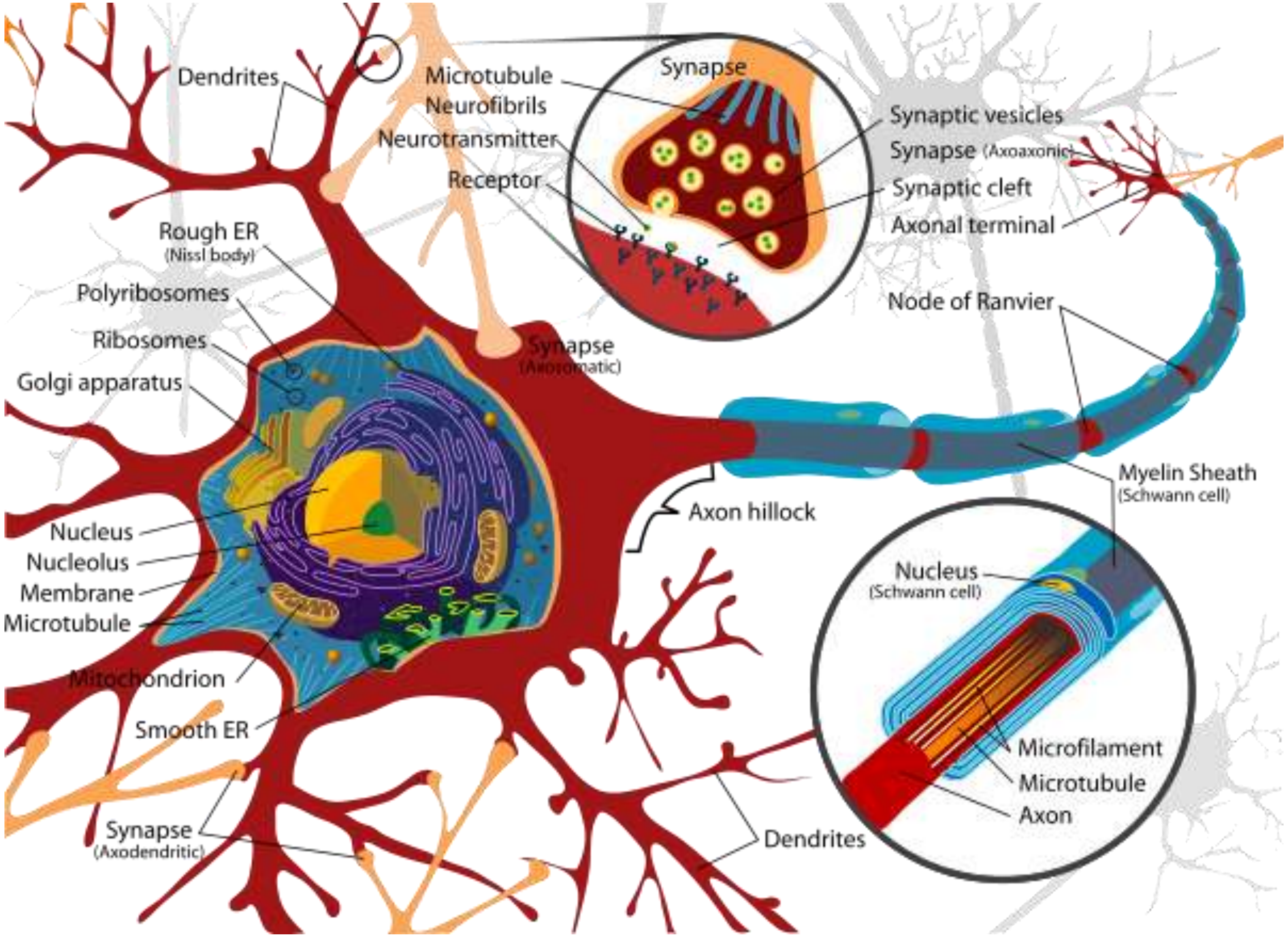
Theoretical & computational Neuroscience:
Programming the Brain

(BM 6140)

