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Mathematical Models In the Interaction Between the Immune System and Cancer

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Overview

- Define Cancer and interaction between immune system
- Biology of Immune system
- Mathematical approaching
- Interaction between cancer cells, immune cells and cytokines
- Numerical solving
- The output in graphs
  - Low antigenic tumours
  - Microscopic tumours
  - Highly antigenic tumors
- Experiment results

What is Cancer ?

- Which means "uncontrolled growth".
  - Which is defined as a new and abnormal growth of tissue in some part of the body.

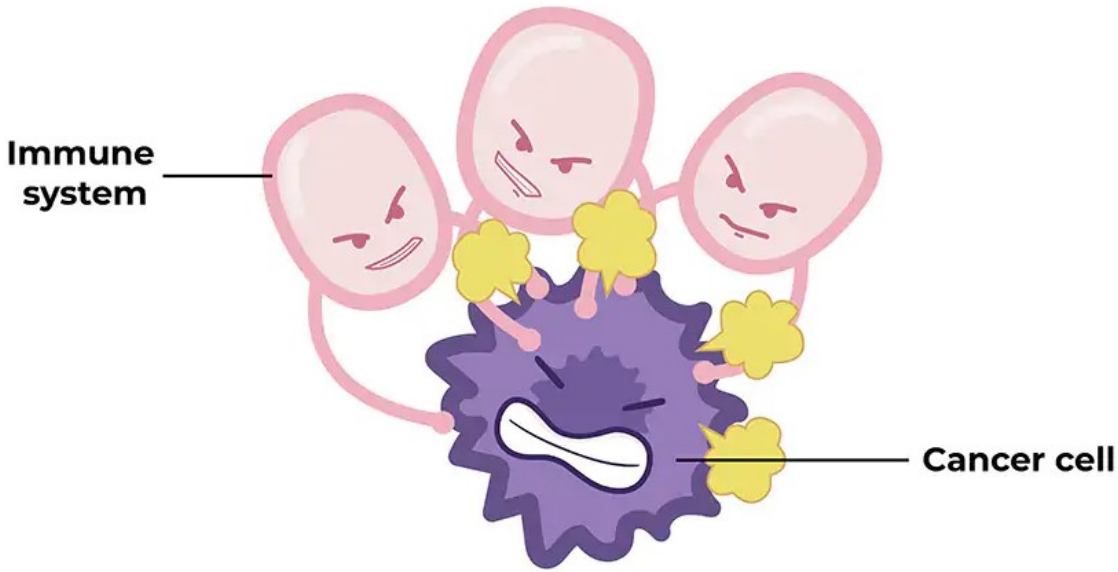

Figure 1: Immune system attacks the cancer cells

Immune system

The immune system is the body's natural defense system.

- Tumour cells
  - Tumor cells are rapidly multiplying self cells.
- Immune cells (Effector cells)
  - A number of different cells work together within the immune system to fight infections and disease. (T cells, B cells, CD8+ killer T cells, CD4+ helper T cells)
- Cytokines (IL-2)
  - Cytokines are messenger molecules that help immune cells work together to coordinate the correct immune response to any given invader, infection, or tumor.

Predator - Prey ?

$$x' = r_2 y(1 - by) - \frac{axy}{g_2 + y}$$

$$y' = cy - \mu_2 x + \frac{p_1 xz}{g_1 + z} + s_1$$

$$z' = \frac{p_2 xy}{g_3 + y} - \mu_3 z + s_2$$

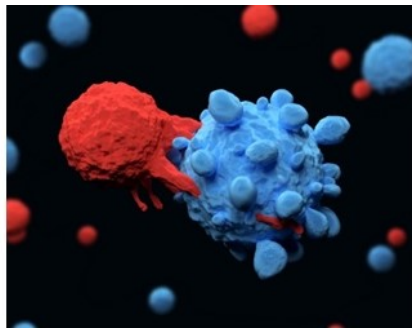

Figure 2: Predator-Prey interaction nature and cancer

$$x' = r_2 y(1 - by) - \frac{axy}{g_2 + y} \tag{1}$$

$$y' = cy - \mu_2 x + \frac{p_1 xz}{g_1 + z} + s_1 \tag{2}$$

$$z' = \frac{p_2 xy}{g_3 + y} - \mu_3 z + s_2 \tag{3}$$

Immune cells represents predator and Cancer cells will be prey.

