Matrices

An arrangement of certain numbers in an array of m rows and n columns is known as m×n matrix.

E.g:-$A_{m\times n}$=$\begin{bmatrix} a_{11} & a_{1n} \\ a_{m1} & a_{mn} \end{bmatrix}_{m\times n}$

Transpose of matrix

Matrix obtained by interchanging rows and columns is known as transpose of matrix and denoted by A^T or A' .

A=$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 6 & 3 & 9 \end{bmatrix}$ then $A^T = \begin{bmatrix} 1 & 0 & 6 \\ 2 & 2 & 3 \\ 3 & 5 & 9 \end{bmatrix}$

Types of Matrices

1) Row Matrix

Matrix which contain only one row

E.g $:A = \begin{bmatrix} 1 & 2 \end{bmatrix}$

2) Column Matrix

Matrix which contain only one Column.

E.g:- B= $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

3) Zero or Null Matrix

All the elements of the matrix are zero.

E.g:- A=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4) Square Matrix

The matrix whose rows and columns are equal i.e,m=n

E.g:- A=$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}_{2\times2}$

5) Symmetric Matrix

a_{ij}=a_{ji} i.e,A=A^T

$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

If

E.g:-

6)Skew Symmetric Matrix

If a_{ij}=$-a_{ji}$ for i$\neq j$ and diagonal elements are zero.

7)Diagonal Matrix

Matrix in which elements other than diagonal elements are zero.

A=$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

8)Scalar Matrix

Diagonal Matrix in which all the diagonal elements are same.

A=$\begin{bmatrix} 2 & 2 & 3 \\ 3 & 2 & 6 \\ 1 & 2 & 2 \end{bmatrix}$

9)Identity Matrix

Scalar Matrix in which all the diagonal elements are 1

A= $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 6 \\ 1 & 2 & 1 \end{bmatrix}$

10)Upper Triangular Matrix

All the elements below the diagonal are zero.

E.g:-A= $\begin{bmatrix} 2 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 2 \end{bmatrix}$

11)lower Triangular Matrix

All the elements above the diagonal are zero.

E.g:-A= $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 5 & 5 & 2 \end{bmatrix}$

12)Singular Matrix

If det A i.e, $|A| = 0$

13)Non Singular

Matrix i.e $|A| \neq 0$ Inverse exist.

14)Orthogonal Matrix

AA^T =I

If matrix A orthogonal then $A^{-1} = A$

- System of ‘m’linear euation in ‘n’ unknowns

$x_1, x_2, x_3, \ldots\ldots x_n$

$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$ . . . . .

$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = b_2$.

$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n = b_m$

The system of equation can be written in compact from by using matrix notation.

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}=\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

i.e AX=B where

coefficient matrix A

is

$$=\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, B=\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

where Ax=B has solution i.e.set of values $x_1, x_2, \ldots\ldots x_n$ satisfy simultaneous all m equation then

system is said to consistent otherwise system is inconsistent.

- Augmented Matrix(A,B):- If AX=B is system of m equation in n unknowns then matrix written as (A,B) is called as the augmented matrix. Hence,

$$(A,B)=\begin{bmatrix} a_{11} & \cdots & a_{1n} & \vdots & b_1 \\ \vdots & \ddots & & \vdots & \\ a_{m1} & \cdots & a_{mn} & \vdots & b_m \end{bmatrix}$$

1)Non Homogeneous Equation :-for the system of
equation AX+B if B is not a null or Zero
matrix then the system is known as non

Homogeneous system equations.

2) Homogeneous Equation :-for the system of equation AX+B if B is a null or Zero matrix then the system is known as
Homogeneous system equation

- Consistency:-System is linear euation is said to be consistentif it has solution otherwise it is non consistent. Eigen Values and Eigen Vectors

The characterstic equation of Matrix is

$|A-\lambda I|$=0

Properties of Eigen Values are:-

1)Trace of A-Sum of all diagonal elements of Square matrix

Trace of A=a_{11}+$a_{22}+a_{33}+\cdots+a_{mn}$

2)sum of eigen values of a matrix is the sum of diagonal elements of matrix

$$a_{11}+a_{22}+a_{33}+\cdots+a_{mn}=\lambda_1+\lambda_2+\lambda_3+\cdots+\lambda_n$$

3)The elgen values of upper and lower triangular matrix are their diagonal elements

4)The product of the eigen values of a matrix equals to the determinant of the matrix

$$\lambda_1 \times \lambda_2 \times \lambda_3 \times \ldots \times \lambda_n = |A|$$

5)If $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n$ are the eigen values of A then $\frac{1}{\lambda_1}$, $\frac{1}{\lambda_2}$, $\frac{1}{\lambda_3}$ ……. $\frac{1}{\lambda_n}$ are the eigen values of A^{-1}.

6)The matrix KA has eigen values $k\lambda_1, k\lambda_2, k\lambda_3, \ldots k\lambda_n$. 7)The matrix A^m has eigen values

$\lambda_1{}^m, \lambda_2{}^m, \lambda_3{}^m \ldots \ldots \lambda_n{}^m$.

Method of finding eigen values of Matrix A

For 2x2 Square matrix

Equation $\lambda^2 - s_1\lambda + |A| = 0$

where s_1 =Sum of all diagonal elements of matrix A, i e,

$s_1 = a_{11} + a_{22}$

$|A|$ =determinant of matrix.

For 3x3 Square matrix

Equation $\lambda^3 - s_1\lambda^2 + s_2\lambda - |A| = 0$

where $s_1 =$ sum of all diagonal elements of matrix A.

ie.

"a" +a'a, $s_2 =$sum of minors of diagonal elements $s_2 =$

minor ofa_{11} +minor of a minor of

a_{22}+ minor of a minor of a_{33}. $|A| =$

determinant of matrix