2-credit

The Neuron

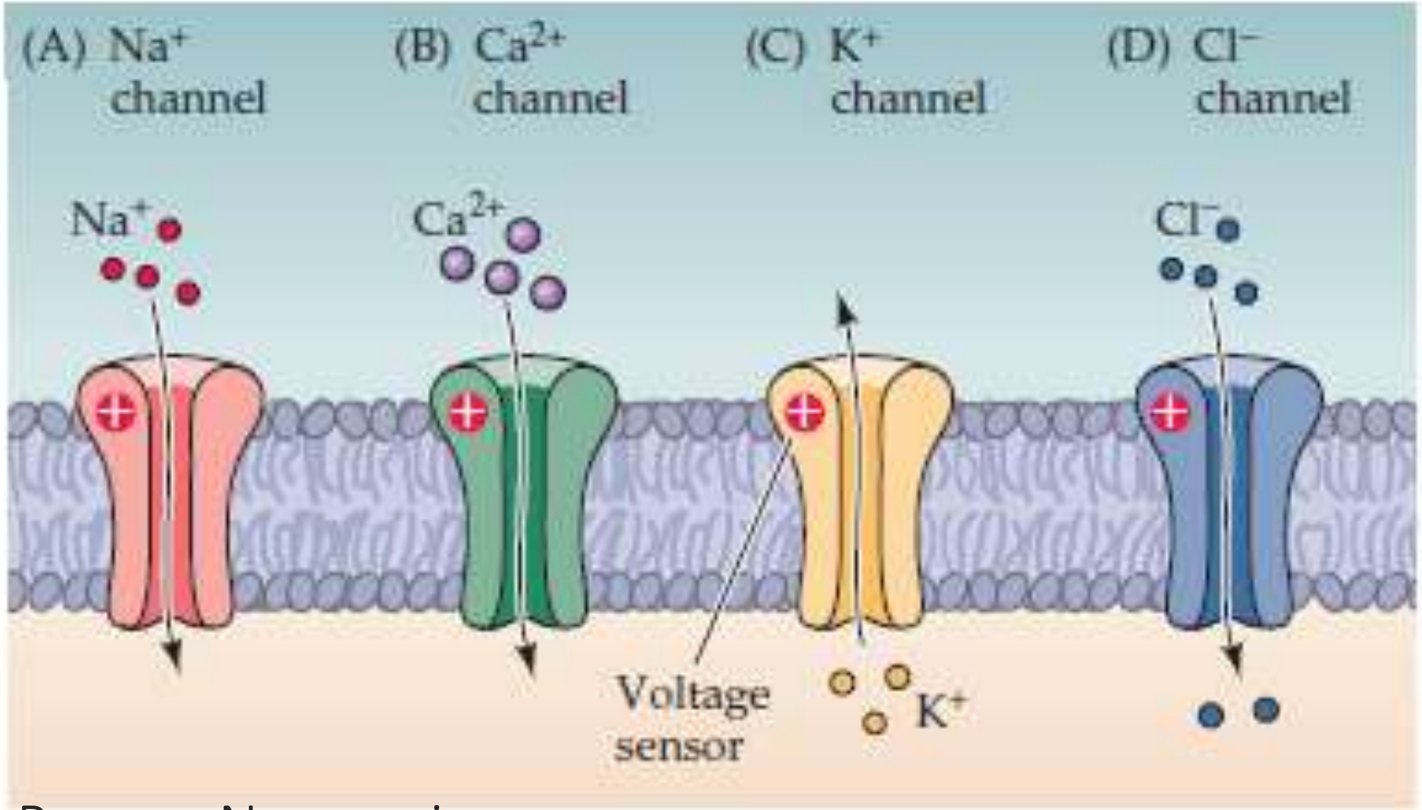


Wikipedia

Ions and Ion channels

Intra cellular fluids : Extra cellular fluids = 2:1

K+ abundant | Na+ abundant



Purves, Neuroscience

Ficks law of diffusion

$$J_{\text{diff}} = -D\frac{\partial[\mathrm{C}]}{\partial x}, \qquad (2.2.1)$$

where J is diffusion flux (molecules/sec-cm^2); D is the diffusion coefficient (cm^2/sec); and [C] is the concentration of ion (molecules/cm^3). The negative sign indicates that J flows from high to low concentration.