Interaction between cancer cells, immune cells and cytokines

- Tumor cells grow logistically to a fixed carrying capacity. $r_2 y(1 - by)$
- $\frac{axy}{g_2+y}$ Tumour cells are killed at a rate by immune cells.
- r_2 and b are parameters that define how the tumour cells will grow and a and g_2 are parameters to adjust the model.
- Effector cells grow at a rate directly proportional to both the size of the tumor and its antigenicity.
  Effector cells grow based on recruitment cy and proliferation $\frac{p_1 xz}{g_1+z} + s_1$.
- Effector cells are also activated by the cytokine IL-2.
- The parameter c represents the antigenicity of the tumour cells and s_1 is the treatment that will boost the number of effector cells.
- Interleukin-2 (IL-2) is created by the effector cells, at a rate that approaches a maximum as the tumor grows.

Runge Kutta method (Fourth order)

Suppose the differential equation $y' = f(t, y)$ with initial condition $y(t_0) = y_0$ is given.
Here y is the unkown function of t.

$$y_{k+1} = y_k + \frac{h}{6}(k_0 + 2k_1 + 2k_2 + k_3)$$

$$t_{n+1} = t_n + h$$

$k = 0,1,2,3...$ using.

$$k_1 = f(t_k, y_k)$$

$$k_2 = f(t_k + h/2, y_k + hk_0/2)$$

$$k_3 = f(t_k + h/2, y_k + hk_1/2)$$

$$k_4 = f(t_k + h, y_k + hk_2)$$

Persistence of large tumours (for low antigenic tumours)

