Assignment 3

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Download all python codes from

https://github.com/sachinkarumanchi/EE3900/blob/main/assignment2.pdf

and latex codes from

https://github.com/sachinkarumanchi/EE3900/blob/main/assignment2.tex

Problem(Matrices-2.22(2)

Draw a line segment AB of length 8units. Taking **A** as center, draw a circle of radius 4 units, taking **B** as center draw another circle of radius 3 units. Construct tangents to each circle from the center of another circle.

Solution

Given, AB of lenght 8 units

	Symbol	Circle1	Circle2
Center	$\mathbf{A}, \mathbf{B}$	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}8\\0\end{pmatrix}$
Radius	r1,r2	4	3

Let BP,BQ be the tangents at to the circle with center **A**
Now,**AP** and **PB** are perpendicular so

$$(\mathbf{A}-\mathbf{P})^T(\mathbf{P}-\mathbf{B}) = 0 \tag{0.0.1}$$

Since $\|\mathbf{P}^2\| = 16$

$$\implies \mathbf{B}^T\mathbf{P} = 16 \tag{0.0.2}$$

$$\implies \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{P} = 2 \tag{0.0.3}$$

Now re-write **P** as

$$\implies \mathbf{P} = \mathbf{q_1} + \lambda_1\mathbf{m_1} \tag{0.0.4}$$

$$\implies \mathbf{P} = \begin{pmatrix}2\\0\end{pmatrix} + \lambda_1\begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.5}$$

where $\mathbf{q_1} = \begin{pmatrix}2\\0\end{pmatrix}, \mathbf{m_1} = \begin{pmatrix}0\\1\end{pmatrix}$ We know,

$$\|\mathbf{q_1} + \lambda\mathbf{m_1}\|^2 = r_1^2 \tag{0.0.6}$$

$$(\mathbf{q_1} + \lambda_1\mathbf{m_1})^T(\mathbf{q_1} + \lambda_1\mathbf{m_1}) = r_1^2 \tag{0.0.7}$$

$$\lambda_1^2 = \frac{r_1^2 - \|\mathbf{q_1}\|^2}{\|\mathbf{m_1}\|^2} \tag{0.0.8}$$

$$\lambda_1 = \pm 3.46410 \tag{0.0.9}$$

from (0.0.5), The Points **P** and **Q** are

$$\mathbf{P} = \begin{pmatrix}2\\3.46410\end{pmatrix} \tag{0.0.10}$$

$$\mathbf{Q} = \begin{pmatrix}2\\-3.46410\end{pmatrix} \tag{0.0.11}$$

Let AR,AS be the tangents to the circle with center **B**
Shift the origin from **A** to **B**
The new co-ordinates of **A** and **B** would be

$$\mathbf{A} = \begin{pmatrix}-8\\0\end{pmatrix} \tag{0.0.12}$$

$$\mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix} \tag{0.0.13}$$

Now similarly, as above **BR** is perpendicular to **RA**

$$(\mathbf{B}-\mathbf{R})^T(\mathbf{R}-\mathbf{A}) = 0 \tag{0.0.14}$$

Since $\|\mathbf{R}^2\| = 9$

$$\implies \mathbf{A}^T\mathbf{R} = 16 \tag{0.0.15}$$

$$\implies \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{R} = -1.125 \tag{0.0.16}$$

Now re-write **P** as

$$\implies \mathbf{R} = \mathbf{q_2} + \lambda_2\mathbf{m_2} \tag{0.0.17}$$

$$\implies \mathbf{R} = \begin{pmatrix}-1.125\\0\end{pmatrix} + \lambda_2\begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.18}$$

where $\mathbf{q_2} = \begin{pmatrix} -1.125 \\ 0 \end{pmatrix}, \mathbf{m_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ We know,

$$\|\mathbf{q_2} + \lambda \mathbf{m_2}\|^2 = r_2^2 \tag{0.0.19}$$

$$(\mathbf{q_2} + \lambda_2 \mathbf{m_2})^T (\mathbf{q_2} + \lambda_2 \mathbf{m_2}) = r_2^2 \tag{0.0.20}$$

$$\lambda_2^2 = \frac{r_2^2 - \|\mathbf{q_2}\|^2}{\|\mathbf{m_2}\|^2} \tag{0.0.21}$$

$$\lambda_2 = \pm 2.78017 \tag{0.0.22}$$

from (0.0.18), The Points $\mathbf{R}$ and $\mathbf{S}$ are

$$\mathbf{R} = \begin{pmatrix} -1.125 \\ 2.78107 \end{pmatrix} \tag{0.0.23}$$

$$\mathbf{S} = \begin{pmatrix} -1.125 \\ -2.78107 \end{pmatrix} \tag{0.0.24}$$

Now shifting the origin to $\mathbf{A}$, The co-ordinates of $\mathbf{A}, \mathbf{B}, \mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ would be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.25}$$

$$\mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{0.0.26}$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3.46140 \end{pmatrix} \tag{0.0.27}$$

$$\mathbf{Q} = \begin{pmatrix} 2 \\ -3.46140 \end{pmatrix} \tag{0.0.28}$$

$$\mathbf{R} = \begin{pmatrix} 6.875 \\ 2.78107 \end{pmatrix} \tag{0.0.29}$$

$$\mathbf{S} = \begin{pmatrix} 6.875 \\ -2.78107 \end{pmatrix} \tag{0.0.30}$$