Ph: 844-844-0102

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E-Mail: gatecse@appliedroots.com

NUMBER SYSTEM

1. Some properties of Square and Square Root

(i) Complete square of a no. is possible if its last digit is 0, 1, 4, 5, 6 & 9. If last digit of a no. is 2, 3, 7, 8 then complete square root of this no. is not possible.
(ii) If last digit of a no. is 1, then last digit of its complete square root is either 1 or 9.
(iii) If last digit of a no. is 4, then last digit of its complete square root is either 2 or 8.
(iv) If last digit of a no. is 5 or 0, then last digit of its complete square root is either 5 or 0.
(v) If last digit of a no. is 6, then last digit of its complete square root is either 4 or 6.
(vi) If last digit of a no. is 9, then last digit of its complete square root is either 3 or 7.

2. Prime Number

(i) Find the approx square root of given no. Divide the given no. by the prime no. less than approx square root of no. If given no. is not divisible by any of these prime no. then the no. is prime otherwise not.

For example : To check 359 is a prime number or not.

Sol. Approx sq. root = 19
Prime no. < 19 are 2, 3, 5, 7, 11, 13, 17
359 is not divisible by any of these prime nos. So 359 is a prime no.
For example: Is $2^{5001} + 1$ is prime or not?

$$\frac{2^{5001}+1}{2+1} \Rightarrow \text{Reminder} = 0,$$

$\therefore$ $2^{5001} + 1$ is not prime.

(ii) There are 15 prime no. from 1 to 50.
(iii) There are 25 prime no. from 1 to 100.
(iv) There are 168 prime no. from 1 to 1000.

3. If a no. is in the form of $x^n + a^n$, then it is divisible by $(x + a)$; if n is odd.

4. If $x^n \div (x - 1)$, then remainder is always 1.

5. If $x^n \div (x + 1)$

(i) If n is even, then remainder is 1.
(ii) If n is odd, then remainder is x.

6.

(i) Value of $\sqrt{P+\sqrt{P+\sqrt{P+.........\infty}}} = \frac{\sqrt{4P+1}+1}{2}$

(ii) Value of $\sqrt{P-\sqrt{P-\sqrt{P-.........\infty}}} = \frac{\sqrt{4P+1}-1}{2}$

(iii) Value of $\sqrt{P.\sqrt{P.\sqrt{P.\,.........\infty}}} = P$

(iv) Value of $\sqrt{P\sqrt{P\sqrt{P\sqrt{P\sqrt{P}}}}} = P^{\left(2^n-1\right)\div 2^n}$

[Where $n \rightarrow$ no. of times P repeated].

Note: If factors of P are n & (n + 1) type then value of $\sqrt{P+\sqrt{P+\sqrt{P+....\infty}}} = (n+1)$ and $\sqrt{P-\sqrt{P-\sqrt{P-....\infty}}} = n$.

7. Number of Divisors

(i) If N is any no. and $N = a^n \times b^m \times c^p \times ....$ where a, b, c are prime no.
No. of divisors of $N = (n+1)(m+1)(p+1)....$
e.g. Find the no. of divisors of 90000.
$N = 90000 = 2^2 \times 3^2 \times 5^2 \times 10^2 = 2^2 \times 3^2 \times 5^2 \times (2 \times 5)^2 = 2^4 \times 3^2 \times 5^4$
So, the no. of divisors $= (4+1)(2+1)(4+1) = 75$

(ii) $N = a^n \times b^m \times c^p$, where a, b, c are prime
Then set of co-prime factors of $N = [(n+1)(m+1)(p+1) - 1 + nm + mp + pn + 3mnp]$

(iii) If $N = a^n \times b^m \times c^p ...$, where a, b & c are prime no. Then sum of the divisors $= \frac{\left(a^{n+1}-1\right)\left(b^{m+1}-1\right)\left(c^{p+1}-1\right)}{(a-1)(b-1)(c-1)}$

8. To find the last digit or digit at the unit's place of a^n

(i) If the last digit or digit at the unit's place of a is 1, 5 or 6, whatever be the value of n, it will have the same digit at unit's place, i.e.,

