This file contains the MATLAB code and the output of the examples given in Remark 3.3 and Examples 4.1, 4.2, 4.3 from the manuscript.

We use spectral norm for the matrices involved in Remark 3.3.

Remark 3.3 (1).

The following example suggests that we cannot determine which method gives a better bound between Theorems 2.2 and 2.5 and Theorems 2.2 and 2.11. Let $T(\lambda) = -B_0 + I\lambda + \dfrac{B_1}{\lambda - a}$ be a 2×2 rational matrix.

First we compare bounds given in Theorems 2.2 and 2.5.

If $a = 0.1$, $B_0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and $B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

```
tic
a= 0.1;
B0 = [0 0; 1 0];
B1 = [1 0; 0 0];
Z=[0 0; 0 0];
E=[Z -eye(2); B1 B0];
u1=norm(E)+abs(a);
r=[1 -(abs(a)+norm(B0)) (abs(a)*norm(B0)-norm(B1))];
v1=max(roots(r));
```

If $a = 1$, $B_0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and $B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

```
a= 1;
u2=norm(E)+abs(a);
r=[1 -(abs(a)+norm(B0)) (abs(a)*norm(B0)-norm(B1))];
v2=max(roots(r));
```

```
for j=1
fprintf('If a=0.1 the bound due to Theorem 2.2 is  %.2f  \n',u1)
fprintf('and the bound due to Theorem 2.5 is  %.2f  \n',v1)
fprintf('***');
fprintf('If a=1 the bound due to Theorem 2.2 is  %.2f  \n',u2)
fprintf('and the bound due to Theorem 2.5 is  %.2f  \n',v2)
end
```

```
If a=0.1 the bound due to Theorem 2.2 is  1.51
and the bound due to Theorem 2.5 is  1.65
***
If a=1 the bound due to Theorem 2.2 is  2.41
and the bound due to Theorem 2.5 is  2.00
```

Note that if $a = 0.1$, the bound due to Theorem 2.2 is better. But when $a = 1$, the bound due to Theorem 2.5 is better.

Now we compare bounds given in Theorems 2.2 and 2.11.

```
a=0.1;
w1=(abs(a)+norm(B0)+sqrt((abs(a)-norm(B0))^2+4*norm(B1)))/2;
```

```
a=1;
w2=(abs(a)+norm(B0)+sqrt((abs(a)-norm(B0))^2+4*norm(B1)))/2;
```

```
for j=1
fprintf('If a=0.1 the bound due to Theorem 2.2 is  %.2f  \n',u1)
fprintf('and the bound due to Theorem 2.11 is  %.2f  \n',w1)
fprintf('***');
fprintf('If a=1 the bound due to Theorem 2.2 is  %.2f  \n',u2)
fprintf('and the bound due to Theorem 2.11 is  %.2f  \n',w2)
end
```

```
If a=0.1 the bound due to Theorem 2.2 is  1.51
and the bound due to Theorem 2.11 is  1.65
***
If a=1 the bound due to Theorem 2.2 is  2.41
and the bound due to Theorem 2.11 is  2.00
```

Note that if $a = 0.1$, the bound due to Theorem 2.2 is better. But when $a = 1$, the bound due to Theorem 2.11 is better.

Remark 3.3 (2).

We cannot conclude which theorem gives a better bound between Theorems 2.3 and 2.9. Consider

$T(\lambda) = -B_0 + I\lambda + \frac{B_1}{\lambda - a}$, where $B_0 = B_1 = I$, the 2×2 identity matrix.

First we find the bound using Theorem 2.9, for $a = 1$ and $a = i$.

```
a= 1;
B0 = eye(2);
B1 = eye(2);
u1 = [1 -norm(B0+a*eye(2)) -norm(a*B0+B1)];
```

```
max(roots(u1));
```

```
a= i;
u2 = [1 -norm(B0+a*eye(2)) -norm(a*B0+B1)];
max(roots(u2));
```

