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A Individual Task on

"Activation and Synaptic dynamic-1 Equations"

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Submitted By,

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CERTIFICATE

This is to certify that SACHIN BASALINGAPPA KOTI(01SU24AI089) has satisfactorily completed the assessment (Individual-Task – Module 2) in **"ARTIFICIAL NEURAL NETWORK "** prescribed by the Srinivas University for the 4st semester B. Tech course during the year **2025-26**.

MARKS AWARDED

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1. Introduction

Artificial Neural Networks (ANNs) are computational models inspired by the biological nervous system. They consist of interconnected processing elements called neurons, whose behavior is governed by activation dynamics and synaptic dynamics. Activation dynamics describe how the internal state (activation) of a neuron changes over time in response to inputs, while synaptic dynamics describe how the connection weights between neurons adapt during learning. These processes are commonly represented using first-order differential equations, which mathematically express how a neuron's activation or weight changes with respect to time.

Artificial Neural Networks (ANNs) are systems of interconnected processing units whose behavior is governed by two fundamental mechanisms: **activation dynamics** and **synaptic dynamics**.

- **Activation dynamics** describe how neuron states (activations) evolve with time for a fixed input.
- **Synaptic dynamics** describe how connection weights change during learning.

Mathematically, both are represented using **first-order differential equations**, which model the rate of change of activation or weight with respect to time.

2. Passive Decay Model

The Passive Decay Model describes the natural tendency of a neuron's activation to decrease over time in the absence of external input. It represents a leakage mechanism where the activation exponentially decays toward zero. The rate of decay is controlled by a positive constant known as the decay or leakage constant. This model reflects the biological reality that neurons cannot maintain activation indefinitely without stimulation.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t)$$

Symbols

- $x_i(t)$: activation (state) of neuron indexed by $i$at time t

- i: index identifying the neuron
- t: time variable
- $\frac{dx_i(t)}{dt}$: derivative of $x_i(t)$with respect to time (rate of change)
- A_i: positive decay (leakage) constant for neuron i
- $-$: indicates reduction (decay) of activation

3. Modified Passive Decay Model

The Modified Passive Decay Model extends the basic passive decay concept by incorporating membrane capacitance. Here, the decay rate is influenced by both the decay constant and the capacitance of the neuron. The capacitance represents the neuron's ability to store electrical charge, thereby affecting how quickly the activation changes. This model provides a more realistic physical interpretation of neuronal behavior by considering electrical properties.

$$\frac{dx_i(t)}{dt} = -\frac{A_i}{C_i} x_i(t)$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- C_i: membrane capacitance of neuron i
- $\frac{A_i}{C_i}$: effective decay rate
- $-$: decay effect

4. Non-Zero Resting Potential Model

The Non-Zero Resting Potential Model accounts for a constant input or bias term in addition to the decay process. Instead of decaying to zero, the neuron's activation stabilizes at a non-zero resting value determined by the constant input. This model represents neurons that maintain a baseline level of activity even without external stimulation, similar to biological neurons with resting membrane potentials.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + P_i$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- P_i: constant input (resting potential or bias)
- $+$: additive effect

5. External Input Activation Model

The External Input Activation Model includes both decay and external stimulus. The neuron's activation decreases due to leakage but increases proportionally to an external input signal scaled by a gain factor. This model demonstrates how neurons respond dynamically to time-varying external signals while maintaining stability through decay.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + B_i I_i$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t

- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- B_i: input gain (scaling factor)
- I_i: external input signal applied to neuron i
- $+$: additive contribution

6. Additive Auto associative Model

he Additive Auto associative Model describes a network in which each neuron receives contributions from external inputs as well as from other neurons in the same network. The total input is obtained by summing weighted outputs from all interconnected neurons. This model forms the basis of associative memory systems, where stored patterns can be recalled through network interactions.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + B_i I_i + \sum_{j=1}^{N} W_{ij}\, f_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- B_i: input gain
- I_i: external input to neuron i
- $\sum_{j=1}^{N}$: summation over index $j$from 1 to N

- j: index of presynaptic neuron
- N: total number of neurons
- W_{ij}: synaptic weight from neuron $j$to neuron i
- $f_j(\cdot)$: output function of neuron j
- $x_j(t)$: activation of neuron $j$at time t

