

Assignment 2

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Download all python codes from

https://github.com/sachinkarumanchi/probability_and_random_variables/blob/assignment2/assignment2.py

and latex-tikz codes from

https://github.com/sachinkarumanchi/probability_and_random_variables/blob/assignment2/Assignment2.tex

1 PROBLEM

Two Players, A and B, alternately keep rolling a fair dice. The person gets a six first wins the game. Given the Player A starts the game, the probability that A wins the game.

2 SOLUTION

Let $X \in \{1, 2, 3, 4, 5, 6\}$ be the random variable representing out come of a dice.

Probability of getting a six on a fair dice

$$\Pr(X = 6) = \frac{1}{6} \quad (2.0.1)$$

Probability of not getting a six on a fair dice

$$\Pr(X \neq 6) = \frac{5}{6} \quad (2.0.2)$$

The probability of some one wining in their n^{th} trail is

$$\begin{aligned} \Pr(X_n = 6 | X_k \neq 6, k = 1, 2, 3, \dots, n-1) \\ = \frac{1}{6} \left(\frac{5}{6}\right)^{n-1} \end{aligned} \quad (2.0.3)$$

Let the probability of a wining the game is $\Pr(A)$
If A start's the game then A can win on odd numbered trail(n)

$$n = 2m + 1 \quad (2.0.4)$$

Inorder for A to win B must lose in all of it's trails until A gets a six.

Therefore,

$$\Pr(A) = \Pr(X_1 = 6) + \Pr(X_3 = 6) + \Pr(X_5 = 6) + \dots$$

(2.0.5)

$$\Pr(A) = \left(\frac{1}{6}\right) + \left(\frac{1}{6} \left(\frac{5}{6}\right)^2\right) + \left(\frac{1}{6} \left(\frac{5}{6}\right)^4\right) \dots$$

(2.0.6)

$$= \frac{1}{6} \sum_{m=0}^{\infty} \left(\frac{5}{6}\right)^{2m}$$

(2.0.7)

$$= \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2}$$

(2.0.8)

$$= \frac{6}{11}$$

(2.0.9)

$$\Rightarrow \Pr(A) = \frac{6}{11} \quad (2.0.10)$$

Therefore, The probability that A wins the game $= \Pr(A) = \frac{6}{11}$

State diagram of A and B