Note that the bound due to Theorem 2.3 is $\sqrt{\frac{(2m+1)+\sqrt{4m+1}}{2}}+|a|$. Since $m=1$ and $a=1$ or $a=i$ the bound is 2.618 in both the cases.

```
for j=1
fprintf('If a=1 the bound due to Theorem 2.9 is %.2f \n',max(roots(u1)))
fprintf('and the bound due to Theorem 2.3 is  %.2f  \n',2.618)
fprintf('***')
fprintf('If a=i the bound due to Theorem 2.9 is %.2f \n',max(roots(u2)))
fprintf('and the bound due to Theorem 2.3 is  %.2f  \n',2.618)
end
```

```
If a=1 the bound due to Theorem 2.9 is 2.73
and the bound due to Theorem 2.3 is  2.62
***
If a=i the bound due to Theorem 2.9 is 2.09
and the bound due to Theorem 2.3 is  2.62
```

Note that if $a=1$ the bound due to Theorem 2.3 is better. But when $a=i$ the bound due to Theorem 2.9 is better.

Remark 3.3 (3).

Again the same phenomenon happens with Theorems 2.5 and 2.9. Let $T(\lambda)=-B_0+I\lambda+\frac{B_1}{\lambda-a}$, where $B_0=B_1=I$, the 2×2 identity matrix.

If $a=-1.5$

```
a= -1.5;
B0 = eye(2);
B1 = eye(2);
u1 = [1 -norm(B0+a*eye(2)) -norm(a*B0+B1)];
max(roots(u1));
v1 = [1 -(abs(a)+1) abs(a)-1];
max(roots(v1));
```

If $a=1.5$

```
a= 1.5;
u2 = [1 -norm(B0+a*eye(2)) -norm(a*B0+B1)];
max(roots(u2));
v2 = [1 -(abs(a)+1) abs(a)-1];
max(roots(v2));
```

```
for j=1
fprintf('If a=-1.5 the bound due to Theorem 2.9 is  %.2f  \n',max(roots(u1)))
fprintf('and the bound due to Theorem 2.5 is  %.2f  \n',max(roots(v1)))
fprintf('***');
fprintf('If a=1.5 the bound due to Theorem 2.9 is  %.2f  \n',max(roots(u2)))
fprintf('and the bound due to Theorem 2.5 is  %.2f  \n',max(roots(v2)))
end
```

```
If a=-1.5 the bound due to Theorem 2.9 is  1.00
and the bound due to Theorem 2.5 is  2.28
***
If a=1.5 the bound due to Theorem 2.9 is  3.27
and the bound due to Theorem 2.5 is  2.28
```

Note that if $a = -1.5$ the bound due to Theorem 2.9 is better. But when $a = 1.5$ the bound due to Theorem 2.5 is better.

Remark 3.3 (4).

The bounds obtained in Theorems 2.9 and 2.11 are also not comparable. Let $T(\lambda) = -B_0 + I\lambda + \frac{B_1}{\lambda - a}$, where $B_0 = B_1 = I$, the 2×2 identity matrix.

If $a = 1$

```
a= 1;
B0 = eye(2);
B1 = eye(2);
u1 = [1 -norm(B0+a*eye(2)) -norm(a*B0+B1)];
max(roots(u1));
w1 = (abs(a)+norm(B0)+sqrt((abs(a)-norm(B0))^2+4*norm(B1)))/2;
```

If $a = -0.5$

```
a= -0.5;
u2 = [1 -norm(B0+a*eye(2)) -norm(a*B0+B1)];
max(roots(u2));
```

```
w2 = (abs(a)+norm(B0)+sqrt((abs(a)-norm(B0))^2+4*norm(B1)))/2;
```

```
for j=1
fprintf('If a=1 the bound due to Theorem 2.9 is  %.2f  \n',max(roots(u1)))
fprintf('and the bound due to Theorem 2.11 is  %.2f  \n',w1)
fprintf('***');
fprintf('If a=-0.5 the bound due to Theorem 2.9 is  %.2f  \n',max(roots(u2)))
fprintf('and the bound due to Theorem 2.11 is  %.2f  \n',w2)
fprintf('***')
fprintf('***')
end
```

```
If a=1 the bound due to Theorem 2.9 is  2.73
and the bound due to Theorem 2.11 is  2.00
***
If a=-0.5 the bound due to Theorem 2.9 is  1.00
and the bound due to Theorem 2.11 is  1.78
***
***
```

Note that if $a = 1$ the bound due to Theorem 2.11 is better. But when $a = -0.5$ the bound due to Theorem 2.9 is better.