Foundations of Cellular Neurophysiology, Johnston and Wu

Drift

$$
\begin{aligned}
J_{\text{drift}} &= \partial_{el} E \\
&= -\mu z[\mathrm{C}]\frac{\partial V}{\partial x},
\end{aligned} \qquad (2.2.2)
$$

where J_{drift} is the drift flux (molecules/sec-cm^2), ∂_{el} is electrical conductivity (molecules/V-sec-cm), E is electric field (V/cm) $= -\frac{\partial V}{\partial x}$, V is electric potential (V), μ is mobility (cm^2/V-sec), z is the valence of the ion (dimensionless), and [C] is the concentration.

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Einsteins relation : connect diffusion constant and mobility

$$D = \frac{kT}{q}\mu, \tag{2.2.3}$$

where k is Boltzmann's constant (1.38×10^{-23} joule/°K), T is absolute temperature (°K), and q is the charge of the molecule (C).

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Nernst Planck equation : Drift + Diffusion at equilibrium

$$
\begin{aligned}
J &= J_{\mathrm{drift}} + J_{\mathrm{diff}} \\
&= -\mu z[\mathrm{C}]\frac{\partial V}{\partial x} - D\frac{\partial[\mathrm{C}]}{\partial x}.
\end{aligned}
$$

$$J = -\left(\mu z[\mathrm{C}]\frac{\partial V}{\partial x} + \frac{\mu kT}{q}\frac{\partial[\mathrm{C}]}{\partial x}\right).$$

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Nernst potential

$kT/q = RT/F$, Flux = 0, Integrate from x_{in} to x_{out} , $[C]_{in}$ to $[C]_{out}$, V_{in} to V_{out}

$$V_{in} - V_{out} = \frac{RT}{zF} \ln \frac{[C]_{out}}{[C]_{in}}$$

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What kind of ion distributions will result ?

What will be typical values of $[C]_{in}$ - $[C]_{out}$ & $V_{in} - V_{out}$?

What kind of ion distributions will result ?

What will be typical values of $[C]_{in}$ - $[C]_{out}$ & $V_{in} – V_{out}$?

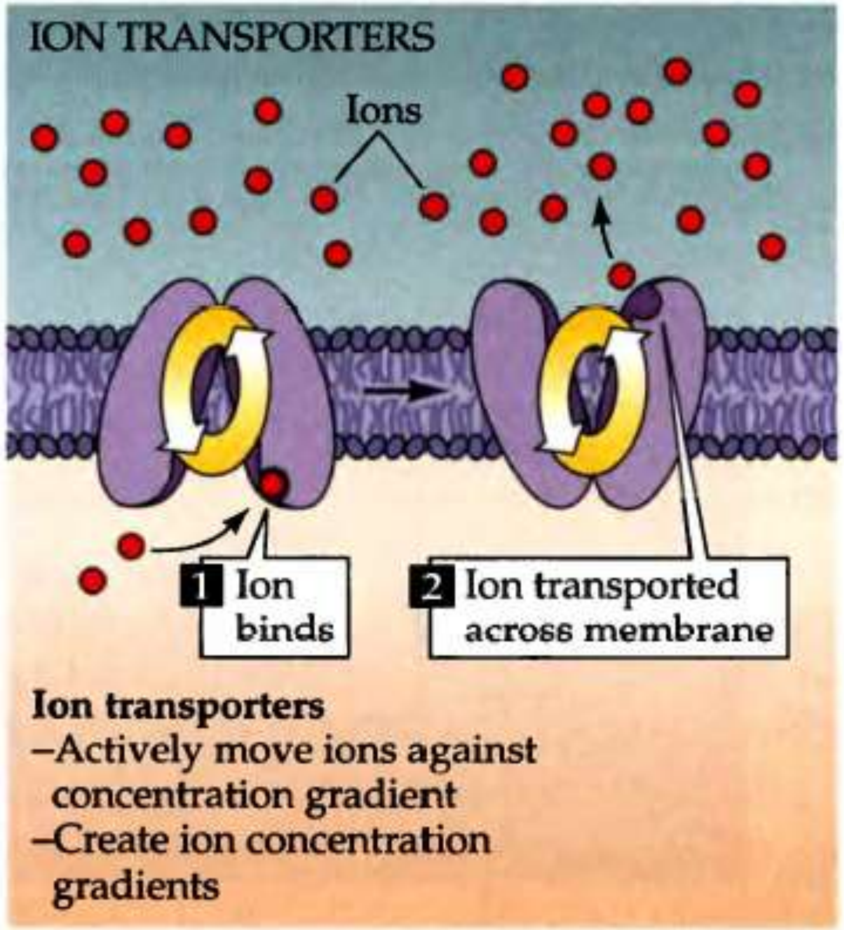
d[C] = dV = 0 ? Most natural… discharged battery

$[K^+]_{in}/[K^+]_{out} = [Cl^-]_{in}/[Cl^-]_{out}$ Equalize by different kinds of ions

Ion distribution ?

