Multiple Integrals:-

Double Integration

An expansion $\int_a^b \int_{h(x)}^{g(x)} f(x,y)dydx$

And $\int_c^d \int_{h(x)}^{g(x)} f(x,y)dxdy$ is a double Integral and is evaluated from inside outward direction.

Consider ,

I= $\iint f(x,y)dxdy$

i) If the strip is parallel to y-axis then the integral is

$$\int_{x=a}^{b} \int_{y=h(x)}^{g(x)} f(x,y)dydx$$

$i.e\ first\ integrate\ with\ resept\ to'y'and\ then\ with$ *respect to 'x'.*

ii) If the strip is parallel to x-axis then the integral is

$$\int_{y=a}^{b} \int_{x=h(y)}^{g(y)} f(x,y)dxdy$$

$i.e\ first\ integrate\ with\ resept\ to'x'and\ then\ with$ *resept to 'y'.*

iii) If limits are constant i.e.

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy$$

Then we are able to separate 'x' and 'y' then Integrate separately.

Tripple integration

An integral I $=\int_{a}^{b}\int_{f_1(x)}^{f_2(x)}\int_{\theta_1(x,y)}^{\theta_2(x,y)} f(x,y,z)dxdydz$.

$is\ called\ tripple\ integration.$

1) Spherical Polar substitution

x= r sinθ cosØ

y = r sinθ sinØ

z = r cosθ

2)For complete sphere $x^2+y^2+z^2=a^2$ varies from 0 to a.

R→ $(0,a)$

$\theta \rightarrow (0,\pi)$

Ø → $(0,2\pi)$

3)For hemisphere

$$R \to (0, a)$$

$$\theta \to (0, \pi/2)$$

$$\emptyset \to (0, 2\pi)$$

4)For positive octant

$$R \to (0, a)$$

$$\theta \to (0, \pi/2)$$

$$\emptyset \to (0, \pi/2)$$

***Dirichlet theorem:-**

1) For plane x+y+z≤ 1

$$\iiint x^{l-1} y^{m-1} z^{n-1} dxdydz = \frac{\lceil l \lceil m \lceil n}{\lceil (l+m+n+1)}$$

For plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$$use\ substitution\ x = au\ y = hv\ z = cw$$

dx = adu , dy = bdv dz = cdw

Volume=$\iiint dxdydz = \iiint r^2 sin\theta drd\theta d\emptyset$