We now consider three different rational matrices and compare our results with the bounds obtained in the refrences [4] and [17] of our manuscript. In all examples we use the spectral norm for matrices involved. As pointed out in the Remark 3.3, one cannot in general determine the best method. But it is possible that for a particular example one of the methods might give a better bound.

Example 4.1 (Table 1).

Let $T(\lambda) = -B_0 + I\lambda + \frac{B_1}{\lambda - 0.1}$, where $B_0 = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$, $B_1 = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$ and I is the identity matrix of size 3. In this example, the bound obtained obtained using Theorem 3.3 from our manuscript gives better bound

```
a= 0.1;
B0 = [2 -1 0; -1 2 -1; 0 -1 1];
B1 = [-1 0 1; 0 -1 1; -1 0 1];
Z=zeros(3);
E=[Z -eye(3); B1 B0];
D=[a*eye(3) -eye(3); B1 B0];
max(abs(eig(D)));
```

Bounds obtained from our results.

Bound using Theorem 2.2

```
S1 = norm(E)+a;
```

Bound using Theorem 2.4

```
S2 = (a+norm(B0)+sqrt((a-norm(B0))^2+4*norm(B1)))/2;
```

Bound using Theorem 2.8

```
S3 = (norm(B0+a*eye(3))+sqrt((norm(B0+a*eye(3)))^2+4*norm(B1+a*B0)))/2;
```

Bound using Theorem 2.11

```
S4= (a+norm(B0)+sqrt(((a-norm(B0))^2)+4*max(1, (norm(B1))^2)))/2;
```

Bounds obtained using results from [4]

Bound using Theorem 3.9 (1)

```
S5 = max(1+a, norm(B0)+norm(B1));
```

Bound using Theorem 3.9 (2)

```
S6 = max(a+norm(B1), 1+norm(B0));
```

Bound using Theorem 3.9 (3)

```
S7 = (a+norm(B0)+ sqrt((a-norm(B0))^2+(1+norm(B1))^2))/2;
```

Bound using Corollary 3.11

```
S8 = max(a, norm(B0))+ (1+norm(B0))/2;
```

Bounds obtained using various results given in [17].

Bound using Theorem 3.2

```
M1=max(norm(B0+a*eye(3)), norm(B1+ a*B0));
roots([1 -(1+M1) 0 M1]);
max(roots([1 -(1+M1) 0 M1]));
S9 = max(1, max(roots([1 -(1+M1) 0 M1])));
```

Bound using Corollary 3.2.1

```
M2 = max([norm(eye(3)+a*eye(3)+B0) norm(a*eye(3)+B0+a*B0+B1) norm(a*B0+B1)]);
roots([1 -(1+M2) 0 0 M2]);
max(roots([1 -(1+M2) 0 0 M2]));
S10 = max(1, max(roots([1 -(1+M2) 0 0 M2])));
```

Bound using Theorem 3.3

```
S11 = 1+M1;
```

Bound using Theorem 3.4

```
S12 = (1+norm(a*eye(3) + B0)+ sqrt((1-norm(a*eye(3) + B0))^2 +4*norm(B1+a*B1)))/2;
```

Bound using Corollary 3.4.2

```
M3 =max(norm(a*eye(3)+B0+a*B0+B1), norm(a*B0+B1));
S13 =(1+norm(a*eye(3)+eye(3)+B0)+ sqrt((1-norm(a*eye(3)+eye(3)+B0))^2 +4*M3))/2;
```

Bound using Corollary 3.4.4

```
M4=max(norm((B0+a*eye(3))*(a*B0+B1)), norm((B0+a*eye(3))^2-(a*B0+B1)));
S14 = (1+sqrt(1+4*M4))/2;
```

Bound using Corollary 3.4.6

```
M5 = max(norm((a*eye(3)+eye(3)+B0)*(a*B0+B1)), norm(-((a*eye(3)+eye(3)+B0)*(a*eye(3)+B0))+(B1+a
S15 = 1+sqrt(M5);
```