Mechanisms exist, that establish and maintain asymmetric distribution of ions and hence, keep the cells alive ! ☺ ….

- Active : pumps .. Need ATP !!! Food keeping you alive…
- Passive : selectively permeable membranes



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GHK

Above derivation assumes movement in fluids……but we have a membrane with finite thickness here !!

Constant field … GHK

$$V = \frac{RT}{F} \ln \frac{P_K[K^+]_{out} + P_{Na}[Na^+]_{out} + P_{Cl}[Cl^-]_{in}}{P_K[K^+]_{in} + P_{Na}[Na^+]_{in} + P_{Cl}[Cl^-]_{out}}. \qquad (2.7.21)$$

Where Px is the membrane permeability of ion x

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Calculate membrane potentials

	Inside (mM)	Outside (mM)
Frog muscle (Conway 1957)		
K^+	124	2.25
Na^+	10.4	109
Cl^-	1.5	77.5
Ca^{2+}	4.9†	2.1
Squid axon (Hodgkin 1964)		
K^+	400	20
Na^+	50	440
Cl^-	40–150	560
Ca^{2+}	0.4†	10

Case 1

$P_K : P_{Na} : P_{Cl} = 1 : 0.03 : 0.1$

Case 2

$P_K : P_{Na} : P_{Cl} = 1 : 15 : 0.1$

Johnston & Wu, Foundations of Cellular Neurophysiology

limitations

Membrane is not the same as an aqueous medium or aqueous pores

Useful only when voltage profiles are not really changing much….and permeability is relatively stable….

To model, we need to understand biology !!

