Assignment 1

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Download all python codes from

https://github.com/sachinkarumanchi/probability_and_random_variables/blob/assignment2/assignment2.py

and latex-tikz codes from

https://github.com/sachinkarumanchi/probability_and_random_variables/blob/assignment2/Assignment2.tex

1 Problem

Two Players,A and B,alternately keep rolling a fair dice.The person gets a six first wins the game.Given the Player A starts the game,the probability that A wins the game.

2 Solution

In order for a player to win eventually the player must get a six.
Therefore,Probability of getting a six on a fair dice $= \frac{1}{6}$
Probability of not getting a six on a fair dice= $\frac{5}{6}$
The probability of some one wining in their n^{th} trail is

$$\Pr\left(X_n = 6 | X_k \neq 6, k = 1, 2, 3..., n-1\right) = \frac{1}{6}\left(\frac{5}{6}\right)^{n-1} \tag{2.0.1}$$

Let the probability of a wining the game is $\Pr(A)$
Inorder for A to win B must lose the game so and since A start the game from the equation (2.0.1) $n = 1, n = 3, n = 5, ...$ have the chances of A wining and $n = 2, n = 4, n = 6, ...$ have the chances of B wining
So,$n = 2m + 1$ having the chances of A wining and $n = 2m$ have the chances of B wining,where $m \in N$
Given,that A starts the game and the probability that A wins is summing over the probabilities that A wins in odd number of trails

$$\Pr(A) = \left(\frac{1}{6}\right) + \left(\frac{1}{6}\left(\frac{5}{6}\right)^2\right) + \left(\frac{1}{6}\left(\frac{5}{6}\right)^4\right) ... \tag{2.0.2}$$

$$= \frac{1}{6}\sum_{m=0}^{\infty}\left(\frac{5}{6}\right)^{2m} \tag{2.0.3}$$

$$\tag{2.0.4}$$

Here it became the sum of infinite terms in Geometric Progression.

$$= \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} \tag{2.0.5}$$

$$= \frac{6}{11} \tag{2.0.6}$$

Therefore, The probability that A wins the game $\Pr(A) = \frac{6}{11}$