1) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$

2) $\log(\infty) = \log(0) = \infty$

3) $\lim_{n\to\infty}(n)^{1/n}$=1

4)Arithmetic Progression

Difference is common between 2c successive term.

Formula for finding n[th] term $t_0 = t_1 + (n-1)d$

5)Geometric Progression

Ratio is common between 2 successive term

For geometric series if $|r| < 1$ then series is convergent.

Important Notes for Online

1)For a series of positive terms $\sum u_n = 0$ is convergent or divergent.

2)For a series of positive term is divergent if $u_n > 0, i.e \log u_n \neq 0.$

3)If sequence has finite limit then it is convergent.

$\lim_{n\to\infty} S_n = Finite$

4) If sequence has infinite limit then it is divergent .

$$\lim_{n\to\infty} S_n = {}^{\infty}/_{-\infty}$$

5)The Convergence of infinite series remains unaltered if

i)the sign of all terms in a series are changed.

ii)Finite number of terms are added or omimited from an infinite series.

iii)Each term of the series is multiplied by non-zero constant.

iv)Infinite number of terms are added to an infinite series.

- **Cauchy's n[th] root test(pow(n))**
  - Convergent $\lim_{n\to\infty} (u_n)^{1/n} < 1$
  - Divergent $\lim_{n\to\infty} (u_n)^{1/n} > 1$
  - Test Fail $\lim_{n\to\infty} (u_n)^{1/n} = 1$
- **P series test** $\left[\sum_{n=1}^{\infty} \frac{1}{n^p}\right]$
  - Convergent p>1
  - Divergent $\leq$**1**
- **Comparison Test**

1)sin x = x$-\frac{x^3}{3!}$+$\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$

2)cos x=1 - $\frac{x^2}{2!}$+$\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$

3)tan x= x+$\frac{x^3}{3!}$+2$\frac{x^5}{15}+17\frac{x^7}{17}+\cdots$

Note :- if series is there then first term if

$\sum v_n$ *is said to be a Auxillary Series*

- **D'Alembert's Ratio Test**[*fact/base,pow(n)]*

1)Convergent $\lim_{n\to\infty}\frac{u_n}{u_{n+1}}>1$

2) Divergent $\lim_{n\to\infty}\frac{u_n}{u_{n+1}}<1$

3)Test Fail $\lim_{n\to\infty}\frac{u_n}{u_{n+1}}=1$

If $\lim_{n\to\infty}\frac{u_n}{u_{n+1}}=\frac{1}{x}$ then put x=1 in u_n & use comparison Test.

- **Raabe's Test**

1) Convergent $\lim_{n\to\infty}\left[n\left(\frac{u_n}{u_{n+1}}\right)\right]>1$

2) Divergent $\lim_{n\to\infty}\left[n\left(\frac{u_n}{u_{n+1}}\right)\right]<1$

3)Test Fail $\lim_{n\to\infty}\left[n\left(\frac{u_n}{u_{n+1}}\right)\right]=1$

- **Leibnitz Test** (*Alternating series*)

i)The terms of given series are alternately positive & negative.

ii)Each term of the series is numerically less than the preceding term.

iii) $\sum\ (-1)^{(n-1)}u_n$

$\lim\limits_{n\to\infty} u_n = 0$[Exclude $(-1)^{(n-1)}$]

Thus all 3 condition of the Leibnitz test are satisfied ,then series is convergent.

- Absolutely Convergent $\sum u_n$ =Convergent &$\sum|u_n|$=Convergent
- Conditionally Convergent $\sum u_n$ =Convergent &$\sum|u_n|$=Divergent

❖ **Successive Differentiation**

1)$\frac{d^n}{dx^n}[e^{ax}]$=$a^n e^{ax}$

2) $\frac{d^n}{dx^n}[a^x]$=$a^x(\log a)^n$

3) $\frac{d^n}{dx^n}[x^n]$=n!

4) $\frac{d^n}{dx^n}[(ax+b)^m]$=$\frac{m!a^n(ax+b)^{m-n}}{(m-n)!}$

5) $\frac{d^n}{dx^n}[(ax+b)^{-m}]$=$\frac{d^n}{dx^n}\left[\frac{1}{(ax+b)^m}\right] =$ $\frac{(-1)^n a^n (m+1)(m+2).....(m+n-1)}{(ax+b)^{m+n}}$

6) $\frac{d^n}{dx^n}[(ax+b)^m]$=n! a^n

7) $\frac{d^n}{dx^n}\left[\frac{1}{(ax+b)}\right]=\frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$

8) $\frac{d^n}{dx^n}[\log(ax+b)]$=$\frac{(-1)^{n-1}(n-1)! a^n}{(ax+b)^n}$

9) $\frac{d^n}{dx^n}[\sin(ax+b)]$=$a^n \sin(ax+b+\frac{n\pi}{2})$

10) $\frac{d^n}{dx^n}[\cos(ax+b)]$=$a^n \cos(ax+b+\frac{n\pi}{2})$

11) $\frac{d^n}{dx^n}[e^{ax}\sin(bx+c)]$=$r^n e^{ax}\sin(bx+n\theta)$

$$where\ r=\sqrt{a^2+b^2}\,,\theta=\tan^{-1}\left({}^{b}/{}_{a}\right)$$

12) $\frac{d^n}{dx^n}[e^{ax}\cos(bx+c)]$=$r^n e^{ax}\cos(bx+c+n\theta)$

13) $\frac{d^n}{dx^n}[\tan^{-1}x]=(-1)^{n-1}(n-1)!\sin[n\theta]\,sin^n\theta$

where $\theta=\cot^{-1}(x)$

14) Leibnitz theorem:-

If y = uv

Then y_n= $nc_0u_nv+nc_1u_{n-1}v_1+nc_2u_{n-2}v_2+\cdots+nc_nuv_n$