

(*Note that w=v^α*)

$$\text{In}[]:= \text{Solve}\left[1^2 w^2 + w^4 \tau^2 + 2 1 w^3 \tau \cos\left[\frac{\pi \alpha}{2}\right] - m^2 == 0, w\right]$$

$$\begin{aligned} \text{Out}[]:= & \left\{ \left\{ w \rightarrow -\frac{1 \cos\left[\frac{\pi \alpha}{2}\right]}{2 \tau} - \frac{1}{2} \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \right.} \right. \right. \\ & \left. \left(2^{1/3} (1^4 - 12 m^2 \tau^2)\right) \Bigg/ \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\ & \left. \left. \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) + \\ & \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \\ & \left. \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \Bigg) - \\ & \frac{1}{2} \sqrt{\left(-\frac{4 1^2}{3 \tau^2} + \frac{2 1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} - \left(2^{1/3} (1^4 - 12 m^2 \tau^2)\right) \Bigg/ \right.} \\ & \left. \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\ & \left. \left. \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) - \\ & \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \\ & \left. \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} - \\ & \left(\frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]}{\tau^3} - \frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]^3}{\tau^3}\right) \Bigg/ \left(4 \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \right. \right. \\ & \left. \left(2^{1/3} (1^4 - 12 m^2 \tau^2)\right) \Bigg/ \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\ & \left. \left. \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) + \\ & \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \end{aligned}$$

$$\begin{aligned}
& \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3} \Bigg) \Bigg) \Bigg) \Bigg\}, \\
& \left\{ \mathbf{w} \rightarrow -\frac{1 \cos\left[\frac{\pi \alpha}{2}\right]}{2 \tau} - \frac{1}{2} \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \left(2^{1/3} \left(1^4 - 12 m^2 \tau^2\right)\right)\right)} \right. \\
& \quad \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3}\right) + \right. \\
& \quad \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3}\right) + \right. \\
& \quad \left. \frac{1}{2} \sqrt{\left(-\frac{4 1^2}{3 \tau^2} + \frac{2 1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} - \left(2^{1/3} \left(1^4 - 12 m^2 \tau^2\right)\right)\right)} \right. \\
& \quad \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3}\right) - \right. \\
& \quad \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3} - \right. \right. \\
& \quad \left. \left. \left(\frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]}{\tau^3} - \frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]^3}{\tau^3}\right) \right) \right. \left(4 \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \right. \right. \right. \\
& \quad \left. \left. \left(2^{1/3} \left(1^4 - 12 m^2 \tau^2\right)\right) \right) \right. \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3}\right) + \right. \\
& \quad \left. \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3}\right) \right) \Bigg) \Bigg) \Bigg) \Bigg\},
\end{aligned}$$