7. Inhibitory Feedback Model

The Inhibitory Feedback Model emphasizes suppression mechanisms in neural networks. In this model, both external inputs and signals from other neurons contribute negatively to the activation of a neuron. Such inhibitory interactions are essential in competitive networks where neurons compete to respond to stimuli, ensuring selectivity and contrast enhancement.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) - B_i I_i - \sum_{j=1}^{N} W_{ij}\, f_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: scaling constant
- I_i: external input
- $\sum_{j=1}^{N}$: summation over neurons
- j: neuron index
- N: total neurons

- W_{ij}: weight from neuron $j$to neuron i
- $f_j(x_j(t))$: output of neuron j

8. Perkel's Model

Perkel's Model represents neural interactions in terms of electrical circuit analogies using resistances and conductance's. The rate of change of activation depends on the conductance of the neuron itself and the conductance's between interconnected neurons. This model provides a biophysically motivated framework for understanding neural dynamics based on electrical network theory.

$$\frac{dx_i(t)}{dt} = \frac{1}{R_i} x_i(t) + \sum_{j=1}^{N} \frac{1}{R_{ij}} \phi_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- R_i: resistance of neuron i
- $\frac{1}{R_i}$: conductance
- $\sum_{j=1}^{N}$: summation over neurons
- j: neuron index
- N: total neurons
- R_{ij}: resistance between neurons $i$and j
- $\phi_j(\cdot)$: output function
- $x_j(t)$: activation of neuron j

9.**Hopfield Model**

The Hopfield Model is a recurrent neural network model used for associative memory. In this model, neurons are symmetrically interconnected, and the system evolves toward stable equilibrium states. The activation changes are governed by decay, weighted feedback from other neurons, and optional external inputs. The network converges to stored patterns, making it useful for pattern recognition and optimization tasks.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + \sum_{j=1}^{N} W_{ij}\, f(x_j(t)) + I_i$$

Symbols

- $x_i(t)$: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- $\sum_{j=1}^{N}$: summation
- j: neuron index
- N: total neurons
- W_{ij}: symmetric connection weight
- $f(\cdot)$: bounded activation function
- $x_j(t)$: activation of neuron j
- I_i: external input

10.**Heteroassociative Network**

A Heteroassociative Network consists of two interconnected layers of neurons. Each layer receives feedback from the other through weighted connections. The activation dynamics in each layer depend on decay, weighted inputs from the opposite layer, and external signals.

This architecture enables the association of patterns from one layer with corresponding patterns in another layer, making it useful for pattern mapping and translation tasks.

Layer-1

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + \sum_{j=1}^{M} f_j\,(y_j(t))V_{ji} + I_i$$

Symbols

- $x_i(t)$: activation of neuron $i$in layer-1
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- $\sum_{j=1}^{M}$: summation over layer-2 neurons
- j: index of neuron in layer-2
- M: number of neurons in layer-2
- $f_j(\cdot)$: output function of neuron j
- $y_j(t)$: activation of neuron $j$in layer-2
- V_{ji}: weight from neuron $j$to neuron i
- I_i: external input

Layer-2

$$\frac{dy_j(t)}{dt} = -B_j y_j(t) + \sum_{i=1}^{N} g_i\,(x_i(t))W_{ij} + J_j$$

Symbols

- $y_j(t)$: activation of neuron $j$in layer-2
- t: time

- $\frac{dy_j(t)}{dt}$: rate of change
- B_j: decay constant
- $\sum_{i=1}^{N}$: summation over layer-1 neurons
- i: neuron index in layer-1
- N: number of neurons in layer-1
- $g_i(\cdot)$: output function of neuron i
- $x_i(t)$: activation of neuron i
- W_{ij}: weight from neuron $i$to neuron j
- J_j: external input

11.Bidirectional Associative Memory (BAM)

Bidirectional Associative Memory (BAM) is a special type of heteroassociative network in which the weight matrices between the two layers are transposes of each other. This symmetry ensures stable bidirectional recall, meaning that presenting a pattern in one layer retrieves its associated pattern in the other layer and vice versa. BAM networks are widely used for pattern association and recall applications.