Bound using Theorem 3.6

```
S16 =1+M1;
```

```
for j=1
fprintf('The maximum of the moduli of eigenvalues is %.2f  \n',max(abs(eig(D))))
fprintf('The bound due to Theorem 2.2 is %.2f  \n',S1)
fprintf('The bound due to Theorem 2.4 (Theorem 3.8, [4]) is %.2f  \n',S2)
fprintf('The bound due to Theorem 2.8 is %.2f  \n',S3)
fprintf('The bound due to Theorem 2.11 is %.2f  \n',S4)
fprintf('***')
fprintf('The bound due to Theorem 3.9 (1) is %.2f  \n',S5)
fprintf('The bound due to Theorem 3.9 (2) is %.2f  \n',S6)
fprintf('The bound due to Theorem 3.9 (3) is %.2f  \n',S7)
fprintf('The bound due to Corollary 3.11 is %.2f  \n',S8)
fprintf('***')
fprintf('The bound due to Theorem 3.2 is %.2f  \n',S9)
fprintf('The bound due to Corollary 3.2.1 is %.2f  \n',S10)
fprintf('The bound due to Theorem 3.3 is %.2f  \n',S11)
fprintf('The bound due to Theorem 3.4 is %.2f  \n',S12)
```

```
fprintf('The bound due to Corollary 3.4.2 is %.2f  \n',S13)
fprintf('The bound due to Corollary 3.4.4 is %.2f  \n',S14)
fprintf('The bound due to Corollary 3.4.6 is %.2f  \n',S15)
fprintf('The bound due to Theorem 3.6 is %.2f  \n',S16)
fprintf('***')
fprintf('***')
end
```

```
The maximum of the moduli of eigenvalues is 3.54
The bound due to Theorem 2.2 is 3.70
The bound due to Theorem 2.4 (Theorem 3.8, [4]) is 3.83
The bound due to Theorem 2.8 is 3.90
The bound due to Theorem 2.11 is 4.36
***
The bound due to Theorem 3.9 (1) is 5.42
The bound due to Theorem 3.9 (2) is 4.25
The bound due to Theorem 3.9 (3) is 3.91
The bound due to Corollary 3.11 is 5.37
***
The bound due to Theorem 3.2 is 4.15
The bound due to Corollary 3.2.1 is 5.32
The bound due to Theorem 3.3 is 4.35
The bound due to Theorem 3.4 is 4.12
The bound due to Corollary 3.4.2 is 5.07
The bound due to Corollary 3.4.4 is 3.99
The bound due to Corollary 3.4.6 is 4.91
The bound due to Theorem 3.6 is 4.35
***
***
```

The calculation suggests that the bound using Theorem 2.2 is better for this example.

Example 4.2 (Table 2).

In the following example we see that few of the bounds obtained from [4] coincide with the bounds obtained from our manuscript. Let $T(\lambda) = -B_0 + I\lambda + \dfrac{B_1}{\lambda - \alpha}$, where $B_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, $B_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $\alpha = 2$.

```
a= 2;
B0 = [1 0 0; 0 1 0; 0 0 2];
B1 = [0 0 0; 0 0 0; 0 0 -1];
Z=zeros(3);
E=[Z -eye(3); B1 B0];
D=[a*eye(3) -eye(3); B1 B0];
eig(D);
max(abs(eig(D)));
max(abs(eig(D)));
```

Bounds obtained from our results.

Bound using Theorem 2.2

```
S1 = norm(E)+a;
```

Bound using Theorem 2.4

```
S2 = (a+norm(B0)+sqrt((a-norm(B0))^2+4*norm(B1)))/2;
```

Bound using Theorem 2.8

```
S3 = (norm(B0+a*eye(3))+sqrt((norm(B0+a*eye(3)))^2+4*norm(B1+a*B0)))/2;
```

Bound using Theorem 2.11

```
S4= (a+norm(B0)+sqrt(((a-norm(B0))^2)+4*max(1, (norm(B1))^2)))/2;
```

