

Confidence Interval

A range computed using sample statistics to estimate an unknown population parameter with a stated level of confidence.

The most common value for α is 0.05 and typically 95% confidence intervals is constructed.

Confidence interval provides an interval, or a range of values, which is expected to cover the true unknown parameter. This provides richer information in comparison to point estimate, where we have only a single value, thus exposing vulnerability in the single estimate.

Confidence Interval Calculation With example:

Confidence Interval is calculated as

$$\text{Xbar} \pm Z * (\text{sigma}/\text{sqrt}(n))$$

Where **Xbar** is the sample mean, **sigma** is the population standard deviation, and **n** is the sample size. $\text{Sigma}/\text{sqrt}(n)$ is the **Standard Error**. Standard Error is the Standard Deviation in a Sampling Distribution. Z value for 95% confidence level from the Z distribution table is **1.96**.

Example:

Assume for a sample size of 1000 students, their average marks are 82 with a population standard deviation of 10 marks.

Solution - Here, $n = 1000$

Xbar = 82

Sigma = 10

Step 1: Calculate the Standard Error

Standard Error, S.E = $10/\text{sqrt}(1000) \Rightarrow 0.316$

Step 2: Calculate the Margin of Error

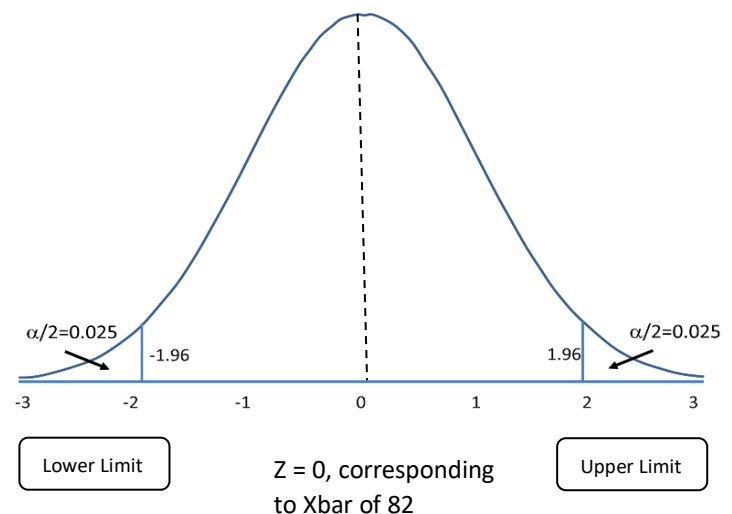
Margin of Error, M.o.E $\Rightarrow \pm (1.96 * 0.316)$

M.o.E $\Rightarrow +0.619$ and -0.619

Step 3: Add the Sample mean to Margin of Error

Lower Confidence Interval, C.I Lower = $82 - 0.619 \Rightarrow 81.381$

Upper Confidence Interval, C.I Upper = $82 + 0.619 \Rightarrow 82.619$



Conclusion:

We are 95% confident that the mean of the student's marks will be between 81.4 and 82.6

Note: For a T-test, instead of Population Standard deviation, we take the sample standard deviation, and by calculating the degrees of freedom ($n-1$), we can get the corresponding T value at 95% confidence level instead of Z value.