Numerical Methods

1)Lagrange's Interpolation Formula :-

Lagrange's Interpolating polynomial

Passing through set of points

(x_0, y_0) ,(x_1,y_1),............(x_n,y_n) is

$$y=\frac{(x-x_1)(x-x_2)----(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)----(x_0-x_n)}y_0+$$

$$\frac{(x-x_0)(x-x_2)(x_0-x_3)----(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_2-x_3)----(x_1-x_n)}y_1+-----+\frac{(x-x_0)(x-x_1)(x_0-x_2)(x_0-x_3)----(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)----(x_n-x_{n-1})}y_n$$

2)Forward Difference : The first forward difference of y=f(x) is given by

$$\Delta f(x) = f(x+h) - f(x) \text{ or } \Delta y_n=y_{n+1}-y_n.$$

3)Backward Difference :-The first backward difference of y=f(x) is given by

$$\nabla f(x) = f(x-h) - f(x) \text{ or } \nabla y_n= y_n- y_{n-1}$$

4)Shift Operator:- The Shift Operator on f(x) is given by Ef(x)=f(x+h)

And inverse shift operator on f(x) is given by

$E^{-1}f(x)=f(x-h)$

5) Central Difference :- The central difference of y=f(x) is given by

$\delta y = y\left(x+\frac{h}{2}\right) - y\left(x-\frac{h}{2}\right)$ Or $f(x)=f(x+\frac{h}{2}) - f(x-\frac{h}{2})$.

6)Average operator : The average operator on y=f(x) is given by

$$\mu f(x) = \frac{1}{2}\left(f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right).$$

7) Newton- Gregory's Interpolation Formula:

i)For Forward Difference:- If polynomial y = f(x) passes through set of points (x_0, y_0) , (x_1, y_1), (x_2, y_2),.............with $x = x_0+wh$ then

$y = y_0+w\Delta y_n+\frac{w(w-1)}{2!}\Delta^2 y_0+\frac{w(w-1)(w-2)}{3!}\Delta^2 y_0+$-------.

ii)For Backward Difference: If polynomial y = f(x) passes through set of points (x_0, y_0) , (x_1,y_1), (x_2,y_2),.............with $x = x_0+wh$ then

$y=y_0+w\nabla y_n+\frac{w(w+1)}{2!}\Delta^2 y_0+\frac{w(w+1)(w+2)}{2!}\nabla^2 y_0+$-------.

8)Euler's Method :-

Suppose $\frac{dy}{dx} = f(x, y), y(0) = y_0$, $x_n = x_0+nh$.

Then $y_1 = y_0+hf(x_0,y_0)$,

$y_2 = y_1+hf(x_1,y_1)$

$y_3 = y_2+hf(x_2,y_2)$

…….

$y_n = y_{n-1}+hf(x_{n-1},y_{n-1})$

$y_{n+1} = y_n +hf(x_n,y_n)$

9) Modification Euler's Method :-

Suppose $\frac{dy}{dx} = f(x, y), y(0) = y_0$, $x_n = x_0+nh$.

Then by Euler's Method we find

$y_1=y_0+hf(x_0, y_0)$

Modified the y_1 are given by

$y_1^{(1)} = y_0+\frac{h}{2}(f(x_0, y_0) + f(x_1, y_1))$

$y_1^{(2)} = y_0+\frac{h}{2}(f(x_0, y_0) + \mathrm{f}(x_1, y_1{}^2)$

$y_1^{(2)} = y_0+\frac{h}{2}(f(x_0, y_0) + \mathrm{f}(x_1, y_1{}^3)$

…………

$y_1^{(n)} = y_0+\frac{h}{2}(f(x_0,y_0)+f(x_1,y_1^{(n-1)}))$

If $y_1^{(n)} = y_1^{(n-1)}$ then $y(x_1)=y_1=y_1^{(n)}$

Then by Euler's method,

$y_2=y_1+hf(x_1,y_1)$

Modified values of y_2 are given by

$y_2^{(2)} = y_1+\frac{h}{2}(f(x1,y1) + f(x2,y2(1)),$

$y_2^{(3)} = y_1+\frac{h}{2}(f(x1,y1) + f(x2,y2(2)),$

…….

$y_2^{(n)} = y_1+\frac{h}{2}(f(x1,y1) + f(x2,y2(n-1)),$

By Euler's Method, we find

$y_n = y_{n-1}+hf(x_{n-1},y_{n-1})$

We Modify above value by using

$y_n^{(n)} = y_n+\frac{h}{2}\left(f(x_{n-1},y_{n-1}) + f(x_n,{y_n}^{(n-1)}\right)$

10)Runge –kutta Fourth Order Method

Suppose $\frac{dy}{dx} = f(x,y), y(0) = y_0,$

Then find K by using

$K_1=hf(x_0, y_0)$

$K_2=hf(x_0+\frac{h}{2}, y_0+\frac{k_1}{2})$

$K_3=hf(x_0+\frac{h}{2}, y_0+\frac{k_2}{2})$

$K_4=hf(x_0+h, y_0+k_3)$

$K=\frac{1}{6}(k_1+2(k_2+k_3)+k_4)$

Then find y_1 using

$y_1=y_0+k.$