

# **DUIS :- DIFFERENTIATION UNDER INTEGRAL SIGN**

## **DUIS rule 1:**

If  $I(\alpha) = \int_a^b f(x, \alpha) dx$ , then  $\frac{dI}{d\alpha} = \int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha) dx$

## **DUIS rule 2:**

If  $I(a) = \int_{a(\alpha)}^{b(\alpha)} f(x, \alpha) dx$  where  $a$  and  $b$  are the  $f(\alpha)$

$$\frac{dI}{d\alpha} = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial \alpha} f(x, \alpha) dx + f(b, \alpha) \frac{db}{d\alpha} - f(a, \alpha) \frac{da}{d\alpha}$$

## **Error Function:-**

1) Error Function :-

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

2) Complementary error function

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du$$

3)  $\text{erf}(\infty) = 1$

4)  $\text{erf}(0) = 0$

$$5) \quad \text{erf}(x) + \text{erfc}(x) = 1$$

$$6) \quad \text{erf}(-x) = -\text{erf}(x)$$

$$7) \quad \text{erf}(x) = \frac{2}{\sqrt{\pi}} \left[ x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \dots \right]$$

## Curve Tracing

Equation of curve	Formula for integral calculus:
$y = f(x)$	$s = \int_{s_1}^{s_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
$x = g(y)$	$s = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
$x = f_1(t),$ $y = f_2(t)$	$s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt$

$r = f(\theta)$	$s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
$\theta = f(r)$	$s = \int_{r_1}^{r_2} \sqrt{1 + r^2 \left(\frac{d\theta}{dr}\right)^2} dr$