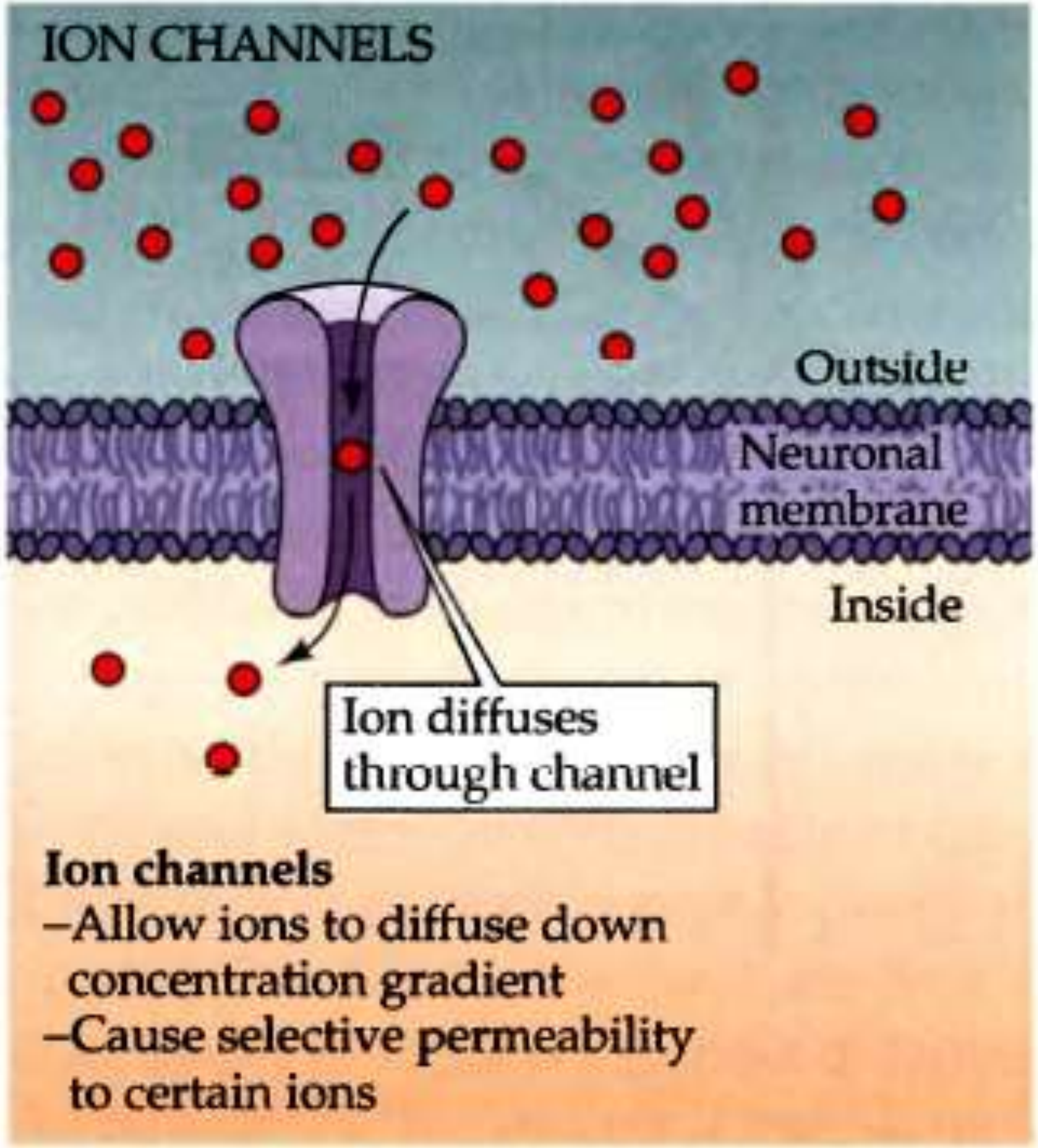
Equivalent circuit

Lipid Bilayer : Insulator, separates charges

Ion channels : Allows ions to flow through

multiple channels ?