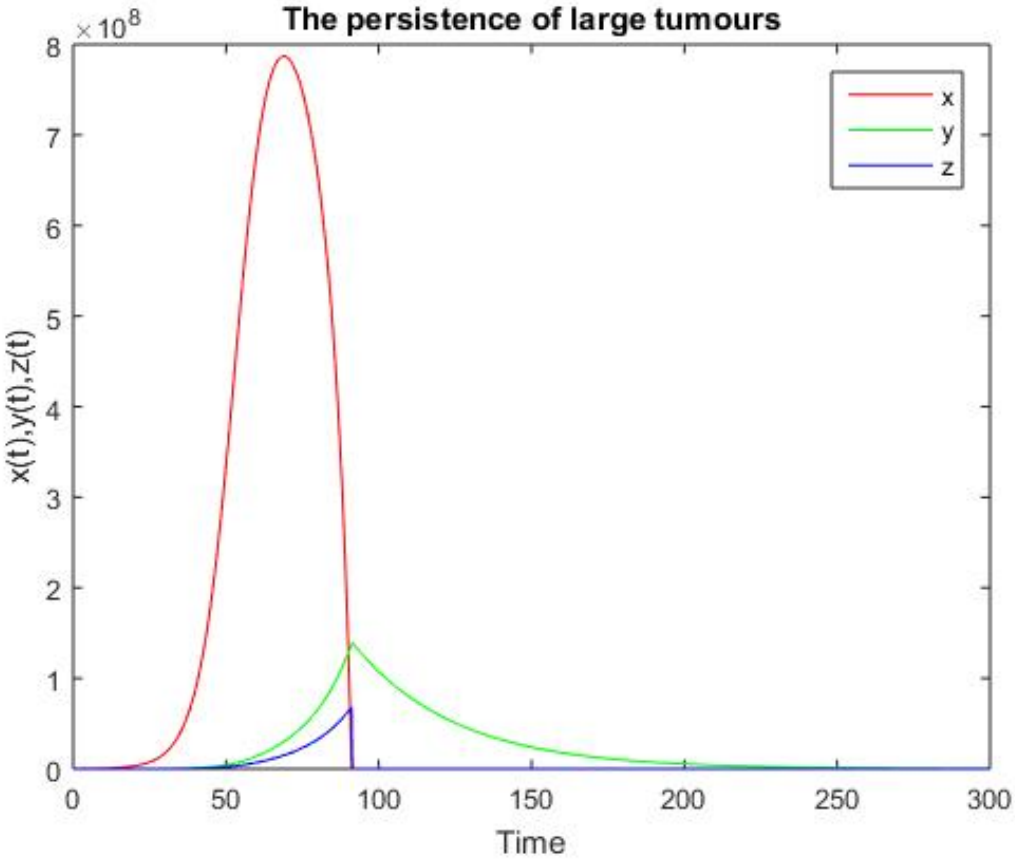

Figure 3: persistence of large tumors

$t_0 = 0, x_0 = 10000, y_0 = 10, z_0 = 0, t_n = 300, stepsize = 300000, c = 0.002$

Persistence of large tumours (for low antigenic tumours) (cont.)

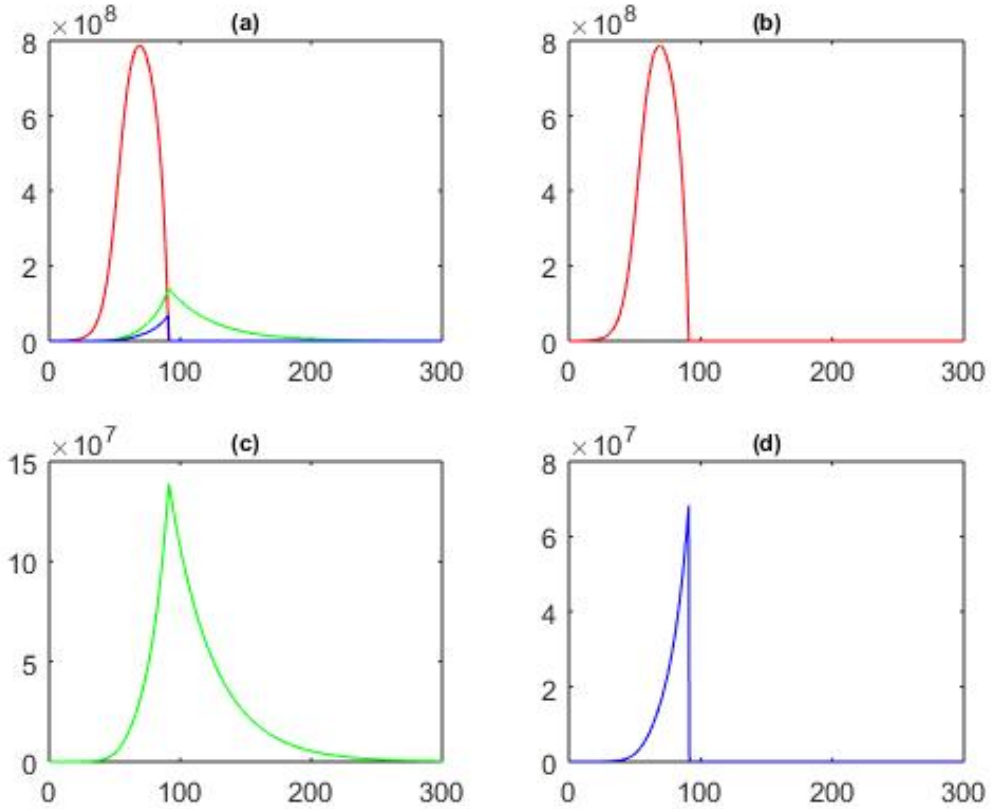

Figure 4: x, y and z behaviours

Source files you can get through this Github link.
https://github.com/sachinkavindaa/My_file1

