ASSIGNMENT 5

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Download all latex codes from

https://github.com/sachinkarumanchi/EE3900/tree/main/assignment5/assignment5.tex

Quadratic forms Q2.34

Find The area enclosed by the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{0.0.1}$$

Solution

Given the equation of ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{0.0.2}$$

On comparing with the standard form we get

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.3}$$

$$\mathbf{D} = \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \tag{0.0.4}$$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 1 \tag{0.0.5}$$

$$\lambda_1 = \frac{1}{a^2} \tag{0.0.6}$$

$$\lambda_2 = \frac{1}{b^2} \tag{0.0.7}$$

Semi major and minor axes of ellipse are

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = \sqrt{\frac{1}{\frac{1}{a^2}}} = a \tag{0.0.8}$$

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = \sqrt{\frac{1}{\frac{1}{b^2}}} = b \tag{0.0.9}$$

The Equation of ellipse can be written as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{0.0.10}$$

Therefore, the area enclosed by the ellipse would be πab