$(.....1)^n = (........1)$
$(.....5)^n = (........5)$
$(.....6)^n = (........6)$

(ii) If the last digit or digit at the units place of a is 2, 3, 5, 7 or 8, then the last digit of a^n depends upon the value of n and follows a repeating pattern in terms of 4 as given below :

n	last digit of $(....2)^n$	last digit of $(....3)^n$	last digit of $(....7)^n$	last digit of $(....8)^n$
4x+1	2	3	7	8
4x+2	4	9	9	4
4x+3	8	7	3	2
4x	6	1	1	6

(iii) If the last digit or digit at the unit's place of a is either 4 or 9, then the last digit of a^n depends upon the value of n and follows repeating pattern in terms of 2 as given below.

n	last digit of $(....4)^n$	last digit of $(....9)^n$
2x	6	1
2x + 1	4	9

9.

(i) Sum of n natural number $= \dfrac{(n)(n+1)}{2}$

(ii) Sum of n even number $= (n)(n+1)$

(iii) Sum of n odd number $= n^2$

10.

(i) Sum of sq. of first n natural no. $= \dfrac{n(n+1)(2n+1)}{6}$

(ii) Sum of sq. of first n odd natural no. $= \dfrac{n(4n^2-1)}{3}$

(iii) Sum of sq. of first n even natural no. $= \dfrac{2n(n+1)(2n+1)}{3}$

11.

(i) Sum of cube of first n natural no. $= \dfrac{n^2(n+1)^2}{4} = \left[\dfrac{n(n+1)}{2}\right]^2$

(ii) Sum of cube of first n even natural no. $= 2n^2(n+1)^2$

(iii) Sum of cube of first n odd natural no. $= n^2(2n^2-1)$

12.

(i) $x^n - y^n$ is divisible by $(x + y)$
When n is even

(ii) $x^n - y^n$ is divisible by $(x - y)$
When n is either odd or even.

13.

For any integer n, $n^3 - n$ is divisible by 3, $n^5 - n$ is divisible by 5, $n^{11} - n$ is divisible by 11, $n^{13} - n$ is divisible by 13.

14. Divisibility

(i) A no. of 3-digits which is formed by repeating a digit 3-times, then this no. is divisible by 3 and 37.
e.g., 111, 222, 333, ...

(ii) A no. of 6-digit which is formed by repeating a digit 6-times then this no. is divisible by 3, 7, 11, 13 and 37.
e.g., 111111, 222222, 333333, 444444, ...

15. Divisible by 7

We use osculator (-2) for divisibility test.

$99995 : 9999 - 2 \times 5 = 9989$

$9989 : 998 - 2 \times 9 = 980$

$980 : 98 - 2 \times 0 = 98$

Now 98 is divisible by 7, so 99995 is also divisible by 7.

16. Divisible by 11

In a number, if difference of sum of digit at even places and sum of digit at odd places is either 0 or multiple of 11, then no. is divisible by 11.

For example, $12342 \div 11$

Sum of even place digit $= 2 + 4 = 6$

Sum of odd place digit $= 1 + 3 + 2 = 6$

Difference $= 6 - 6 = 0$

$\therefore$ 12342 is divisible by 11.

17. Divisible by 13

We use (+ 4) as osculator.

e.g., $876538 \div 13$

$876538: 8 \times 4 + 3 = 35$

$5 \times 4 + 3 + 5 = 28$

$8 \times 4 + 2 + 6 = 40$

$0 \times 4 + 4 + 7 = 11$

$1 \times 4 + 1 + 8 = 13$

13 is divisible by 13.

$\therefore$ 876538 is also divisible by 13.

18. Divisible by 17

We use (– 5) as osculator.

e.g., $294678: 29467 - 5 \times 8 = 29427$

$27427: 2942 - 5 \times 7 = 2907$

$2907: 290 - 5 \times 7 = 255$

$255: 25 - 5 \times 5 = 0$

$\therefore$ 294678 is completely divisible by 17.

