If z=f(x ,y) then $\frac{\partial z}{\partial x}$ =Partial derivative of z w.r.to x.

Composite Function (chain rule)

1)u=f(x,y)

X=$\emptyset(s,t), y = \varphi(s,t)$

u=f(x,y)=f(s,t) $\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s}$,

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

Variables to be treated as constant

$\left(\frac{\partial u}{\partial \theta}\right)_y$ Express u in terms of θ & y and differentiate partially w.r.to θ keeping y constant

Homogeneous Functions

An expression of the form $a_0x^n+a_1x^{n-1}y+a_2x^{n-2}y^2+....+a_ny^n$ in which degree of each term is equal is known as Homogeneous equation

Deduction

1)If u is homogeneous function of degree 'n' in (x,y) then

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial xy} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

2) If $u=f^{-1}(x,y)$ is not homogeneous but f(u) is homogeneous then

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \frac{nf(u)}{f\prime(u)} \text{ and } x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial xy} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)[g'(u)-1]$$

where $g(u)= \dfrac{hf(u)}{f\prime(u)}$

Total Derivative of Implicit Function f(x,y)=0

u=f(x,y)=t

then total derivative , $\dfrac{dy}{dx} = -\dfrac{\partial f}{\partial x}\Big/\dfrac{\partial f}{\partial y}$

$$\frac{d^2 y}{dx^2} = \frac{1}{q^3}\begin{vmatrix} r & s & p \\ s & t & q \\ p & q & 0 \end{vmatrix} \text{ where } p = \frac{\partial f}{\partial x}, q = \frac{\partial f}{\partial y}, r = \frac{\partial f^2}{\partial x^2}$$