Oscillation between macroscopic and microscopic tumours (for moderately antigenic tumours)

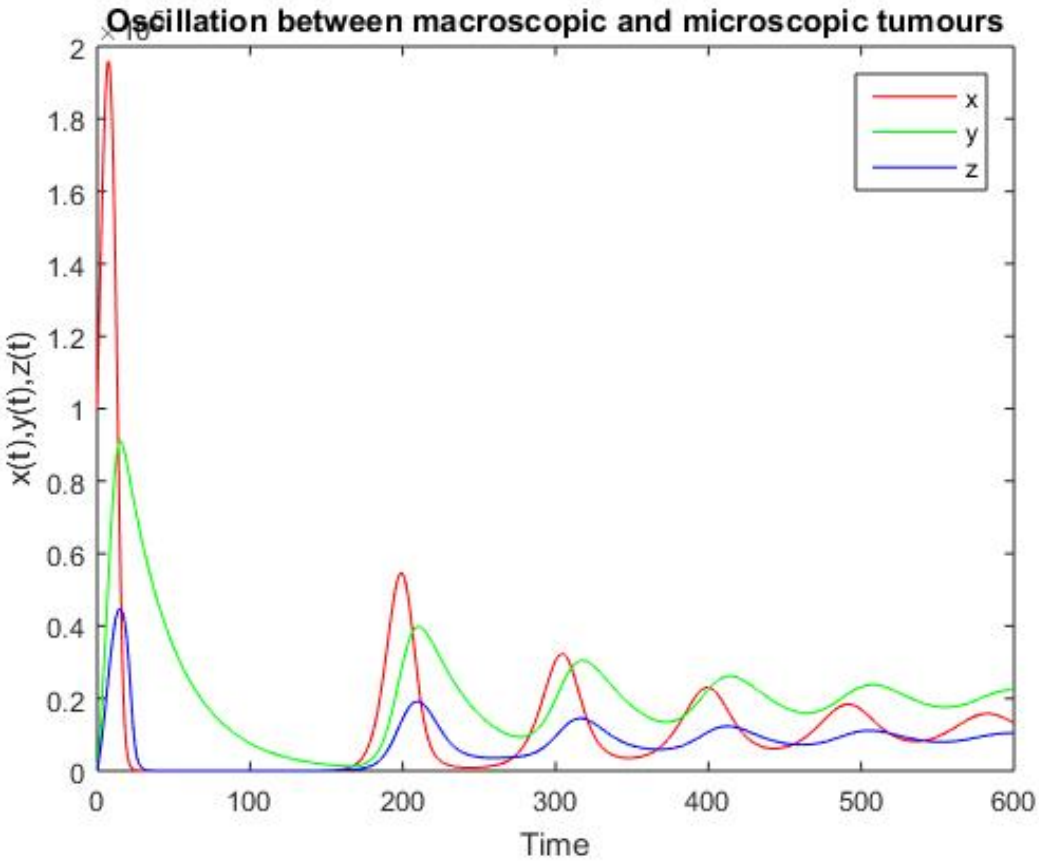

Figure 5: Oscillation between macroscopic and microscopic tumours

$t_0 = 0, x_0 = 10000, y_0 = 10, z_0 = 0, t_n = 600, stepsize = 600000, c = 0.05$

Oscillation between macroscopic and microscopic tumours (for moderately antigenic tumours) (cont.)