19. Divisible by 19

We use (+ 2) as osculator.

e.g: 149264: $4 \times 2 + 6 = 14$

$4 \times 2 + 1 + 2 = 11$

$1 \times 2 + 1 + 9 = 12$

$2 \times 2 + 1 + 4 = 9$

$9 \times 2 + 1 = 19$

19 is divisible by 19

$\therefore$ 149264 is divisible by 19.

20. HCF or GCD (Highest Common Factor or Greatest Common Divisor)

There are two methods to find the HCF–

(a) Factor method (b) Division method

(i) For two no. a and b if $a < b$, then HCF of a and b is always less than or equal to a .

(ii) The greatest number by which x, y and z completely divisible is the HCF of x, y and z.

(iii) The greatest number by which x, y, z divisible and gives the remainder a, b and c is the HCF of (x –a), (y–b) and (z–c).

(iv) The greatest number by which x, y and z divisible and gives same remainder in each case, that number is HCF of (x–y), (y–z) and (z–x).

(v) H.C.F. of $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f} = \frac{\text{H.C.M. of (a, c, e)}}{\text{L.C.M. of (b, d, f)}}$

21. LCM (Least Common Multiple)

There are two methods to find the LCM–

(a) Factor method (b) Division method

(i) For two numbers a and b if $a < b$, then L.C.M. of a and b is more than or equal to b.

(ii) If ratio between two numbers is a : b and their H.C.F. is x, then their L.C.M. = abx.

(iii) If ratio between two numbers is a : b and their L.C.M. is x, then their H.C.F. $= \frac{x}{ab}$

(iv) The smallest number which is divisible by x, y and z is L.C.M. of x, y and z.

(v) The smallest number which is divided by x, y and z give remainder a, b and c, but $(x - a) = (y - b) = (z - c) = k$, then number is (L.C.M. of (x, y and z) – k).

(vi) The smallest number which is divided by x, y and z give remainder k in each case, then number is (L.C.M. of x, y and z) + k.

(vii) L.C.M. of $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f} = \frac{\text{L.C.M. of (a, c, e)}}{\text{H.C.F. of (b, d, f)}}$

(viii) For two numbers a and b –

LCM × HCF = a × b

(ix) If a is the H.C.F. of each pair from n numbers and L is L.C.M., then product of n numbers $= a^{n-1}.L$

22. Arithmetic Progression (AP)

(i) If a, a + d, a + 2d, ... are in A.P., then, nth term of A.P. $a_n = a + (n-1)d$

Sum of n terms of this A.P. $= S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}[a+1]$ where ℓ = last term

a = first term
d = common difference

(ii) $\text{A.M.} = \frac{a+b}{2}$ [$\because$ A.M. = Arithmetic mean]

23. Geometric Progression (GP)

(i) G.P. $\rightarrow a, ar, ar^2, \ldots\ldots\ldots$

Then, nth term of G.P. $a_n = ar^{n-1}$

$$S_n = \frac{a\left(r^n - 1\right)}{(r-1)}, r > 1$$

$$= \frac{a(1-r^n)}{(1-r)}, r < 1$$

$S_\infty = 1 - r$ [where r = common ratio, a = first term]

(ii) $\text{G.M.} = \sqrt{ab}$

24.

If a, b, c are in H.P., $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$n^{th} \text{ term of H.M.} = \frac{1}{n^{th} \text{ term of A.P.}}$$

$$\text{H.M.} = \frac{2ab}{a+b}$$

Note : Relation between A.M., G.M. and H.M.

(i) $\text{A.M.} \times \text{H.M.} = \text{G.M.}^2$

(ii) $\text{A.M.} > \text{G.M.} > \text{H.M.}$

A.M. $\rightarrow$ Arithmetic Mean
G.M. $\rightarrow$ Geometric Mean
H.M. $\rightarrow$ Harmonic Mean

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AVERAGE

1.

(i) Average of first n natural no. = $\frac{n+1}{2}$

(ii) Average of first n even no. = (n + 1)

(iii) Average of first n odd no. = n

2.

