Period function:-A function repeat itself after specific period of time is called as periodic function .

The periodic function f(x) is period T is

f(x+T) = f(x)

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos \mathrm{nx} + b_n \sin nx)$$

$a_0=\frac{1}{\pi}\int_0^{2\pi} f(x)dx$

$a_n=\frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)dx$

$b_n=\frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)dx$

$$-\pi \leq x \leq \pi$$

Case 1:Neither even nor odd

$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx$

$$-\pi \le x \le \pi$$

Case 2:even

$a_0=\frac{2}{\pi}\int_{0}^{\pi} f(x)dx$

$a_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\cos(nx)dx$

$b_n=0$

$$-\pi \le x \le \pi$$

Case 3:odd

$a_0=0$

$a_n=0$

$b_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\sin(nx)dx$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$-L \le x \le L$$

Case 1: Neither even nor odd

$a_0=\frac{1}{L}\int_{-L}^{L} f(x)dx$

$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos(\frac{nx\pi}{L})dx$

$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\sin(\frac{nx\pi}{L})dx$

$$-L \leq x \leq L$$

Case 2: Even

$a_0=\frac{2}{L}\int_{0}^{L} f(x)dx$

$a_n=\frac{2}{L}\int_{0}^{L} f(x)\cos(\frac{nx\pi}{L})dx$

$b_n=0$

$$-L \leq x \leq L$$

Case 3:odd

$a_0=0$

$a_n=0$

$b_n=\frac{2}{L}\int_{0}^{L} f(x)\sin(\frac{nx\pi}{L})dx$

Practical Harmonic Analysis

$a_1\cos x+b_1\sin x$ is fundamental or first harmonic &

Amplitude = $\sqrt{{a_1}^2+{b_1}^2}$

$a_2\cos 2x+b_2\sin 2x$ is called second harmonic .And so on….

$a_o=2 \text{ X } \frac{\sum y}{n}$

$a_n=2 \text{ X } \frac{\sum y\cos(nx)}{n}$

$b_n=2 \text{ X } \frac{\sum y\sin(nx)}{n}$

Reduction Formulae:- $\boldsymbol{\int_0^{\pi} \sin^n x dx}$

$I_n=\frac{n-1}{n} I_{n-2}$

$$\int_0^{\pi/2} \sin^n x dx = \frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}........\frac{3}{4}.\frac{1}{2}\text{xP}$$

Where $p=\frac{\pi}{2}$ if n=even & p=1 otherwise.

$$\int_0^{\pi/2} \sin^n x dx = 2\int_0^{\pi/2} \sin^n x dx \ \ for\ all\ n.$$

$\int_0^{\pi/2} cos^n xdx = 2\int_0^{\pi/2} cos^n xdx \;\; if\ n = even$

=0 if n=odd

$$\int_0^{2\pi} sin^n xdx \;\; = 4\int_0^{\pi/2} sin^n xdx \quad if\ n = even.$$

= 0 if n=odd.

$$\int_0^{2\pi} cos^n xdx = 4\int_0^{\pi/2} cos^n xdx\ if\ n = even.$$

=0 if n=odd.

Reduction Formula :- $\int_0^{\pi/2} sin^n xcos^n dx$

$I_{m.n}=\frac{n-1}{m+n} I_{mn-2}$

$$\int_0^{\pi/2} sin^n xcos^n dx \quad = [\frac{\{(n-1)(n-3) \ldots ..2or1\}\{(m-1)(m-3) \ldots \ldots .2or1)\}}{(m+n)(m+n-2) \ldots \ldots 2or\ 1)}]$$

Where P=$\frac{\pi}{2}$ when m & n both are even

P=1 otherwise

$\int_0^{\pi} sin^m xcos^n dx = 2\int_0^{\pi/2} sin^n xcos^n dx$,

if n=even m=even /odd

=0

if n=odd m=even/odd

$\int_0^{2\pi} sin^m x cos^n dx = 2\int_0^{\pi/2} sin^m x cos^n dx$

if m , n=even

=0 otherwise

Gamma Function:-

1)$\lceil n = \int_0^{\infty} e^{-x} x^{n-1}\, dx (n > 0)$

2) $\int_0^{\infty} e^{-ky} y^{n-1}\, dy = \frac{\lceil n}{k^n}$

3) $\lceil p \lceil (1-p) = \frac{\pi}{sinp\pi}$

4) $\lceil (n+1) = n \lceil n$

5) $\lceil (n+1) = n!\ or\ \lceil n = n(n-1)!$

6) $\lceil I = 1\ \&\ \left\lceil \frac{1}{2} \right. = \sqrt{\pi}$

7) $\int_0^{\infty} \frac{x^n}{n^x}\, dx = \frac{n!}{(\log n)^{n+1}}$

Beta Function

1)B(m ,n)= $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

2) B(m ,n)= $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$

3)$\int_0^{\pi/2} sin^p\theta cos^q\theta \; d\theta = \frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

4)B(m, n)=B(n, m)

5)B(m ,n)=$\frac{\lceil m \lceil n}{\lceil (m+n)}$

6)Duplication Formula for Gamma Function:-

$\lceil m \left| (1+\frac{m}{n}) \right.$=$\frac{\sqrt{\pi}}{2^{2m-1}}\lceil 2m$

DUIS rule 1:

If I(α)=$\int_a^b f(x,\alpha)dx,\; then\; \frac{dI}{d\alpha} = \int_a^b \frac{\partial}{\partial \alpha} f(x,\alpha)dx$

DUIS rule 2:

If I(a)= $\int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)$dx where a and b are the f(α)

$$\frac{dI}{da} = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial a} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$$

Error Function:-

1) Error Function :-

   erf(x)=$\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

2) Complementary error function

   erfc(x)=$\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$

3) erf(∞)=1

4) erf(0)=0

5) erf(x)+erfc(x)=1

6) erf(-x)=-erf(x)

7) erf(x)= $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3}+\frac{x^5}{10}-\frac{x^7}{42}+\cdots\right]$

Curve tracing

Equation of curve	**Formula for integral calculus:**
$\boldsymbol{y = f(x)}$	$s=\int_{s1}^{s2}\sqrt{1+(\frac{dy}{dx})^2dx}$
$\boldsymbol{x = g(y)}$	$s=\int_{y1}^{y2}\sqrt{1+(\frac{dx}{dy})^2dy}$
$\boldsymbol{x = f_1(t)}$, $\boldsymbol{y = f_2(t)}$	$s=\int_{t1}^{t2}\sqrt{(\frac{dy}{dt})^2+(\frac{dx}{dt})^2dt}$
$\boldsymbol{r = f(\theta)}$	$s=\int_{\theta1}^{\theta2}\sqrt{r^2+(\frac{dr}{d\theta})^2d\theta}$
$\boldsymbol{\theta = f(r)}$	$s=\int_{r1}^{r2}\sqrt{1+r^2(\frac{d\theta}{dr})^2dr}$