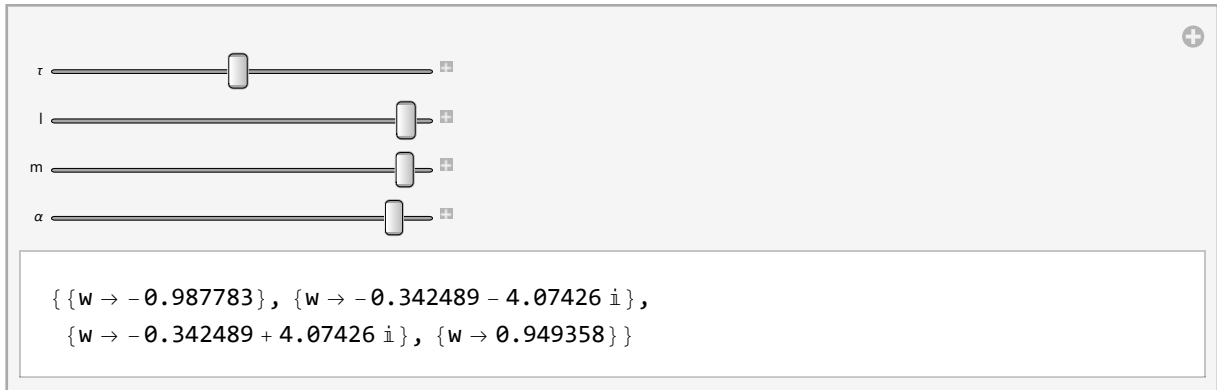
$$\begin{aligned}
& \left\{ \mathbf{w} \rightarrow -\frac{1 \cos\left[\frac{\pi \alpha}{2}\right]}{2 \tau} + \frac{1}{2} \sqrt{\left(-\frac{2 \mathbf{l}^2}{3 \tau^2} + \frac{\mathbf{l}^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \left(2^{1/3} \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)\right)\right)} \right. \\
& \quad \left(3 \tau^2 \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)^3 + \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) + \\
& \quad \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \\
& \quad \left. \sqrt{-4 \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)^3 + \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) - \\
& \quad \frac{1}{2} \sqrt{\left(-\frac{4 \mathbf{l}^2}{3 \tau^2} + \frac{2 \mathbf{l}^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} - \left(2^{1/3} \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)\right)\right)} \right. \\
& \quad \left(3 \tau^2 \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)^3 + \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) - \\
& \quad \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \\
& \quad \left. \sqrt{-4 \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)^3 + \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} + \\
& \quad \left(\frac{8 \mathbf{l}^3 \cos\left[\frac{\pi \alpha}{2}\right]}{\tau^3} - \frac{8 \mathbf{l}^3 \cos\left[\frac{\pi \alpha}{2}\right]^3}{\tau^3} \right) \left/ \left(4 \sqrt{\left(-\frac{2 \mathbf{l}^2}{3 \tau^2} + \frac{\mathbf{l}^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \right. \right. \right. \\
& \quad \left. \left. \left(2^{1/3} \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)\right)\right) \right/ \left(3 \tau^2 \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)^3 + \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) + \\
& \quad \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)^3 + \left(2 \mathbf{l}^6 + 72 \mathbf{l}^2 \mathbf{m}^2 \tau^2 - 108 \mathbf{l}^2 \mathbf{m}^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right) \right) \right\}, \\
& \left\{ \mathbf{w} \rightarrow -\frac{1 \cos\left[\frac{\pi \alpha}{2}\right]}{2 \tau} + \frac{1}{2} \sqrt{\left(-\frac{2 \mathbf{l}^2}{3 \tau^2} + \frac{\mathbf{l}^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \left(2^{1/3} \left(\mathbf{l}^4 - 12 \mathbf{m}^2 \tau^2\right)\right)\right)} \right/
\end{aligned}$$

$$\begin{aligned}
& \left(3 \tau^2 \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 (l^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 \right)^2} \right)^{1/3} \right) + \\
& \quad \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 + \right. \\
& \quad \left. \sqrt{-4 (l^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 \right)^2} \right)^{1/3} \right) + \\
& \quad \frac{1}{2} \sqrt{\left(-\frac{4 l^2}{3 \tau^2} + \frac{2 l^2 \cos \left[\frac{\pi \alpha}{2} \right]^2}{\tau^2} - (2^{1/3} (l^4 - 12 m^2 \tau^2)) \right) /} \\
& \quad \left(3 \tau^2 \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 (l^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 \right)^2} \right)^{1/3} \right) - \\
& \quad \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 + \right. \\
& \quad \left. \sqrt{-4 (l^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 \right)^2} \right)^{1/3} + \\
& \quad \left(\frac{8 l^3 \cos \left[\frac{\pi \alpha}{2} \right]}{\tau^3} - \frac{8 l^3 \cos \left[\frac{\pi \alpha}{2} \right]^3}{\tau^3} \right) / \left(4 \sqrt{\left(-\frac{2 l^2}{3 \tau^2} + \frac{l^2 \cos \left[\frac{\pi \alpha}{2} \right]^2}{\tau^2} + \right. \right. \\
& \quad \left. \left. (2^{1/3} (l^4 - 12 m^2 \tau^2)) \right) / \left(3 \tau^2 \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{-4 (l^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 \right)^2} \right)^{1/3} \right) \right) + \\
& \quad \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{-4 (l^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos \left[\frac{\pi \alpha}{2} \right]^2 \right)^2} \right)^{1/3} \right) \right) \right) \} \}
\end{aligned}$$

(*Only Fourth root is positive for different l, m, τ and α *)

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In[4]:= Manipulate[NSolve[1^2 w^2 + w^4 τ^2 + 2 l w^3 τ Cos[ $\frac{\pi \alpha}{2}$ ] - m^2 == 0, w],
  {τ, 0, 5}, {l, 0, 10}, {m, 0, 10}, {α, 0, 1}]
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Out[4]=



(*Expression for g(τ) in terms of l, m, α and τ*)