(i) Average of sum of square of first n natural no. = $\frac{(n+1)(2n+1)}{6}$

(ii) Average of sum of square of first n even no. = $\frac{2(n+1)(2n+1)}{3}$

(iii) Average of sum of square of first odd no. = $\left(\frac{4n^2-1}{3}\right)$

3.

(i) Average of cube of first n natural no. = $\frac{n(n+1)^2}{4}$

(ii) Average of cube of first n even natural no. = $2n(n+1)^2$

(iii) Average of cube of first n odd natural no. = $n(2n^2-1)$

Average of first n multiple of m = $\frac{m(n+1)}{2}$

E-Mail: gatecse@appliedroots.com

4.

(i) If average of some observations is x and a is added in each observations, then new average is $(x + a)$.

(ii) If average of some observations is x and a is subtracted in each observations, then new average is $(x - a)$.

(iii) If average of some observations is x and each observations multiply by a, then new average is ax.

(iv) If average of some observations is x and each observations is divided by a, then new average is $\frac{x}{a}$.

(v) If average of n_1 is A_1, & average of n_2 is A_2, then Average of $(n_1 + n_2)$ is $\frac{n_1A_1 + n_2A_2}{n_1 + n_2}$ and

Average of $(n_1 - n_2)$ is $\frac{n_1A_1 - n_2A_2}{n_1 - n_2}$

5.

When a person is included or excluded the group, then age/weight of that person = No. of persons in group × (Increase / Decrease) in average $\pm$ New average.

For example : In a class average age of 15 students is 18 yrs. When the age of teacher is included their average increased by 2 yrs, then find the age of teacher.

Sol. Age of teacher $= 15 \times 2 + (18 + 2) = 30 + 20 = 50$ yrs.

6.

When two or more than two persons included or excluded the group, then average age of included or excluded person is

$$= \frac{\text{No. of person} \times (\text{Increase / Decrease}) \text{ in average} \pm \text{New average} \times (\text{No. of person included or excluded})}{\text{No. of included or person}}$$

For example : Average weight of 13 students is 44 kg. After including two new students their average weight becomes 48 kg, then find the average weight of two new students.

Sol. Average weight of two new students

$$= \frac{13 \times (48 - 44) + 48 \times 2}{2} = \frac{13 \times 4 + 48 \times 2}{2} = \frac{52 + 96}{2} = 74 \text{ kg}$$

7.

If a person travels two equal distances at a speed of x km/h and y km/h, then average speed $= \frac{2xy}{x + y}$ km/h

If a person travels three equal distances at a speed of x km/h, y km/h and z km/h, then average speed $= \frac{3xyz}{xy + yz + zx}$ km/h.

RATIO AND PROPORTION

1.

(i) If $\frac{a}{K_1}=\frac{b}{K_2}=\frac{c}{K_3}=....$, then $\frac{a+b+c+....}{c}=\frac{K_1+K_2+K_3+.....}{K_3}$

For example: If $\frac{P}{3}=\frac{Q}{4}=\frac{R}{7}$, then find $\frac{P+Q+R}{R}$

Sol. $P=3, Q=4, R=7$

Then $\frac{P+Q+R}{R}=\frac{3+4+7}{7}=2$

(ii) If $\frac{a_1}{a_2}=\frac{a_2}{a_3}=\frac{a_3}{a_4}=\frac{a_4}{a_5}=....\frac{a_n}{a_{n+1}}=K$, then $a_1 : a_{n+1}=(K)^n$

2.

A number added or subtracted from a, b, c & d, so that they are in proportion $=\frac{ad-bc}{(a+d)-(b+c)}$

For example : When a number should be subtracted from 2, 3, 1 & 5 so that they are in proportion. Find that number.

Sol. Req. No. $=\frac{2\times 5-3\times 1}{(2+5)-(3+1)}=\frac{10-3}{7-4}=\frac{7}{3}$

3.

If X part of A is equal to Y part of B, then $A : B = Y : X$.

For example: If 20% of A = 30% of B, then find A : B.

Sol. $A : B=\frac{30\%}{20\%}=\frac{3}{2}=3:2$

4.

When X^{th} part of P, Y^{th} part of Q and Z^{th} part of R are equal, then find A : B : C.

