

Vector Calculus plays an important role in Engineering to understand the concept of Electrodynamics

- Dot Product: Let $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$, $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ be two vectors then the dot product of two vectors is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

- Cross product: Let $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$, $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ be two vectors then the Cross product of two

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

- Scalar Triple product/ Box product:

$$|\vec{a} \quad \vec{b} \quad \vec{c}| = \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

- Vector Triple Product: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{a})\vec{c}$

- Angle between $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$, $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

- Gradient of Scalar Function ϕ : $\nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$

- The D.D of ϕ at (x, y, z) along the direction of vector \vec{a} is $= \nabla_{(x, y, z)} \cdot \hat{a}$

- Divergence of Vector Field \vec{F} : $\nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

- A vector field \vec{F} is Solenoidal if and only if $\nabla \cdot \vec{F} = 0$

- Curl of Vector Field \vec{F} : $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

- A vector field \vec{F} is Irrotational if and only if

$\nabla \times \vec{F} = 0$, we find scalar function or scalar potential $\phi = \int F_1 dx + \int_{\text{Terms free from } x} F_2 dx + \int_{\text{Terms free from } (x,y)} F_3 dx + C$

$\nabla \times \vec{r} = 0, \nabla \times \vec{a} = 0,$ $\nabla (\vec{a}, \vec{r}) = \vec{a}$	$\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r),$ $\nabla f(r) = f'(r) \frac{\vec{r}}{r}$
$\nabla r = \frac{\vec{r}}{r}, \nabla \cdot \vec{r} = 3$	$\nabla \cdot (\phi \vec{u}) = \nabla \phi \cdot \vec{u} + \phi (\nabla \cdot \vec{u})$
$\nabla \cdot (\vec{u} \times \vec{v}) =$ $\vec{v} \cdot (\nabla \times \vec{u}) - \vec{u} \cdot (\nabla \times \vec{v})$	$\nabla \times (\phi \vec{u}) = \nabla \phi \times \vec{u} + \phi (\nabla \times \vec{u})$
$\nabla \times (\nabla \phi) = 0$	$\nabla \cdot (\nabla \times \vec{u}) = 0$