Gate Assignment 1

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Download all python codes from

https://github.com/sachinkarumanchi/EE3900/tree/Gate_assignment1.ipynb

and latex codes from

https://github.com/sachinkarumanchi/EE3900/tree/Gate_assignment1.tex

Problem

Let $h[n]$ be a length-7 discrete-time finite impulse response filter, given by $h[0] = 4$, $h[1] = 3$, $h[2]=2$, $h[3] = 1$, $h[-1] = -3$, $h[-2] = -2$, $h[-3] = -1$, and $h[n]$ is zero for $|n| \geq 4$. A length-3 finite impulse response approximation $g[n]$ of $h[n]$ has to be obtained such that

$$E(h, g) = \int_{-\pi}^{\pi} \left|H(e^{j\omega}) - G(e^{j\omega})\right|^2 d\omega$$

is minimized, where $H(e^{j\omega})$ and $G(e^{j\omega})$ are the discrete-time Fourier transforms of $h[n]$ and $g[n]$ respectively, For the filter minimizes $E(h, g)$, the value of $10g[-1]+g[1]$, rounded off to 2 decimal places, is

Solution

Consider $y[n]$ such that

$$y[n] = h[n] - g[n] \tag{0.0.1}$$

Let $Y(e^{j\omega})$ be the Discrete-time Fourier transform of $y[n]$.
From Parseval's theorem,

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega \tag{0.0.2}$$

from (0.0.2) we can say that

$$\int_{-\pi}^{\pi} \left|Y(e^{j\omega})\right|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |y[n]|^2 \tag{0.0.3}$$

$$\int_{-\pi}^{\pi} \left|H(e^{j\omega}) - G(e^{j\omega})\right|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |y[n]|^2 \tag{0.0.4}$$

$$E(h, g) = 2\pi \sum_{n=-\infty}^{\infty} |y[n]|^2 \tag{0.0.5}$$

Given that $h[n]$ is of length 7 and $g[n]$ is of length 3, also that $E(h, g)$ to be minimum.
Therefore, $E(h, g)$ transforms into

$$E(h, g) = 2\pi \sum_{n=-3}^{3} |h[n] - g[n]|^2 \tag{0.0.6}$$

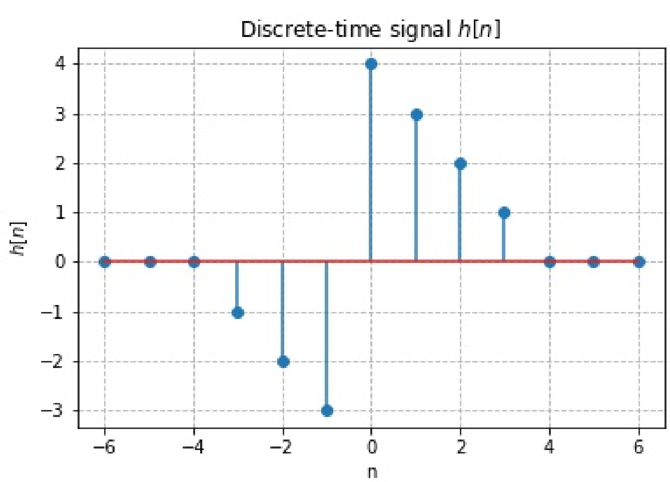
$$E(h, g) = 2\pi(|4 - g[0]|^2 + |3 - g[1]|^2 + |-3 - g[-1]|^2 + 10) \tag{0.0.7}$$

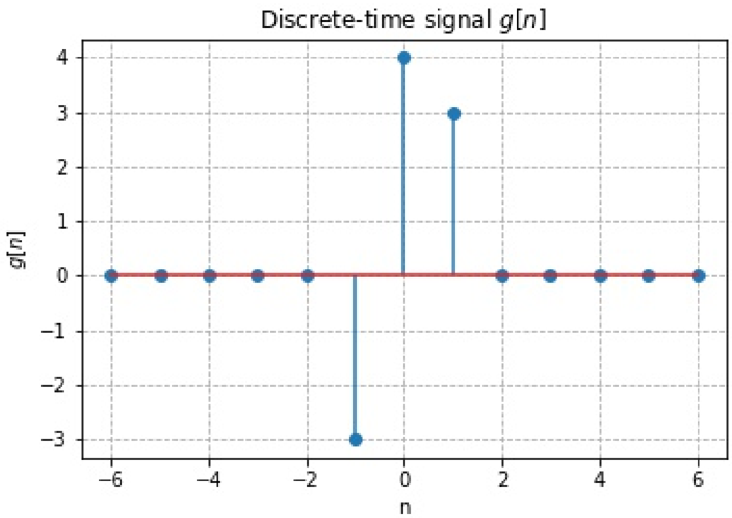
The values of $g[0] = 4$, $g[1] = 3$, $g[-1] = -3$ for $E(h, g)$ to be minimum.
Now, finding the value of $10g[-1]+g[1]$

$$10g[-1] + g[1] = 10 \times (-3) + 3 \tag{0.0.8}$$

$$= -27 \tag{0.0.9}$$

The value of $10g[-1]+g[1]$ is -27.00




Discrete-time signal $g[n]$
4
3
2
1
0
−1
−2
−3
$g[n]$
−6
−4
−2
0
2
4
6
n