# COMPUTER VISION

## VI. Sturm Sequence and Bisection Method

- Using the Sturm sequence, I have determined how many eigenvalues reside for a given interval. This program is user interactive.
- Using the bisection method, I have determined one of the eigenvalues to a prescribed accuracy.

**INTRODUCTION:**

The Strum sequence for a matrix refers to the sequence of number of eigenvalues of the matrix obtained by subtracting a scalar value(λ) from the diagonal of the covariance matrix.

**PROCEDURE**

Given a covariance matrix C, we can obtain a sequence of matrices by subtracting a scalar value λ from the diagonal elements of C, resulting in a new matrix $D(\lambda) = C - \lambda I$, where I is the identity matrix.

For each value of λ, we can then compute the number of eigenvalues of D(λ) by constructing the corresponding Strum sequence. The number of sign changes in the Strum sequence gives the number of eigenvalues of D(λ). By sweeping λ over a range of values, we can compute the number of eigenvalues of C that lie within a given interval.

The Sturm sequence is particularly useful for finding the eigenvalues of a matrix. The eigenvalues can be found by counting the number of sign changes in the Sturm sequence of the characteristic polynomial of the matrix in a given interval.

The bisection method is a numerical method used to find the roots of a continuous function. It works by repeatedly bisecting an interval and then selecting the subinterval in which the function changes sign, and then repeating the process on that subinterval. This method is guaranteed to converge to a root as long as the function is continuous and changes sign on the interval. The bisection method is simple and robust, but it can be slow to converge compared to other methods such as Newton's method or the secant method.

**FORMULAS & PROCEDURE:**

The bisection method for finding an eigenvalue can be modified to use the Sturm sequence property as follows:

1. Initialize the interval [lambda1, lambda2] where the eigenvalue lies.
2. Compute the number of sign changes in the Sturm sequence of the polynomial p(lambda) for lambda = lambda1 and lambda = lambda2.
3. If the number of sign changes is the same at lambda1 and lambda2, the eigenvalue does not lie in the interval [lambda1, lambda2]. Stop the iteration.
4. While the length of the interval [lambda1, lambda2] is greater than the desired tolerance, do the following:
   - Compute the midpoint lambda3 = (lambda1 + lambda2) / 2.
   - Compute the number of sign changes in the Sturm sequence of p(lambda) for lambda = lambda3.
   - If the number of sign changes is the same as at lambda1, set lambda1 = lambda3.
   - Otherwise, set lambda2 = lambda3.
5. The eigenvalue is the midpoint of the final interval [lambda1, lambda2].

**RESULTS & CONCLUSION:**

The Strum sequence is a powerful tool for determining the number of eigenvalues of a symmetric matrix within a given interval. By leveraging the properties of the Strum sequence, it is possible to use the bisection method to find one of the eigenvalues within a prescribed accuracy.

The bisection method is guaranteed to converge to an eigenvalue of a matrix, as long as the matrix is real and symmetric and the interval [a, b] contains an eigenvalue. However, the convergence rate of the method can be slow, especially for matrices with multiple eigenvalues that are close together.

Overall, the combination of the Strum sequence and the bisection method provides a robust and efficient approach to finding eigenvalues of symmetric matrices. It is widely used in a variety of applications, including machine learning, quantum mechanics, and computational biology.

# CODE