Nernst potential : Ion imbalance at rest



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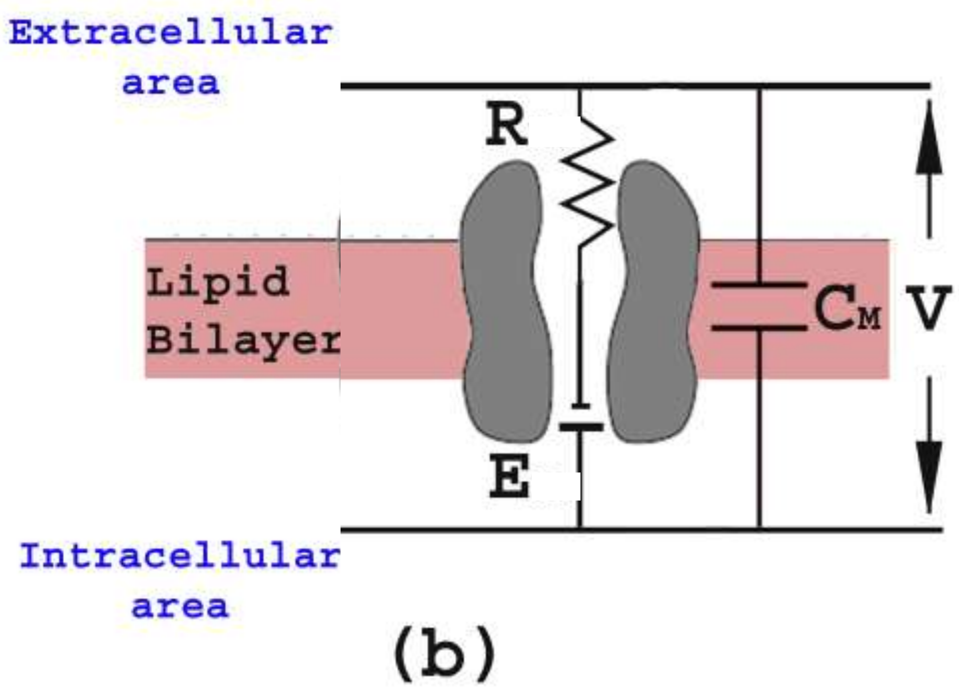
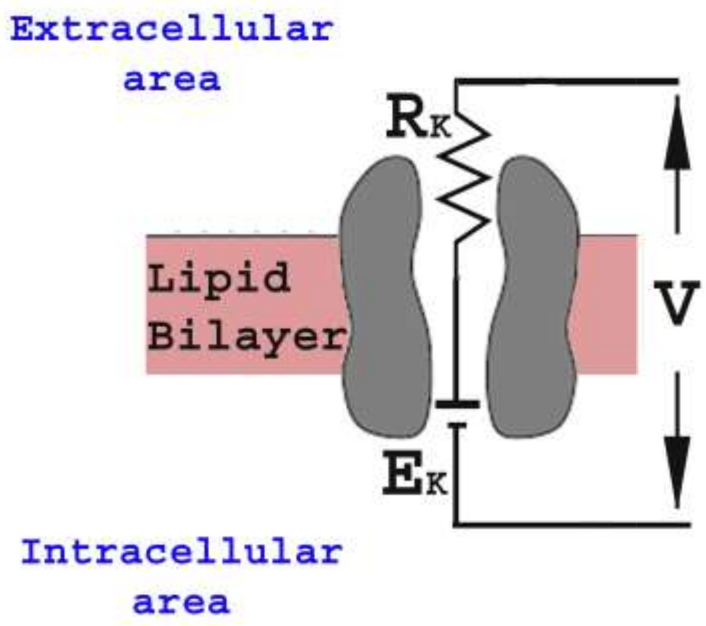
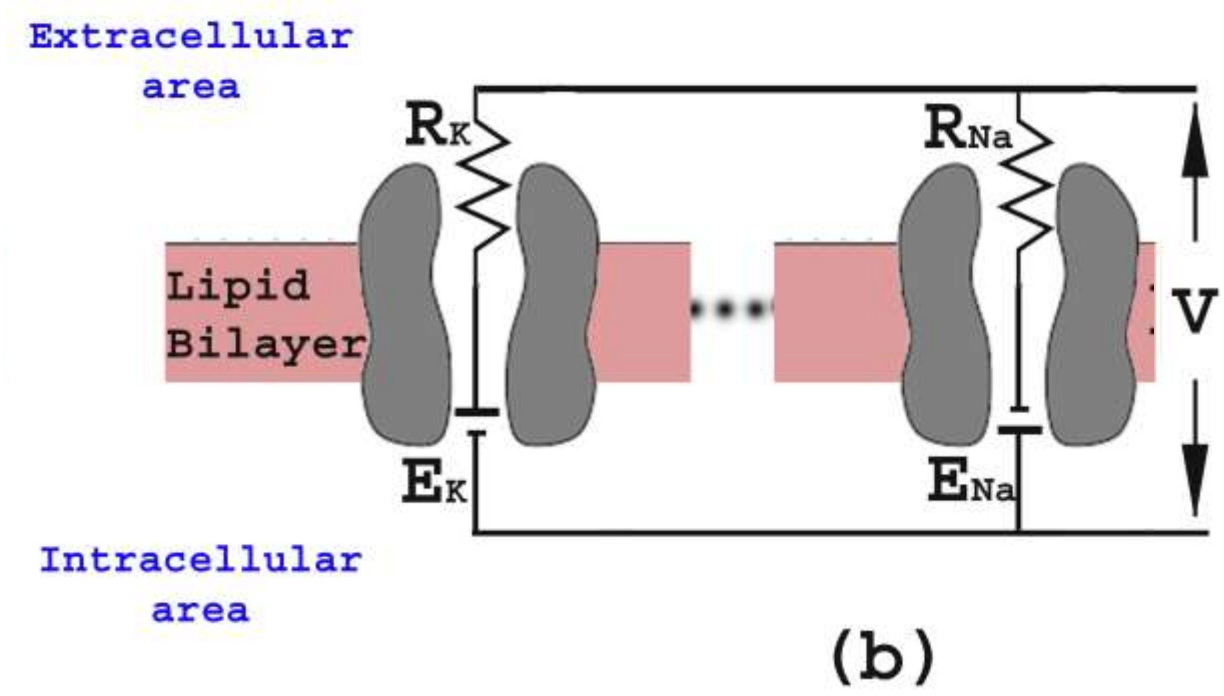
Equivalent circuit



Modeling an ion channel : a potassium channel



Add another channel for Sodium



Parallel conductance model

$$I_{inj} = i_c + i_{Na} + i_K$$

$$I_{inj} = C_m \frac{dV}{dt} + (V - E_K)g_K + (V - E_{Na})g_{Na}$$