

GATE ASSIGNMENT 2

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Download all latex codes from

<https://github.com/sachinkarumanchi/EE3900/tree/main/Gateassignment2/Gateassignment2.tex>

GATE EC 2008 Q.84

The impulse response $h(t)$ of linear time - invariant continuous time system is given by $h(t) = e^{-2t}u(t)$, where $u(t)$ denotes the unit step function.

The frequency response $H(\omega)$ of this system in terms of angular frequency ω , is given by $H(\omega)$

(a) $\frac{1}{1+j2\omega}$

(b) $\frac{\sin(\omega)}{\omega}$

(c) $\frac{1}{2+j\omega}$

(d) $\frac{j\omega}{2+j\omega}$

SOLUTION

Given,

$$h(t) = e^{-2t}u(t) \quad (0.0.1)$$

Finding the laplace transform

$$\mathcal{L}(h(t)) = \int_0^{\infty} h(t)e^{s-t} dt \quad (0.0.2)$$

$$= \int_0^{\infty} e^{-2t} e^{-st} dt \quad (0.0.3)$$

$$= \int_0^{\infty} e^{-(2+s)t} dt \quad (0.0.4)$$

$$= \frac{1}{2+s} \quad (0.0.5)$$

The Fourier transform is nothing but the Laplace transform evaluated at $s = j\omega$

$$H(\omega) = |\mathcal{L}(f(t))|_{s=j\omega} \quad (0.0.6)$$

$$\Rightarrow H(\omega) = \frac{1}{2+j\omega} \quad (0.0.7)$$

Therefore, frequency response $H(\omega) = \frac{1}{2+j\omega}$

Hence, option (c) is correct answer