Bound using Theorem 2.7

```
l=roots([1 -(a+1/norm(inv(B0))) (a*(1/norm(inv(B0))))-norm(B1)]);
L=min(l);
```

Bounds obtained using results from [4]

Bound using Theorem 3.9 (1)

```
S5 = max(1+a, norm(B0)+norm(B1));
```

Bound using Theorem 3.9 (2)

```
S6 = max(a+norm(B1), 1+norm(B0));
```

Bound using Theorem 3.9 (3)

```
S7 = (a+norm(B0)+ sqrt((a-norm(B0))^2+(1+norm(B1))^2))/2;
```

Bound using Corollary 3.11

```
S8 = max(a, norm(B0))+ (1+norm(B0))/2;
```

Bounds obtained using various results given in [17].

Bound using Theorem 3.2

```
M1=max(norm(B0+a*eye(3)), norm(B1+ a*B0));
roots([1 -(1+M1) 0 M1]);
max(roots([1 -(1+M1) 0 M1]));
S9 = max(1, max(roots([1 -(1+M1) 0 M1])));
```

Bound using Corollary 3.2.1

```
M2 = max([norm(eye(3)+a*eye(3)+B0) norm(a*eye(3)+B0+a*B0+B1) norm(a*B0+B1)]);
roots([1 -(1+M2) 0 0 M2]);
max(roots([1 -(1+M2) 0 0 M2]));
S10 = max(1, max(roots([1 -(1+M2) 0 0 M2])));
```

Bound using Theorem 3.3

```
S11 = 1+M1;
```

Bound using Theorem 3.4

```
S12 = (1+norm(a*eye(3) + B0)+ sqrt((1-norm(a*eye(3) + B0))^2 +4*norm(B1+a*B1)))/2;
```

Bound using Corollary 3.4.2

```
M3 =max(norm(a*eye(3)+B0+a*B0+B1), norm(a*B0+B1));
S13 =(1+norm(a*eye(3)+eye(3)+B0)+ sqrt((1-norm(a*eye(3)+eye(3)+B0))^2 +4*M3))/2;
```

Bound using Corollary 3.4.4

```
M4=max(norm((B0+a*eye(3))*(a*B0+B1)), norm((B0+a*eye(3))^2-(a*B0+B1)));
S14 = (1+sqrt(1+4*M4))/2;
```

Bound using Corollary 3.4.6

```
M5 = max(norm((a*eye(3)+eye(3)+B0)*(a*B0+B1)), norm(-((a*eye(3)+eye(3)+B0)*(a*eye(3)+B0))+(B1+a
S15 = 1+sqrt(M5);
```

Bound using Theorem 3.6

```
S16 =1+M1;
```

```
for j=1
fprintf('The maximum of the moduli of eigenvalues is %.2f  \n',max(abs(eig(D))))
fprintf('The minimum of the moduli of eigenvalues is %.2f  \n',min(abs(eig(D))))
fprintf('The bound due to Theorem 2.2 is %.2f  \n',S1)
fprintf('The bound due to Theorem 2.4 (Theorem 3.8, [4]) is %.2f  \n',S2)
fprintf('The bound due to Theorem 2.8 is %.2f  \n',S3)
fprintf('The bound due to Theorem 2.11 is %.2f  \n',S4)
fprintf('The bound due to Theorem 2.7 is %.2f  \n',L)
fprintf('***')
fprintf('The bound due to Theorem 3.9 (1) is %.2f  \n',S5)
fprintf('The bound due to Theorem 3.9 (2) is %.2f  \n',S6)
fprintf('The bound due to Theorem 3.9 (3) is %.2f  \n',S7)
fprintf('The bound due to Corollary 3.11 is %.2f  \n',S8)
fprintf('***')
```

```
fprintf('The bound due to Theorem 3.2 is %.2f  \n',S9)
fprintf('The bound due to Corollary 3.2.1 is %.2f  \n',S10)
fprintf('The bound due to Theorem 3.3 is %.2f  \n',S11)
fprintf('The bound due to Theorem 3.4 is %.2f  \n',S12)
fprintf('The bound due to Corollary 3.4.2 is %.2f  \n',S13)
fprintf('The bound due to Corollary 3.4.4 is %.2f  \n',S14)
fprintf('The bound due to Corollary 3.4.6 is %.2f  \n',S15)
fprintf('The bound due to Theorem 3.6 is %.2f  \n',S16)
fprintf('***')
fprintf('***')
end
```

```
The maximum of the moduli of eigenvalues is 3.00
The minimum of the moduli of eigenvalues is 1.00
The bound due to Theorem 2.2 is 4.41
The bound due to Theorem 2.4 (Theorem 3.8, [4]) is 3.00
The bound due to Theorem 2.8 is 4.65
The bound due to Theorem 2.11 is 3.00
The bound due to Theorem 2.7 is 0.38
***
The bound due to Theorem 3.9 (1) is 3.00
The bound due to Theorem 3.9 (2) is 3.00
The bound due to Theorem 3.9 (3) is 3.00
The bound due to Corollary 3.11 is 3.50
***
The bound due to Theorem 3.2 is 4.83
The bound due to Corollary 3.2.1 is 7.99
The bound due to Theorem 3.3 is 5.00
The bound due to Theorem 3.4 is 4.79
The bound due to Corollary 3.4.2 is 6.32
The bound due to Corollary 3.4.4 is 4.14
The bound due to Corollary 3.4.6 is 5.12
The bound due to Theorem 3.6 is 5.00
***
***
```

