Unit 2
Computer Arithmetic

Lecture 1

Number Systems

Here we discuss positional number systems with Positive radix (or base) ***r***. A number with radix ***r*** is represented by a string of digits as below i.e. wherever you see numbers of whatever bases, all numbers can be written in general as:

Radix point

$$\underbrace{A_{n-1}A_{n-2}\dots A_1A_0}_{\text{Integer portion (n digits)}} \,.\, \underbrace{A_{-1}A_{-2}\dots A_{-m+1}A_{-m}}_{\text{Fractional portion (m digits)}}$$

Most Significant Digit (MSD) Least Significant Digit (LSD)

Decimal Number System (Base-10 system)

- Radix (r) = 10
- Symbols = 0 through r-1 = 0 through 10-1 = {0, 1, 2... 8, 9}
- starting from base-10 system since it is used vastly in everyday arithmetic besides computers to represent numbers by strings of digits or symbols defined above, possibly with a *decimal point*. Depending on its position in the string, each digit has an associated value of an integer raised to the power of 10.
- Example: decimal number 724.5 is interpreted to represent 7 hundreds plus 2 tens plus 4 units plus *5*
- tenths.
  - 724.5 = 7 X 10^2 + 2 X 10^1 + 4 X 10^0 *+5* X 10^-1

Binary Number System (Base-2 system)

- Radix (r) = 2
- Symbols = 0 through r-1 = 0 through 2-1 = {0, 1}
- A binary numbers are expressed with a string of 1'sand 0's and, possibly, a *binary point* within it. The decimal equivalent of a binary number can be found by expanding the number into a power series with a base of 2.
- Example: $(11010.01)_2$ can be interpreted using power series as:
- $(11010.01)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} = (26.25)_{10}$
- Digits in a binary number are called bits (**B**inary dig**it**s).
- When a bit is equal to 0, it does not contribute to the sum during the conversion. Therefore, the conversion to decimal can be obtained by adding the numbers with powers of 2 corresponding to the bits that are equal to 1.
- Looking at above example, $(11010.01)_2 = 16 + 8 + 2 + 0.25 = (26.25)_{10}$.

In computer work,

- 2^{10} is referred to as K (kilo),
- 2^{20} as M (mega),
- 2^{30} as G (giga),
- 2^{40} as T (tera) and so on.

Octal Number System (Base-8 system)

- Radix (r) = 8
- Table: Numbers obtained from 2 to the power of n
- Symbols = 0 through r-1 = 0 through 8-1 = {0, 1, 2…6, 7}
- An octal numbers are expressed with a strings of symbols defined above, possibly, an *octal point* within it. The decimal equivalent of a octal number can be found by expanding the number into a power series with a base of 8.
- Example: $(40712.56)_8$ can be interpreted using power series as:
- $(40712.56)_8 = 4 \times 8^4 + 0 \times 8^3 + 7 \times 8^2 + 1 \times 8^1 + 2 \times 8^0 + 5 \times 8^{-1} + 6 \times 8^{-2} = (16842.1)_{10}$

- **Hexadecimal Number System (Base-16 system)**
- Radix (r) = 16
- Symbols = 0 through r-1 = 0 through 16-1 = {0, 1, 2…9, A, B, C, D, E, F}
-
- A hexadecimal numbers are expressed with a strings of symbols defined above, possibly, a *hexadecimal point* with in it. The decimal equivalent of a hexadecimal number can be found by expanding the number into a power series with a base of 16.
- Example: $(4D71B.C6)_{16}$ can be interpreted using power series as:

$= 4 \times 16^4 + 13 \times 16^3 + 7 \times 16^2 + 1 \times 16^1 + 11 \times 16^0 + 12 \times 16^{-1} + 6 \times 16^{-2}$

$= (317211.7734375)_{10}$

Complements

- Complements are used in digital computers for simplifying the subtraction operation and for logical manipulation.
- There are two types of complements for each base-r system: r's complement and the second as the (r - 1)'s complement.
- When the value of the base *r* is substituted, the two types are referred to as the 2's complement and 1's complement for binary numbers, the 10's complement and 9's complement for decimal numbers etc.

(r-1)'s Complement (diminished radix compl.)

(r-1)'s complement of a number N is defined as $(r^n - 1) - N$

Where **N** is the given number
r is the base of number system
n is the number of digits in the given number

To get the (r-1)'s complement fast, subtract each digit of a number from (r-1).

Example:

- 9's complement of 835_{10} is 164_{10} (Rule: $(10^n - 1) - N$)
- 1's complement of 1010_2 is 0101_2 (bit by bit complement operation)

r's Complement (radix complement)

r's complement of a number N is defined as $r^n - N$

Where **N** is the given number
r is the base of number system
n is the number of digits in the given number

To get the r's complement fast, add 1 to the low-order digit of its (r-1)'s complement.

Example:

- 10's complement of 835_{10} is $164_{10} + 1 = 165_{10}$
- 2's complement of 10102 is $0101_2 + 1 = 0110_2$

Subtraction with complements

- The direct method of subtraction in elementary schools uses the borrow concept.
- When subtraction is implemented with digital hardware, this method is found to be less efficient than the method that uses complements.
- The subtraction of two n-digit unsigned numbers *M* - *N* in base-*r* can be done as follows:

1. Add the minuend *M* to the *r's* complement of the subtrahend *N.* This performs
   - *M + (r^n–N) = M -N + r".*
2. If *M >= N,* the sum will produce an end carry, *r",* which is discarded; what is left is the result *M - N.*
3. If *M < N,* the sum does not produce an end carry and is equal to *r^n - (N - M),* which is
   - the *r's* complement of *(N - M).* To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

Example I:

Using 10's complement, subtract 72532 − 3250.

$$
\begin{aligned}
M &= 72532 \\
10\text{'s complement of } N &= +\ \underline{96750} \\
\text{Sum} &= 169282 \\
\text{Discard end carry } 10^5 &= -\underline{100000} \\
\text{Answer} &= 69282
\end{aligned}
$$

Example II:

Using 10's complement, subtract 3250 − 72532.

$$
\begin{aligned}
M &= \quad 03250 \\
\text{10's complement of } N &= +\ \underline{27468} \\
\text{Sum} &= \quad 30718
\end{aligned}
$$

There is no end carry.

Answer: −(10's complement of 30718) = −69282

Perform the following (r’s complement)

1. 234 – 1278
2. 1234-567
3. 345-1356
4. 1789-367