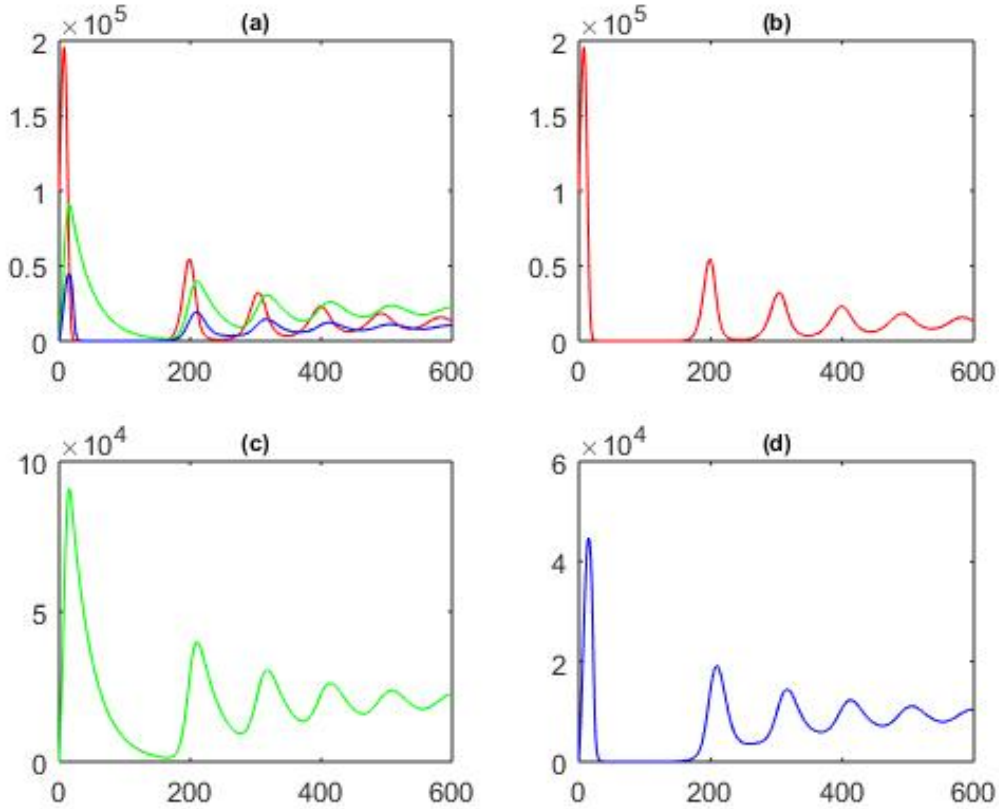

Figure 6: x, y and z behaviours

Source files get through
https://github.com/sachinkavindaa/My_file2

Persistence of dormant tumors, that is persistence of residual tumor cells (for highly antigenic tumors)

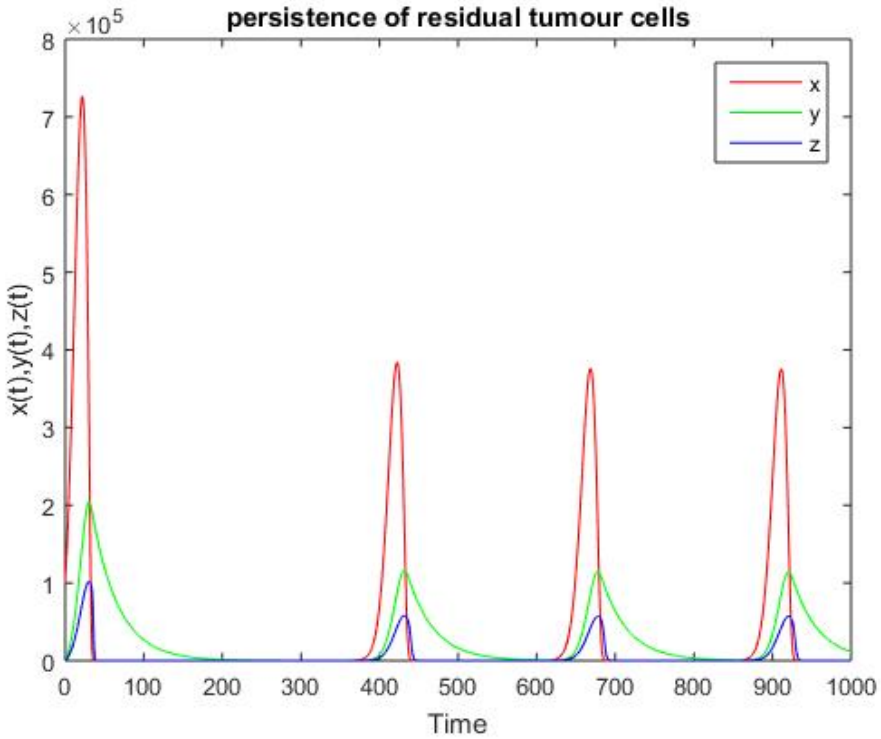

Figure 7: Persistence of dormant tumors, that is persistence of residual tumor cells

