Jacobian

If u& v are function of independent variables x&y

then Jacobian 'J' is defined as J=$\frac{\partial(u,v)}{\partial(x,y)}$ = $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$

J'=$\frac{\partial(x,y)}{\partial(u,v)}$ where JJ'=1

Jacobian of Implicit function

1)$f_1(x,y,u,v)=0$

$f_2(x,y,u,v)=0$ are two implicit function then

$$\frac{\partial(u,v)}{\partial(x,y)} = (-1)^2\,\frac{\partial(f_1,f_2)}{\partial(x,y)}\frac{\partial(f_1,f_2)}{\partial(u,v)}$$

Evaluation of partial derivative

Let $f_1(x,y,u,v)=0$

$f_2(x,y,u,v)=0$

from this we can find partial derivatives

$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ and so on…

i) $$\frac{\partial u}{\partial x} = (-1)\,\frac{\frac{\partial(f_1,f_2)}{\partial(x,v)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$$

ii) $$\frac{\partial u}{\partial y} = (-1)\,\frac{\frac{\partial(f_1,f_2)}{\partial(y,v)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$$

iii) $\frac{\partial y}{\partial u} = (-1)\frac{\frac{\partial(f_1,f_2)}{\partial(x,u)}}{\frac{\partial(f_1,f_2)}{\partial(x,y)}}$

iv) $\frac{\partial x}{\partial u} = (-1)\frac{\frac{\partial(f_1,f_2)}{\partial(u,y)}}{\frac{\partial(f_1,f_2)}{\partial(x,y)}}$ Similarly you can do others

Error And Approximation

i)If z=f(x ,y)   dz =$\frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

ii) dx,dy,dz are the actual errors in x,y,z

iii)$\frac{dx}{x}, \frac{dy}{y}, \frac{dz}{z}$ are the relative error's in x,y,z

iv)$100\frac{dx}{x}, 100\frac{dy}{y}, 100\frac{dz}{z}$ are the Percent error's in x,y,z

Maxima and Minima

1)find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

2)Equating $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y}$=0

we get(a_1,b_1), (a_2,b_2),..... as stationary point.

3)find r=$\frac{\partial^2 f}{\partial x^2}$ $s = \frac{\partial^2 f}{\partial x \partial y}$ t=$\frac{\partial^2 f}{\partial y^2}$

4)rt-s^2>0 & r<0 Maxima

rt-s^2>0 & r>0 Minima

rt-s^2<0 Neither maxima nor Minima

rt-s^2=0 No conclusion

Lagrange’s Method of undetermined Multiplier’s

1)u=f(x,y,z) and $\emptyset(x,y,z)=0$

F=u+$\lambda\emptyset$

2)Find $\frac{\partial F}{\partial x} = 0, \frac{\partial F}{\partial y} = 0, \frac{\partial F}{\partial z} = 0$

3)Eliminate x,y,z& λ using above two points and then we get equation in terms of u.