Then, $A : B : C = yz : zx : xy$

PERCENTAGE

1.

Simple Fraction	Their Percentage
1	100%
$\frac{1}{2}$	50%
$\frac{1}{3}$	33.3%
$\frac{1}{4}$	25%
$\frac{1}{5}$	20%
$\frac{1}{6}$	16.67%
$\frac{1}{7}$	14.28%

Simple Fraction	Their Percentage
$\frac{1}{8}$	12.5%
$\frac{1}{9}$	11.11%
$\frac{1}{10}$	10%
$\frac{1}{11}$	9.09%
$\frac{1}{12}$	8.33%

2.

(i) If A is $\left(x\% = \frac{a}{b}\right)$ more than B, then B is $\left(\frac{a}{a+b}\%\right)$ less than A.

(ii) If A is $\left(x\% = \frac{a}{b}\right)$ less than B, then B is $\left(\frac{a}{a-b}\%\right)$ more than A

if $a > b$, we take $a - b$

if $b > a$, we take $b - a$.

3.

If price of a article increase from ₹ a to ₹ b, then its expenses decrease by $\left(\frac{b-a}{b}\times 100\right)\%$ so that expenditure will be same.

4.

Due to increase/decrease the price x%, A man purchase *a* kg more in ₹ y, then

Per kg increase or decrease $= \left(\frac{xy}{100\times a}\right)$

Per kg starting price $=$ ₹$\frac{xy}{(100\pm x)a}$

5. For two articles, if price

Ist	IInd	Overall
Increase (x%)	Increase (y%)	Increase $\left(x+y+\frac{xy}{100}\right)\%$
Increase (x%)	Decrease (y%)	$\left(x-y-\frac{xy}{100}\right)\%$ If +ve (Increase) If –ve (Decrease)
Decrease (x%)	Decrease (y%)	Decrease $\left(x+y-\frac{xy}{100}\right)\%$
Increase (x%)	Decrease (x%)	Decrease $\left(\frac{x^2}{100}\right)\%$

6.

If the side of a square or radius of a circle is x% increase/decrease, then its area increase/decrease = $\left(2x \pm \frac{x^2}{100}\right)\%$

If the side of a square, x% increase/decrease then x% its perimeter and diagonal increase/decrease.

7.

(i) If population P increase/decrease at r% rate, then after t years population = $P\left(\frac{100 \pm R}{100}\right)^t$

(ii) If population P increase/decrease r_1% first year, r_2% increase/decrease second year and r_3% increase/decrease third year, then after 3 years population = $P\left(1 \pm \frac{r_1}{100}\right)\left(1 \pm \frac{r_2}{100}\right)\left(1 \pm \frac{r_3}{100}\right)$

8.

If a man spend x% of this income on food, y% of remaining on rent and z% of remaining on cloths. If he has ₹ P remaining, then total income of man is = $\frac{P \times 100 \times 100 \times 100}{(100 - x)(100 - y)(100 - z)}$

[**Note:** We can use this table for area increase/decrease in mensuration for rectangle, triangle and parallelogram].

GEOMETRY

1. Polygon

(i) Sum of all the exterior angle of a polygon = $360°$

(ii) Each exterior angle of a regular polygon = $\frac{360°}{n}$

(iii) Sum of all the interior angles of a polygon = $(n-2) \times 180°$

(iv) Each interior angle of a regular polygon = $\frac{(n-2)}{n} \times 180°$

(v) No. of diagonals of a polygon = $\frac{n(n-3)}{2}$, $n \rightarrow$ no. of sides.

(vi) The ratio of sides a polygon to the diagonals of a polygon is $2 : (n-3)$

(vii) Ratio of interior angle to exterior angle of a regular polygon is $(n-2) : 2$

2. Triangle

(i) When one side is extended in any direction, an angle is formed with another side. This is called the exterior angle. There are six exterior angles of a triangle.

(ii) Interior angle + corresponding exterior angle = $180°$.

(iii) An exterior angle = Sum of the other two interior opposite angles.

(iv) Sum of the lengths of any two sides is greater than the length of third side.

