Numerical simulations for selected 1D and 2D Mass-spring models and Applications

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1 Problem statement

Considering the our project title, in this section will be described nvestigate the motion for undamped linear systems with coupled many degrees of freedom. We were given below figure 1 and we had to create a MATLAB model considering that.

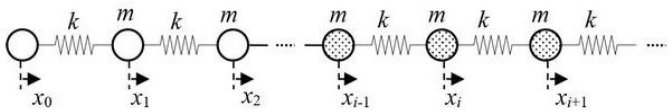

Figure 1: Linear chain of coupled oscillators

2 Approach

In this section, I have described some of the theories and assumptions I have already used. The vibrations of the extreme left particle are gradually transferred to the other particles of the chain via bonds. After some time, all the particles of the system oscillate in harmonic oscillating motion with the same angular frequency of ω, but different amplitudes and different phases.

2.1 The equation of motion

Consider three adjacent particles with indexes $i-1, i, i+1$ indicated by a dotted fill in the figure 1. The equation of motion of the $i^{th}, (i-1)^{th}, (i+1)^{th}$ particle has the form below.

$$m_1 \ddot{x_{i-1}} = k(2x_{i-1} + x_{i-2} + x_i) = 0 \tag{1}$$

$$m_1 \ddot{x_i} = k(x_{i-1} + 2x_i - x_{i+1}) = 0 \tag{2}$$

$$m_1 \ddot{x_{i+1}} = k(x_{i-1} + x_i - 2x_{i+1}) = 0 \tag{3}$$

periodic solutions to the equation of motion, so we might expect the solution to have the form.

$$x_{i-1} = Aexp[i(\omega t - k_p(c-1)d)] \tag{4}$$

$$x_i = Aexp[i(\omega t - k_p cd)] \tag{5}$$

$$x_{i+1} = Aexp[i(\omega t - k_p(c+1)d)] \tag{6}$$

A = number of particles in the system
ω = angular frequency
k_p = propagation constant
k = spring constant

Note that the mass are situated on equally spaced sites with a separation distance d. Substitution of above equation,

2.2 Angular frequency

$$\omega = \sqrt{\frac{4k}{m}} sin(\frac{k_p d}{2}) \tag{7}$$

$$\omega_0 = \sqrt{\frac{4k}{m}} \tag{8}$$

The maximum angular velocity is ω_0.

2.3 Phase velocity

And phase velocity is v_p and maximum phase velocity we can say v_{max} by substitution (9).

$$v_P = \frac{\omega}{k_p} \tag{9}$$

$$v_{max} = v_0 = d\sqrt{\frac{k}{m}} \tag{10}$$

2.4 Group velocity

In general, a wave is a superposition of its harmonic components. The envelope or the group of waves are seen to move forward with a velocity $d\omega/dt$ which is termed the group velocity.

$$v_G = \frac{d\omega}{dt} \tag{11}$$

$$V_G = v_0 cos(\frac{k_p d}{2}) \tag{12}$$

2.5 Results

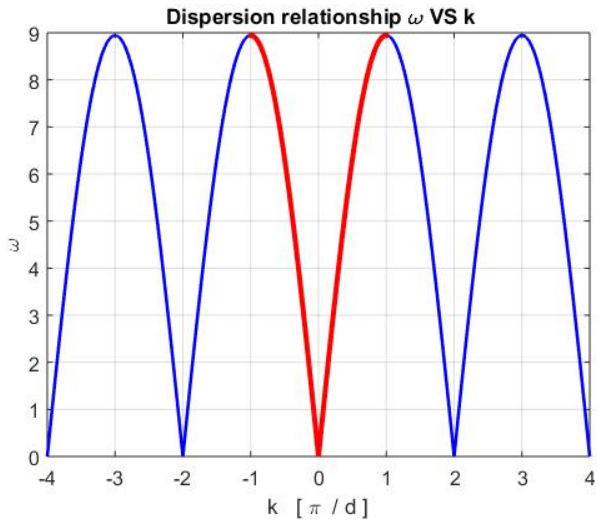

Figure 2: Dispersion relationships for the monatomic lattice.

