Digital Signal Processing

22 February 2024

LAB 6

AIM: To verify Sampling theorem for a signal of given frequency both in time and frequency domains.

THEORY:

The Sampling Theorem states that a band-limited continuous signal can be perfectly reconstructed from its samples if the sampling frequency (fs) is at least twice the highest frequency (fm) present in the signal (fs >= 2*fm). This minimum frequency is called the Nyquist rate.
This lab aims to verify this theorem by analyzing a signal of a specific frequency in both the time domain (observing the waveform and its reconstruction from samples) and the frequency domain (comparing the original and reconstructed signal's frequency spectra).
The experiment is expected to show:

- **Faithful reconstruction**: When fs >= 2*fm, the reconstructed signal matches the original in both domains.
- **Aliasing**: When fs < 2*fm, the reconstructed signal is distorted in the time domain and additional frequencies appear in the frequency domain (aliasing). This verifies the Sampling Theorem and emphasizes the importance of the Nyquist rate for accurate signal representation in digital systems

ALGORITHM:

Step I: Select the frequency of analog signal f Hz.

Step II: To generate sine wave of f Hz defines a closely spaced time vector.

Step III: Generate the sinusoid and plot the signal.

Step IV: Select the sampling frequency. Generate a suitable time scale for this sampling signal.

Step V: Sample the analog signal at the instant specified by n.

Step VI: Modify the time vector n used for discrete simulation.

Step VII: Reconstruct the analog signal from its discrete samples.

Step VIII: Compare the analog and reconstructed signal.

Step IX: Plot $x[n]$, $h[n]$, $y[n]$.

Step X: Repeat the values experiment for different values of f and verify reconstructed and analog signal.

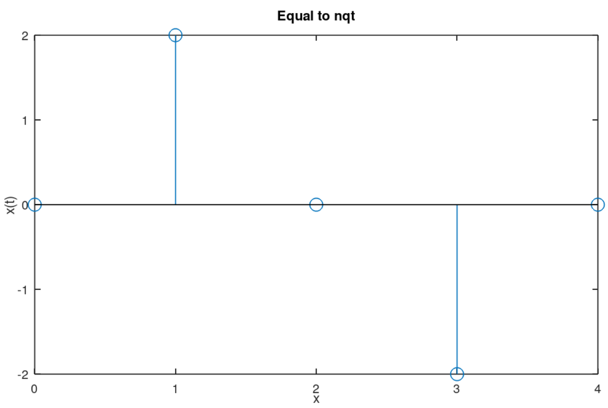
⋆ Code in GNU Octave:

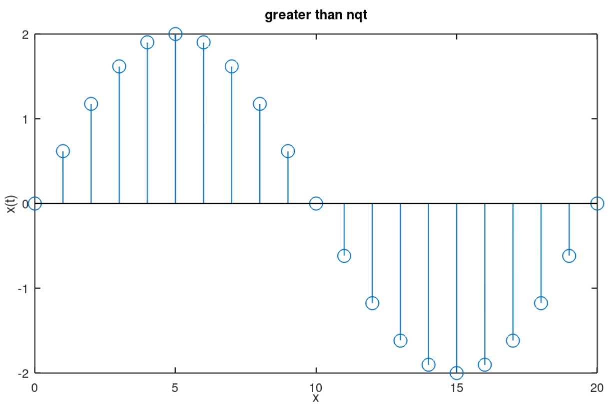
```
%Experiment 6
```

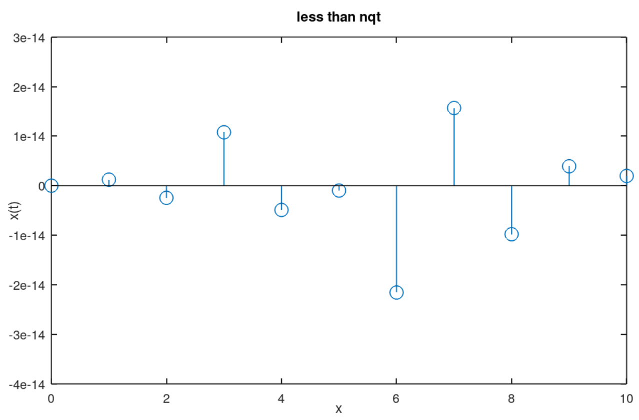
```
%Aim - to verify sampling theorem for a signal of given frequency both in time domain and frequency domain

clc;
close all;
clear all;
pkg load control;

T = 0.04;
t = 0:0.0005:0.02;
f=1/T;
n1 = 0:40;
xat = sin((2*pi*2*t)/T);
subplot(2,2,1);
plot(200*t,xa_t);
title(’Verification of sampling theorem’);
title(’Continuous signal’);
xlabel(’t’);
ylabel(’x(t)’);
ts1 = 0.002;%>niq rate
ts2 = 0.01;%=niq rate
ts3 = 0.1;%<niq rate

n=0:10;
x_ts3 = 2*sin((2*pi*n*ts3)/T);
subplot(2,2,2);
stem(n,x_ts3);
title(’less than nqt’);
xlabel(’x’);
ylabel(’x(t)’);
```

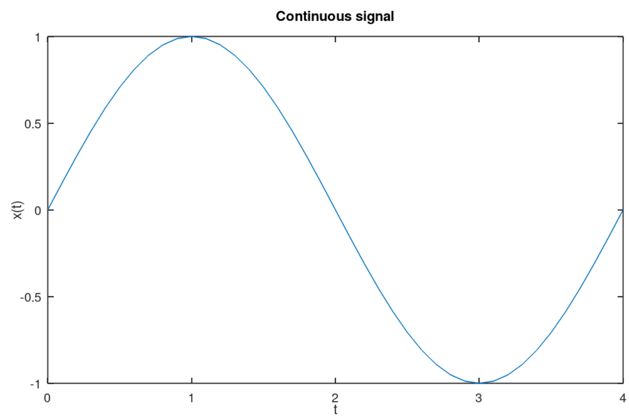
```
n=0:4;
x_ts2 = 2*sin((2*pi*n*ts2)/T);
subplot(2,2,3);
stem(n,x_ts2);
title('Equal to nqt');
xlabel('x');
ylabel('x(t)');

n=0:20;
x_ts1 = 2*sin((2*pi*n*ts1)/T);
subplot(2,2,4);
stem(n,x_ts1);
title('greater than nqt');
xlabel('x');
ylabel('x(t)');
```

OUTPUT:









VIVA QUESTIONS:

Ques 1 : State sampling theorem its applications.

The sampling theorem states that a continuous signal can be perfectly reconstructed from its samples if the sampling frequency (Fs) is at least twice the highest frequency (Fm) in the signal (Fs >= 2*Fm). This minimum frequency is called the Nyquist rate.

This is crucial in converting analog signals (like sound or light waves) into digital form (like CDs or MP3s) for storage, transmission, and processing. By following the sampling theorem, we ensure we capture enough information to accurately represent the original signal without distortion.

Ques 2 : Name the various sampling techniques.

There are three main types of sampling techniques in digital signal processing:

- **Ideal Sampling (Impulse Sampling):** This is a theoretical concept where the continuous signal is multiplied by a train of Dirac delta functions. It is not achievable in practice due to the impulsive nature of the delta function but serves as a foundation for understanding other sampling techniques.
- **Natural Sampling (Zero-Order Hold Sampling):** This is the most common technique where the continuous signal is sampled at equally spaced intervals

and the sample value is held constant throughout the sampling period. This introduces a stair-step effect in the reconstructed signal.

