Assignment 2

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Download all python codes from

https://github.com/sachinkarumanchi/probability_and_random_variables/blob/assignment2/assignment2.py

and latex-tikz codes from

https://github.com/sachinkarumanchi/probability_and_random_variables/blob/assignment2/Assignment2.tex

1 Problem

Two Players,A and B,alternately keep rolling a fair dice.The person gets a six first wins the game.Given the Player A starts the game,the probability that A wins the game.

2 Solution

Let $X \in \{1,2,3,4,5,6\}$ be the random variable representing out come of a dice.
Probability of getting a six on a fair dice

$$\Pr(X = 6) = \frac{1}{6} \tag{2.0.1}$$

Probability of not getting a six on a fair dice

$$\Pr(X \neq 6) = \frac{5}{6} \tag{2.0.2}$$

The probability of some one wining in their n^{th} trail is

$$\Pr(X_n = 6 | X_k \neq 6, k = 1,2,3...,n-1) = \frac{1}{6}\left(\frac{5}{6}\right)^{n-1} \tag{2.0.3}$$

Let the probability of a wining the game is $\Pr(A)$
If A start's the game then A can win on odd numbered trail(n)

$$n = 2m + 1 \tag{2.0.4}$$

Inorder for A to win B must lose in all of it's trails until A gets a six.
Therefore,

$$\Pr(A) = \Pr(X_1 = 6) + \Pr(X_3 = 6) + \Pr(X_5 = 6) + ... \tag{2.0.5}$$

$$\Pr(A) = \left(\frac{1}{6}\right) + \left(\frac{1}{6}\left(\frac{5}{6}\right)^2\right) + \left(\frac{1}{6}\left(\frac{5}{6}\right)^4\right)... \tag{2.0.6}$$

$$= \frac{1}{6}\sum_{m=0}^{\infty}\left(\frac{5}{6}\right)^{2m} \tag{2.0.7}$$

Here it became the sum of infinite terms in Geometric Progression ($r < 1$).

$$= \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} \tag{2.0.8}$$

$$= \frac{6}{11} \tag{2.0.9}$$

$$\implies \Pr(A) = \frac{6}{11} \tag{2.0.10}$$

Therefore, The probability that A wins the game=$\Pr(A) = \frac{6}{11}$

State diagram of A and B