Assignment 4

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Download all latex-tikz codes from

https://github.com/sachinkarumanchi/probability_and_random_variables/blob/assignment4/Assignment4.tex

1 Problem

Cars arrive at a service station according to Poisson's distribution with a mean rate of 5 per hour.The service time per car is exponential with a mean of 10 minutes.At a steady state,the average waiting time in the queue is

2 Solution

Fom the question given,

$$\lambda = 5\text{hr}^{-1} \tag{2.0.1}$$

$$\mu = \frac{1}{10}\text{min}^{-1} = 6\text{hr}^{-1} \tag{2.0.2}$$

Therefore,

$$\text{Utilization rate}(\rho) = \frac{\lambda}{\mu} = \frac{5}{6} \tag{2.0.3}$$

Average number (or) length in queue be L_q

$$L_q = \frac{\rho^2}{1-\rho} \tag{2.0.4}$$

$$= \frac{\left(\frac{5}{6}\right)^2}{1-\frac{5}{6}} \tag{2.0.5}$$

$$= \frac{25}{6} \tag{2.0.6}$$

Let the Average waiting time in queue be W_q

$$W_q = \frac{L_q}{\lambda} \tag{2.0.7}$$

$$= \frac{\frac{25}{6}}{5} \tag{2.0.8}$$

$$= \frac{5}{6}\text{hr} = 50\text{min} \tag{2.0.9}$$

The average waiting time in the queue is 50 min.

Parameter	Value
λ	5hr^{-1}
μ	6hr^{-1}
Utilization rate $(\rho) = \frac{\lambda}{\mu}$	$\frac{5}{6}$
Length in queue $\left(L_q\right) = \frac{\rho^2}{1-\rho}$	$\frac{25}{6}$
Waiting time in queue $\left(W_q\right) = \frac{L_q}{\lambda}$	$\frac{5}{6}$hr

TABLE 0: Parameters of the given question and values.