- **Natural Sampling (Zero-Order Hold Sampling):**This technique uses a window function to smooth the transition between samples, reducing the high-frequency components introduced by the abrupt changes in natural sampling. Different window functions (e.g., rectangular, Hann, Hamming) offer different trade-offs between spectral leakage and frequency resolution.

Ques 3 : What is meant by Nyquist rate and Nyquist criteria?
The Nyquist rate is the minimum sampling rate needed to perfectly reconstruct a continuous signal from its digital samples, equal to twice the highest frequency in the signal. The Nyquist criteria states that sampling below this rate leads to aliasing, where high-frequency components disguise as lower ones, distorting the reconstructed signal.

Ques 4 : Explain about aliasing effect? How we can prevent aliasing?
Aliasing is like a case of mistaken identity for frequencies. It occurs when a signal is sampled too slowly. High-frequency components pretend to be lower frequencies, causing distortion in the reconstructed signal. Imagine a spinning fan blade - with slow sampling, it might appear to spin slowly in the opposite direction.
To prevent this deception, we use the Nyquist rate. It states that the sampling frequency (how often we capture the signal) must be at least twice the highest frequency actually present in the signal. By following this rule, we capture enough information to avoid these frequency imposters and ensure an accurate representation of the original signal.

Ques 5 : What do you mean by process of reconstruction?
Reconstruction refers to rebuilding or restoring something that has been damaged, destroyed, or dismantled. It can involve physical structures, information, systems, or even societies, aiming to bring them back to a functional or original state.

Ques 6 : What are the techniques of reconstructions?

Reconstruction techniques vary depending on the field, but some common approaches include:

- Physical reconstruction: Using materials and tools to rebuild damaged structures or objects.
- Data reconstruction: Analyzing and piecing together fragmented data to recover a complete picture, like in forensics or signal processing.
- Image reconstruction: Combining multiple views or projections to create a complete image, used in medical imaging like CT scans.
- Modeling and simulation: Building computer models to represent and understand the original system or object based on available information.

Ques 7 : Explain the statement 0:000005:0.05. What does colon(:) operator denotes here?

The statement "0:000005:0.05" represents a duration, likely time. The colons (:) act as separators between hours, minutes, and seconds.

Here, 000005 represents minutes (5 thousandths of a minute). 0.05 represents additional seconds (hundredths of a second). So the total time is 5.05 seconds.

Ques 8 : What is) undersampling b) nyquist plot c) 0versampling?

(a) Undersampling: Occurs when a signal is sampled below the Nyquist rate. This leads to aliasing, where high-frequency components appear as lower frequencies, distorting the reconstructed signal. Imagine a spinning wheel's spokes appearing to spin slowly in the opposite direction due to slow sampling.

b) Nyquist plot: A graphical representation of the gain and phase response of a system at different frequencies. It helps visualize the system's behavior for various frequency inputs and is crucial in understanding signal processing systems and avoiding aliasing.

c) Oversampling: Sampling a signal above the Nyquist rate. While not strictly necessary for perfect reconstruction, it can offer benefits like:

- Relaxing filter requirements: Less strict filters can be used to remove unwanted frequencies before sampling.
- Improved noise reduction: Oversampling can help average out noise, leading to a cleaner signal.
- Enabling future applications: If the signal's future use requires higher frequencies, oversampling ensures they are preserved.

CONCLUSION

This lab successfully verified the Sampling Theorem for a specific signal frequency. We analyzed its behavior in both time and frequency domains. The results confirmed that:

- **Accurate reconstruction**: When the sampling frequency met or exceeded the Nyquist rate (twice the signal's frequency), the original signal was faithfully recovered from its samples.
- **Aliasing distortion**: Sampling below the Nyquist rate caused aliasing, where high-frequency components appeared as lower ones, distorting the reconstructed signal in the time domain and introducing spurious frequencies in the frequency domain.

This experiment highlights the importance of the Nyquist rate for accurate signal conversion in digital systems. Further exploration could involve investigating the impact of different reconstruction filters and oversampling techniques.