$t_0 = 0, x_0 = 10000, y_0 = 10, z_0 = 0, t_n = 1000, stepsize = 1000000, c = 0.02$

Persistence of dormant tumors, that is persistence of residual tumor cells (for highly antigenic tumors)

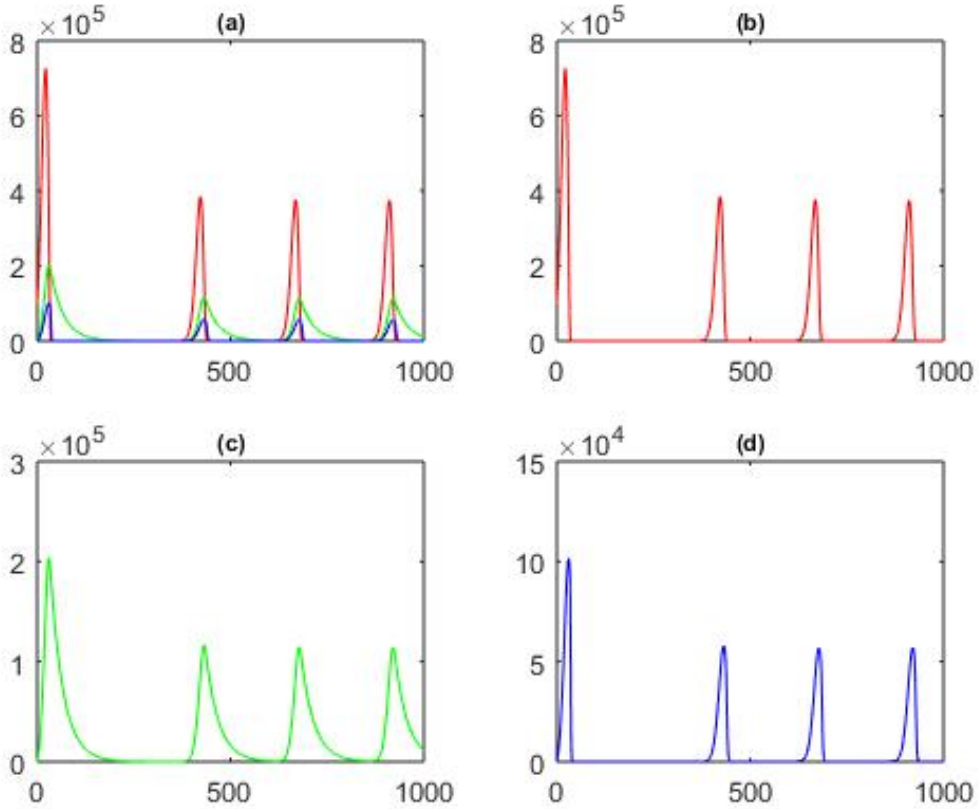

Figure 8: x, y and z behaviours

Source files get through
https://github.com/sachinkavindaa/My_file3

Experiment results

Without treatment ($s_1 = 0$ and $s_2 = 0$) which shows corresponding to the difference antigenicity values of the tumour cells (c), we can get three types of results.

- Persistence of large tumors (for low antigenic tumors).
- Oscillation between macroscopic and microscopic tumors (for moderately antigenic tumors).
- Persistence of dormant tumors, that is persistence of residual tumor cells (for highly antigenic tumors).

Q&A

THANK YOU !!!