(v) Difference of any two sides is less than the third side.
Side opposite to the greatest angle is greatest and vice versa.

(vi) A triangle must have at least two acute angles.

(vii) Triangles on equal bases and between the same parallels have equal areas.

(viii) If a, b, c denote the sides of a triangle then

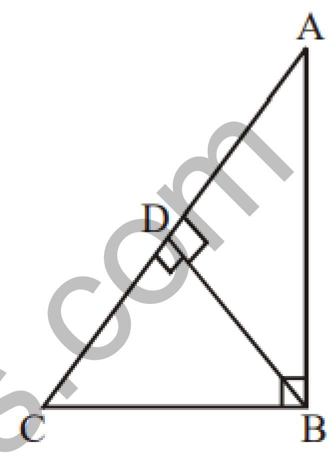
(i) if $c^2 < a^2 + b^2$, Triangle is acute angled.

(ii) if $c^2 = a^2 + b^2$, Triangle is right angled.

(iii) if $c^2 > a^2 + b^2$, Triangle is obtuse angled.

(ix) If 2 triangles are equiangular, their corresponding sides are proportional. In triangles ABC and XYZ, if $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$, then

$$\frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ}..$$



(i) In ΔABC, $\angle B = 90°$ $BD \perp AC$

$\therefore BD \times AC = AB \times BC$

(ii) $$\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$$

(iii) $BD^2 = AD \times DC$

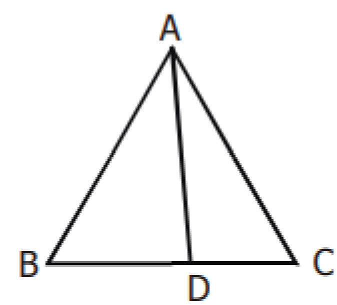
(x) The perpendiculars drawn from vertices to opposite sides (called altitudes) meet at a point called **Orthocentre** of the triangle.

(xi) The line drawn from a vertex of a triangle to the opposite side such that it bisects the side is called the **Median** of the triangle. A median bisects the area of the triangle.

(xii) When a vertex of a triangle is joined to the midpoint of the opposite side, we get a median. The point of intersection of the medians is called the **Centroid** of the triangle. The centroid divides any median in the ratio 2 : 1.

(xiii) Angle Bisector Theorem–

In the figure if AD is the angle bisector (interior) of $\angle BAC$. Then,



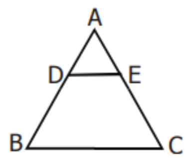
1. AB/AC = BD/DC.
2. AB x AC – BD x DC = AD^2.

(xiv) Midpoint Theorem –

In a triangle, the line joining the mid points of two sides is parallel to the third side and half of it.

(xv) Basic Proportionality Theorem

A line parallel to any one side of a triangle divides the other two sides proportionally. If DE is parallel to BC, then



$\frac{AD}{BD} = \frac{AE}{EC}, \frac{AB}{AD} = \frac{AC}{AE}, \frac{AD}{DE} = \frac{AB}{BC}$ and so on.

3. Circle

(i) Only one circle can pass through three given points.

(ii) There is one and only one tangent to the circle passing through any point on the circle.

(iii) From any exterior point of the circle, two tangents can be drawn on to the circle.

(iv) The lengths of two tangents segment from the exterior point to the circle, are equal.

(v) The tangent at any point of a circle and the radius through the point are perpendicular to each other.

(vi) When two circles touch each other, their centres & the point of contact are collinear.

(vii) If two circles touch externally, distance between centres = sum of radii.

(viii) If two circles touch internally, distance between centres = difference of radii

(ix) Circles with same centre and different radii are **concentric circles**.

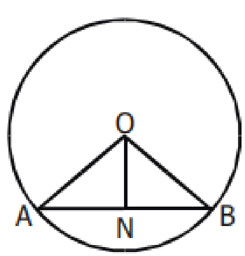
(x) Points lying on the same circle are called **concyclic points**.

(xi) Measure of an arc means measure of central angle.
m(minor arc) + m(major arc) = 360°.