In this example, the bound obtained from Theorem 2.11 is better and it is same as the bounds obatined from Theorems 3.8 and 3.9 of [4]. Also the lower bound using Theorem 2.7 is 0.38.

Example 4.3 (Table 3).

In the following example the bounds obtained from Corollary 3.4.4 of [17] is better. Let

$T(\lambda) = -(A+C)B^{-1} + I\lambda - \frac{CB^{-1}}{\lambda - 1}$ an $n \times n$ rational matrix, where $A = \begin{pmatrix} 6 & -3 & 0 \\ -3 & 6 & -3 \\ 0 & -3 & 3 \end{pmatrix}$, $B = \frac{1}{18}\begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ and

$C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

```
a= 1;
A=[6 -3 0; -3 6 -3; 0 -3 3];
B=(1/18)*[4 1 0; 1 4 1; 0 1 2];
x=[0; 0; 1];
C=x*x';
B0 = (A+C)*inv(B);
B1 = -a*C*inv(B);
Z=zeros(3);
E=[Z -eye(3); B1 B0];
D=[a*eye(3) -eye(3); B1 B0];
max(abs(eig(D)));
```

Bounds obtained from our results.

Bound using Theorem 2.2

```
S1 = norm(E)+a;
```

Bound using Theorem 2.4

```
S2 = (a+norm(B0)+sqrt((a-norm(B0))^2+4*norm(B1)))/2;
```

Bound using Theorem 2.8

```
S3 = (norm(B0+a*eye(3))+sqrt((norm(B0+a*eye(3)))^2+4*norm(B1+a*B0)))/2;
```

Bound using Theorem 2.11

```
S4= (a+norm(B0)+sqrt(((a-norm(B0))^2)+4*max(1, (norm(B1))^2)))/2;
```

Bounds obtained using results from [4]

Bound using Theorem 3.9 (1)

```
S5 = max(1+a, norm(B0)+norm(B1));
```

Bound using Theorem 3.9 (2)

```
S6 = max(a+norm(B1), 1+norm(B0));
```

Bound using Theorem 3.9 (3)

```
S7 = (a+norm(B0)+ sqrt((a-norm(B0))^2+(1+norm(B1))^2))/2;
```

Bound using Corollary 3.11

```
S8 = max(a, norm(B0))+ (1+norm(B0))/2;
```

Bounds obtained using various results given in [17].