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In[5]:= g[l_, m_, α_, τ_] :=
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$$\left\{ \text{ArcCos}\left[-\frac{1}{m} l \cos\left[\frac{\pi \alpha}{2}\right] \left(-\frac{1 \cos\left[\frac{\pi \alpha}{2}\right]}{2 \tau} + \frac{1}{2} \sqrt{\left(-\frac{2 l^2}{3 \tau^2} + \frac{l^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + (2^{1/3} (1^4 - 12 m^2 \tau^2))\right)}\right] \right. \right.$$

$$\left. \left(3 \tau^2 \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 \right)^2} \right)^{1/3} \right) + \right.$$

$$\left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 \right)^2} \right)^{1/3} \right) +$$

$$\frac{1}{2} \sqrt{\left(-\frac{4 l^2}{3 \tau^2} + \frac{2 l^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} - (2^{1/3} (1^4 - 12 m^2 \tau^2))\right)} \left(3 \tau^2 \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 \right)^2} \right)^{1/3} \right) -$$

$$\frac{1}{3 \times 2^{1/3} \tau^2} \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \sqrt{-4 (1^4 - 12 m^2 \tau^2)^3 + \left(2 l^6 + 72 l^2 m^2 \tau^2 - 108 l^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 \right)^2} \right)^{1/3} \right)$$

$$\begin{aligned}
& \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3} + \\
& \left(\frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]}{\tau^3} - \frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]^3}{\tau^3}\right) / \left(4 \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \right. \right. \\
& \left. \left. (2^{1/3} (1^4 - 12 m^2 \tau^2))\right) / \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} \right. \\
& \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} \right) \right) \\
& \frac{1}{m} \tau \left(-\frac{1 \cos\left[\frac{\pi \alpha}{2}\right]}{2 \tau} + \frac{1}{2} \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + (2^{1/3} (1^4 - 12 m^2 \tau^2)) / \right. \right. \\
& \left. \left. \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} + \right. \right. \\
& \left. \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} + \right. \\
& \left. \frac{1}{2} \sqrt{\left(-\frac{4 1^2}{3 \tau^2} + \frac{2 1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} - (2^{1/3} (1^4 - 12 m^2 \tau^2)) / \left(3 \tau^2 \right. \right. \right. \\
& \left. \left. \left. \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} - \right. \\
& \left. \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3} + \\
& \left(\frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]}{\tau^3} - \frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]^3}{\tau^3}\right) / \left(4 \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + \right. \right. \\
& \left. \left. (2^{1/3} (1^4 - 12 m^2 \tau^2))\right) / \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} \right. \\
& \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} \right) \right) \\
& \cos[\pi \alpha] \left[-\frac{1 \cos\left[\frac{\pi \alpha}{2}\right]}{2 \tau} + \frac{1}{2} \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} + (2^{1/3} (1^4 - 12 m^2 \tau^2))\right) / \right. \right. \\
& \left. \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} \right) + \right. \\
& \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} \right) + \right. \\
& \left. \frac{1}{2} \sqrt{\left(-\frac{4 1^2}{3 \tau^2} + \frac{2 1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2} - (2^{1/3} (1^4 - 12 m^2 \tau^2))\right) / \right. \right. \\
& \left. \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3} \right)^{1/3} \right) - \right. \\
& \left. \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}^{1/3} + \\
& \left(\frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]}{\tau^3} - \frac{8 1^3 \cos\left[\frac{\pi \alpha}{2}\right]^3}{\tau^3}\right) / \left(4 \sqrt{\left(-\frac{2 1^2}{3 \tau^2} + \frac{1^2 \cos\left[\frac{\pi \alpha}{2}\right]^2}{\tau^2}\right)^2 + \right. \\
& \left. \left(2^{1/3} \left(1^4 - 12 m^2 \tau^2\right)\right) / \left(3 \tau^2 \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \right. \right. \right. \\
& \left. \left. \left. \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2\right)^2}\right)^{1/3}\right)}\right) + \\
& \frac{1}{3 \times 2^{1/3} \tau^2} \left(2 1^6 + 72 1^2 m^2 \tau^2 - 108 1^2 m^2 \tau^2 \cos\left[\frac{\pi \alpha}{2}\right]^2 + \sqrt{-4 \left(1^4 - 12 m^2 \tau^2\right)^3 + \left(2 1^6 + \right. \right. \right.
\end{aligned}$$

(*If we have to find out the root where $g(\tau)$ intersect with τ then suppose if we fix $l=0.9$, $m=3$, $\alpha=0.2$ then if we want to find the intersection point of τ near 1 then, *)

```
In[7]:= FindRoot[g[0.9, 3, 0.2,  $\tau$ ] -  $\tau$ , { $\tau$ , 1}]
```

```
Out[7]= { $\tau \rightarrow 1.75459$ }
```