Same equations as heteroassociative network with

- $V = W^T$: weight matrix Vequals transpose of weight matrix W

12.Basic Shunting Model

The Basic Shunting Model introduces multiplicative interaction between activation and input. Instead of simple addition, the input is scaled by the remaining activation capacity. This ensures that the activation remains bounded between lower and upper limits. The model prevents unbounded growth of activation and more closely resembles biological neuron behavior.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + (B_i - x_i(t))I_i$$

Symbols

- $x_i(t)$: activation
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper saturation limit
- I_i: external input
- $(B_i - x_i(t))$: remaining activation capacity

13.On-Centre Off-Surround Model

The On-Center Off-Surround Model incorporates both excitatory and inhibitory interactions. A neuron receives excitatory input from its own stimulus (on-center) and inhibitory input from neighboring neurons (off-surround). This competitive mechanism enhances contrast and sharpens pattern recognition, similar to processes observed in sensory systems such as vision.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + (B_i - x_i(t))I_i - x_i(t) \sum_{j \neq i} I_j$$

Symbols

- $x_i(t)$: activation
- t: time

- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- I_i: excitatory input
- $\sum_{j \neq i}$: summation over all neurons except i
- $j \neq i$: index condition
- I_j: inhibitory inputs

14.Modified Shunting Model

The Modified Shunting Model extends the shunting concept by introducing both upper and lower bounds for activation. Excitatory inputs are limited by the upper bound, while inhibitory inputs are influenced by a lower bound parameter. This ensures stable, bounded activation dynamics and improves the biological realism of the model.

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + (B_i - x_i(t))I_i - (E_i + x_i(t)) \sum_{j \neq i} I_j$$

Symbols

- $x_i(t)$: activation
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- E_i: lower bound parameter
- I_i: excitatory input
- $\sum_{j \neq i}$: summation excluding neuron i

- I_j: inhibitory inputs

15.Shunting Model with Feedback

The Shunting Model with Feedback incorporates both external inputs and feedback signals from other neurons. Excitatory and inhibitory influences are modulated by scaling constants and bounded by upper and lower limits. This model captures complex neural interactions involving self-feedback and lateral feedback, providing a comprehensive representation of dynamic neural processing.

$$\frac{dx_i(t)}{dt} = -A_i x_i + (B_i - C_i x_i)[I_i + f_i(x_i)] - (E_i + D_i x_i)[J_i + \sum_{j \neq i} f_j\,(x_j) w_{ji}]$$

Symbols

- x_i: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- C_i: scaling constant for excitatory term
- I_i: external excitatory input
- $f_i(x_i)$: feedback function of neuron i
- E_i: lower bound parameter
- D_i: scaling constant for inhibitory term
- J_i: external inhibitory input
- $\sum_{j \neq i}$: summation over neurons except i
- $f_j(x_j)$: output of neuron j

- w_{ji}: feedback weight from neuron $j$to neuron i

16.Synaptic Dynamics Model

The Synaptic Dynamics Model describes how connection weights between neurons evolve over time. The change in synaptic weight depends on its current value and the product of the outputs of the connected neurons. This reflects Hebbian learning principles, often summarized as “neurons that fire together wire together.” The model forms the mathematical foundation for learning and adaptation in neural networks.

$$\frac{dw_{ij}(t)}{dt} = -w_{ij}(t) + f_i(x_i(t))f_j(x_j(t))$$

Symbols

- $w_{ij}(t)$: synaptic weight from neuron $j$to neuron $i$at time t
- $\frac{dw_{ij}(t)}{dt}$: rate of change of weight
- t: time
- $f_i(x_i(t))$: output of neuron i
- $x_i(t)$: activation of neuron i
- $f_j(x_j(t))$: output of neuron j
- $x_j(t)$: activation of neuron j

17.Conclusion

Activation and synaptic dynamics models provide the mathematical foundation for understanding the behavior of Artificial Neural Networks. Starting from simple passive decay models to advanced shunting and associative memory models, each framework explains how neuron activations evolve and how networks stabilize, compete, or learn patterns. These differential equation–based models bridge the gap between biological neural systems and computational intelligence, enabling the design of stable, adaptive, and efficient neural architectures for real-world applications.

18.References

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