(xii) Angle in a semicircle is a right angle.

(xiii) Only one circle can pass through three given

(xxv) If ON is ⊥ from the centre O of a circle to a chord AB, then AN = NB.



(⊥ from centre bisects chord)

(xv) If N is the midpoint of a chord AB of a circle with centre O, then ∠ONA = 90°.
(Converse, ⊥ from centre bisects chord)

(xvi) Two congruent figures have equal areas but the converse need not be true.

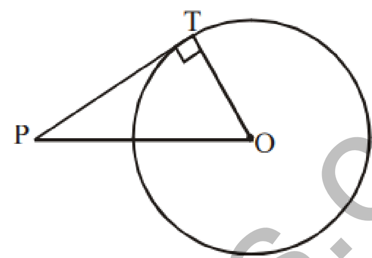
(xvii) A diagonal of a parallelogram divides it into two triangles of equal area.

(xviii) Parallelograms on the same base and between the same parallels are equal in area.

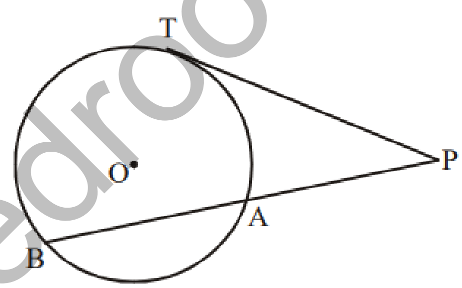
(xix) Triangles on the same bases and between the same parallels are equal in area.

(xx) If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to the half of the parallelogram.

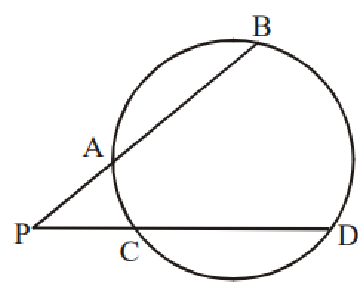
- If PT is a tangent to the circle, then $OP^2 = PT^2 = OT^2$



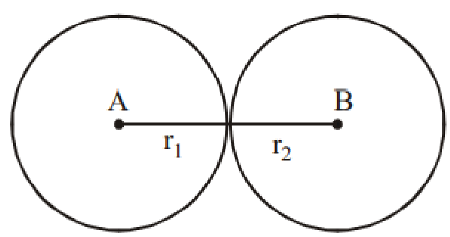
- If PT is tangent and PAB is secant of a circle, then $PT^2 = PA.PB$



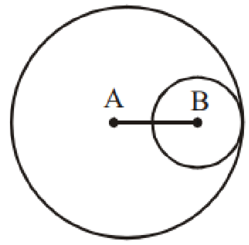
- If PB & PD are two secant of a circle, then PA.PB = PC.PD



- If two circles touch externally, then distance between their centres = $(r_1 + r_2)$



- If two circles touch internally, then distance between their centres = $r_1 - r_2$ where $r_1 > r_2$.



MENSURATION

1.

(i) Area of triangle $= \frac{1}{2} \times \text{base} \times \text{altitude}$

(ii) Area of triangle using heron's formula $= \sqrt{S/S-a\,(S-b)\,(S-c)}$, where $S = \frac{a+b+c}{2}$

2.

In an equilateral triangle with side a, then

$$\boxed{\frac{4A}{\sqrt{3}} = \frac{4h^2}{3} = \frac{P^2}{9} = a^2}$$

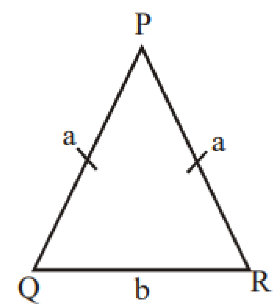
where $A \rightarrow$ Area of triangle
$P \rightarrow$ Perimeter
$h \rightarrow$ Height

3.

In an isosceles triangle PQR

ar $\Delta PQR = \frac{b}{4}\sqrt{4a^2 - b^2}$

Height $= \sqrt{\frac{4a^2 - b^2}{2}}$

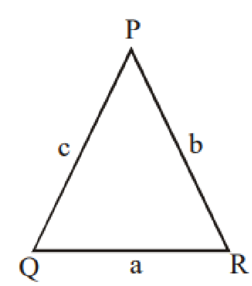


4.