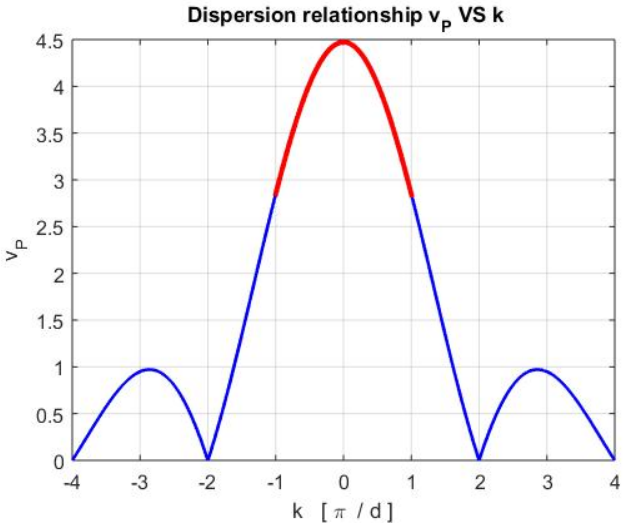

Figure 3: Dispersion relationships for the monatomic lattice.

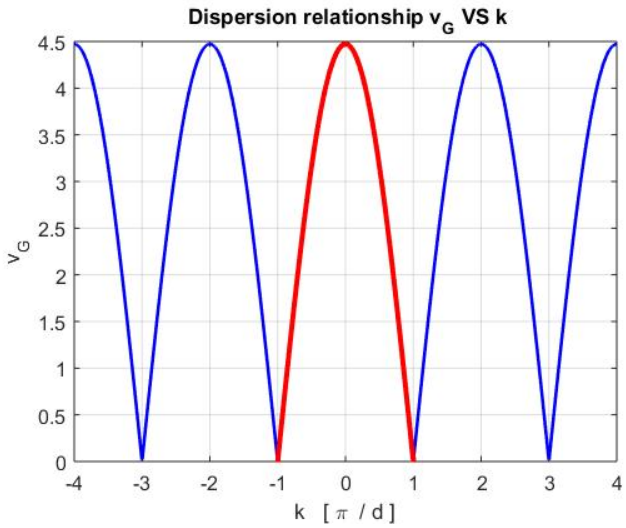

Figure 4: Dispersion relationships for the monatomic lattice.

The dispersion relationship , the phase velocity (equation 7) and the group velocity as functions of the propagation constant k for a linear monatomic lattice are shown in above figures. Note that many values of k are be associated with any given frequency .

Figure 5 shows the atomic displacements on the monatomic lattice as transverse displacements for convenience of illustration (actual displacements are longitudinal ones in which the atoms are displaced along the chain, rather than normal to it).

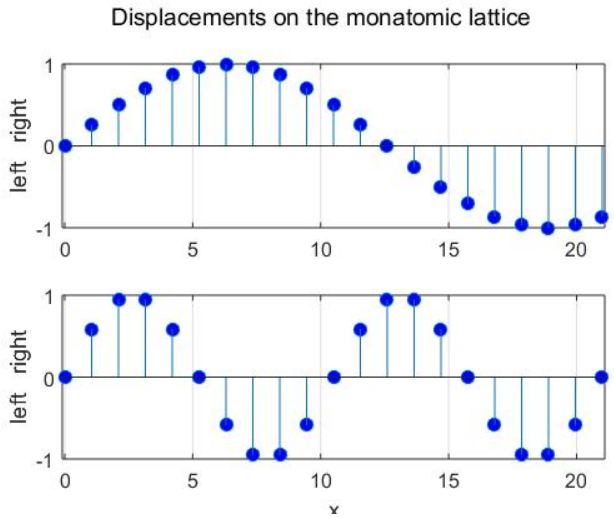

Figure 5: Displacements on the monatomic lattice.

In the results section, all the figures show the results of running the Script MATLAB code with the input parameters: number of atoms N = 21, separation distance between atoms d = 1; mass of each atom m = 0.5, spring constant for each spring kS = 10.