * 0.3 = \cancel{0.06}$ 0.15

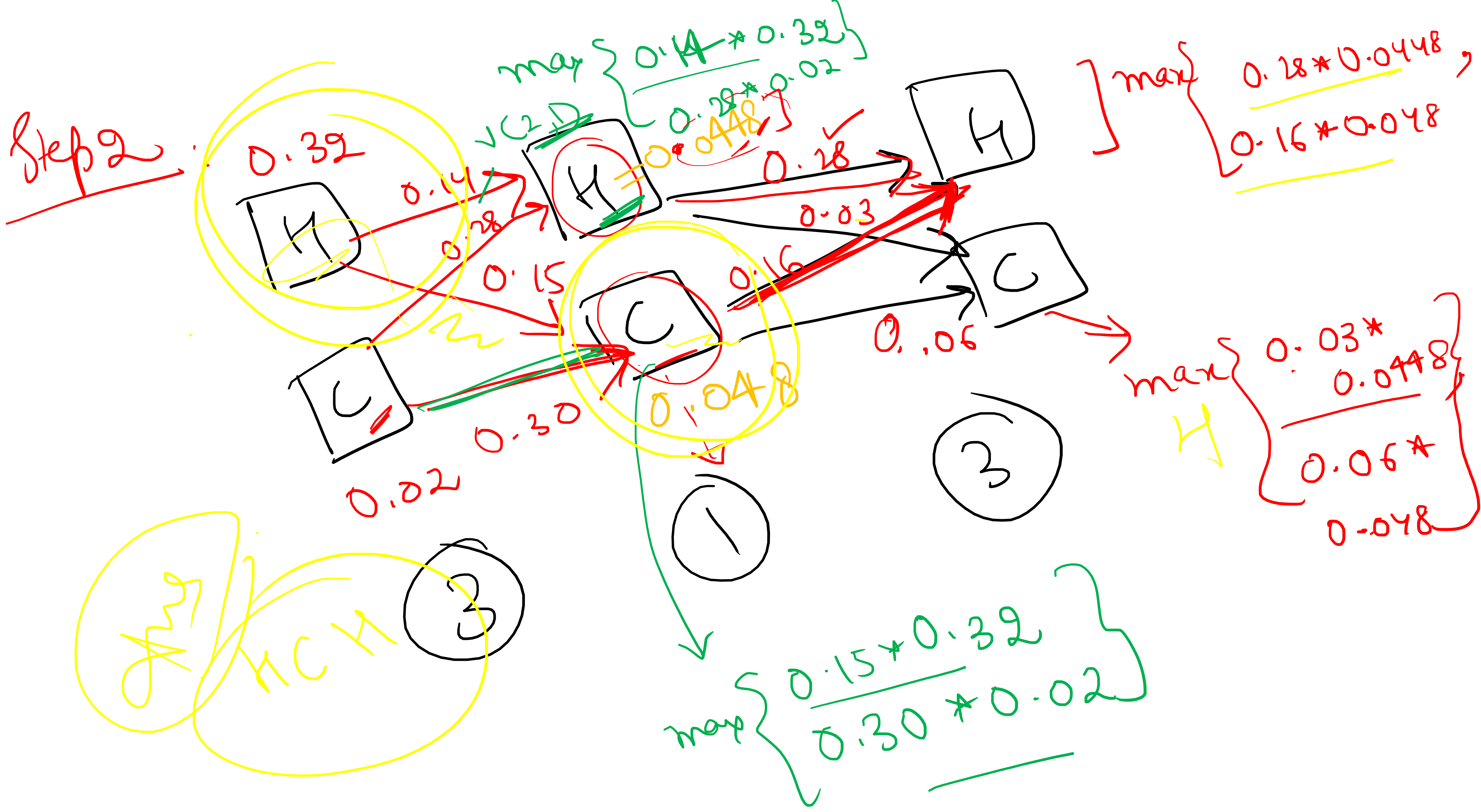
(iii) $P(1, C) = P(1|C) \cdot P(C|C)$

$= 0.5 * 0.6 =$ 0.3

0.08

(iv) $P(1, H) = P(1|H) \cdot P(H|C)$

$= 0.2 * 0.4 =$



Step 2
0.32
H
C
0.14
0.28
0.15
0.30
0.02
max{0.14 * 0.32, 0.28 * 0.02}
0.0448
H
C
0.048
max{0.15 * 0.32, 0.30 * 0.02}
0.28
0.03
0.16
0.06
H
C
max{0.28 * 0.0448, 0.16 * 0.048}
max{0.03 * 0.0448, 0.06 * 0.048}
H
0.048
1
3
Ans
HCH
3

Step 2:

$$P(1,H) = P(1|H) \cdot P(H|H)$$

$$0.2 * 0.7$$

$$= \qquad\qquad \underline{0.14}$$

$$P(1,C) = P(1|C) \cdot P(C|H)$$

$$= 0.5 * 0.3 = 0.15$$

$$P(1, H) = P(1|H) \cdot P(H|C)$$

$$= 0.2 * 0.4 = 0.08$$

$$P(1, C) = P(1|C) \cdot P(C|C)$$

$$= 0.5 * 0.6 = 0.30$$

Step 3 :

$P(3, H) = P(3 \mid H) \cdot P(H \mid H)$

$= 0.4 * 0.7 = 0.28$

$P(3, H) = P(3 \mid H) \cdot P(H \mid C)$

$= 0.4 * 0.4 = 0.16$

$$P(3,C) = P(3|C) \cdot P(C|H)$$

$$= 0.1 * 0.3 = 0.03$$

$$P(3,C) = P(3|C) \cdot P(C|C)$$

$$= 0.1 * 0.6 = \underline{0.06}$$