Bound using Theorem 3.2

```
M1=max(norm(B0+a*eye(3)), norm(B1+ a*B0));
roots([1 -(1+M1) 0 M1]);
max(roots([1 -(1+M1) 0 M1]));
S9 = max(1, max(roots([1 -(1+M1) 0 M1])));
```

Bound using Corollary 3.2.1

```
M2 = max([norm(eye(3)+a*eye(3)+B0) norm(a*eye(3)+B0+a*B0+B1) norm(a*B0+B1)]);
roots([1 -(1+M2) 0 0 M2]);
max(roots([1 -(1+M2) 0 0 M2]));
S10 = max(1, max(roots([1 -(1+M2) 0 0 M2])));
```

Bound using Theorem 3.3

```
S11 = 1+M1;
```

Bound using Theorem 3.4

```
S12 = (1+norm(a*eye(3) + B0)+ sqrt((1-norm(a*eye(3) + B0))^2 +4*norm(B1+a*B1)))/2;
```

Bound using Corollary 3.4.2

```
M3 =max(norm(a*eye(3)+B0+a*B0+B1), norm(a*B0+B1));
S13 =(1+norm(a*eye(3)+eye(3)+B0)+ sqrt((1-norm(a*eye(3)+eye(3)+B0))^2 +4*M3))/2;
```

Bound using Corollary 3.4.4

```
M4=max(norm((B0+a*eye(3))*(a*B0+B1)), norm((B0+a*eye(3))^2-(a*B0+B1)));
S14 = (1+sqrt(1+4*M4))/2;
```

Bound using Corollary 3.4.6

```
M5 = max(norm((a*eye(3)+eye(3)+B0)*(a*B0+B1)), norm(-((a*eye(3)+eye(3)+B0)*(a*eye(3)+B0))+(B1+a
S15 = 1+sqrt(M5);
```

Bound using Theorem 3.6

```
S16 =1+M1;
```

```
for j=1
fprintf('The maximum of the moduli of eigenvalues is %.2f  \n',max(abs(eig(D))))
fprintf('The bound due to Theorem 2.2 is %.2f  \n',S1)
fprintf('The bound due to Theorem 2.4 (Theorem 3.8, [4]) is %.2f  \n',S2)
fprintf('The bound due to Theorem 2.8 is %.2f  \n',S3)
fprintf('The bound due to Theorem 2.11 is %.2f  \n',S4)
```

```
fprintf('***')
fprintf('The bound due to Theorem 3.9 (1) is %.2f  \n',S5)
fprintf('The bound due to Theorem 3.9 (2) is %.2f  \n',S6)
fprintf('The bound due to Theorem 3.9 (3) is %.2f  \n',S7)
fprintf('The bound due to Corollary 3.11 is %.2f  \n',S8)
fprintf('***')
fprintf('The bound due to Theorem 3.2 is %.2f  \n',S9)
fprintf('The bound due to Corollary 3.2.1 is %.2f  \n',S10)
fprintf('The bound due to Theorem 3.3 is %.2f  \n',S11)
fprintf('The bound due to Theorem 3.4 is %.2f  \n',S12)
fprintf('The bound due to Corollary 3.4.2 is %.2f  \n',S13)
fprintf('The bound due to Corollary 3.4.4 is %.2f  \n',S14)
fprintf('The bound due to Corollary 3.4.6 is %.2f  \n',S15)
fprintf('The bound due to Theorem 3.6 is %.2f  \n',S16)
fprintf('***')
fprintf('***')
end
```

```
The maximum of the moduli of eigenvalues is 94.60
The bound due to Theorem 2.2 is 98.46
The bound due to Theorem 2.4 (Theorem 3.8, [4]) is 97.38
The bound due to Theorem 2.8 is 99.18
The bound due to Theorem 2.11 is 98.46
***
The bound due to Theorem 3.9 (1) is 108.04
The bound due to Theorem 3.9 (2) is 98.27
The bound due to Theorem 3.9 (3) is 97.63
The bound due to Corollary 3.11 is 146.40
***
The bound due to Theorem 3.2 is 99.24
The bound due to Corollary 3.2.1 is 191.47
The bound due to Theorem 3.3 is 99.25
The bound due to Theorem 3.4 is 98.47
The bound due to Corollary 3.4.2 is 101.13
The bound due to Corollary 3.4.4 is 97.33
The bound due to Corollary 3.4.6 is 98.33
The bound due to Theorem 3.6 is 99.25
***
***
```

```
toc
```

```
Elapsed time is 0.796563 seconds.
```

For this example, the bound obtained from Corollary 3.4.4 of [17] is better.