(i) Area of $\Delta = \frac{1}{2} bc \operatorname{Sin} P$ where $\angle P = \angle QPR$

(ii) Area of $\Delta = \frac{1}{2} ac \operatorname{Sin} Q$

(iii) Area of $\Delta = \frac{1}{2} ab \operatorname{Sin} R$



5.

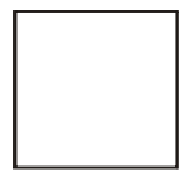
$$\mathrm{Cos}P = \frac{b^2 + c^2 - a^2}{2bc}, \quad \mathrm{Cos}Q = \frac{a^2 + c^2 - b^2}{2ac},$$

$$\mathrm{Cos}R = \frac{a^2 + b^2 - c^2}{2ab}$$

Sine Rule : $\frac{a}{\mathrm{Sin}P} = \frac{b}{\mathrm{Sin}Q} = \frac{c}{\mathrm{Sin}R}$

6.

$$\sqrt{\text{Area of square}} = \frac{\text{Perimeter of square}}{4} = \frac{\text{Diagonal of square}}{4} = \text{side of square}$$



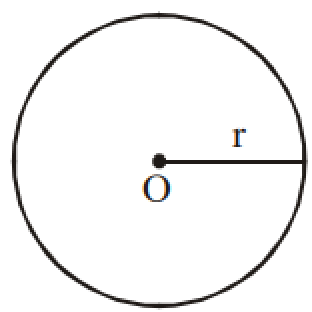
Square

7.

In a circle with radius r.

$\frac{A}{C} = \frac{D}{4}$ where A - Area of circle

C - Circumference of circle
D - Diameter of circle



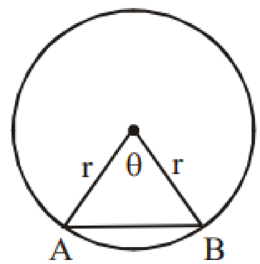
8.

$$\text{If } \theta = 60°, \text{ ar } \Delta AOB = \frac{\sqrt{3}}{4} r^2$$

$$\text{If } \theta = 90°, \text{ ar } \Delta AOB = \frac{1}{2} r^2$$

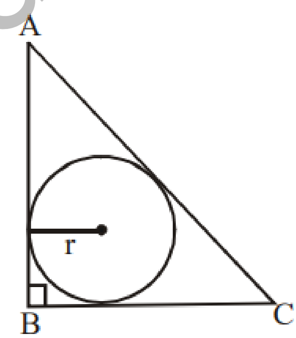
$$\text{If } \theta, \text{ ar } \Delta AOB = \frac{1}{2} r^2$$

$$\sin\theta = r^2 \sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)$$

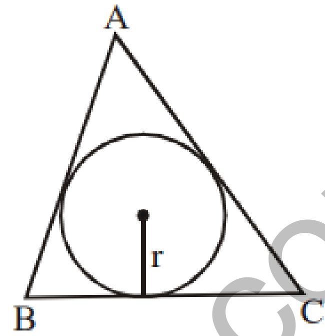


9.

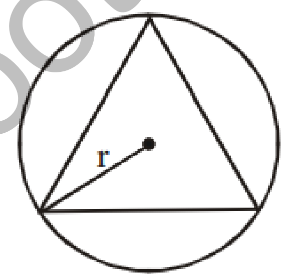
(i) A circle with largest area inscribed in a right angle triangle, then $r = \dfrac{2 \times \text{area of } \Delta ABC}{\text{Perimeter of } \Delta ABC}$



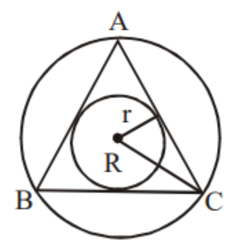
(ii) If ABC is an equilateral triangle with side a, then Area of circle = $\frac{\pi a^2}{12}$



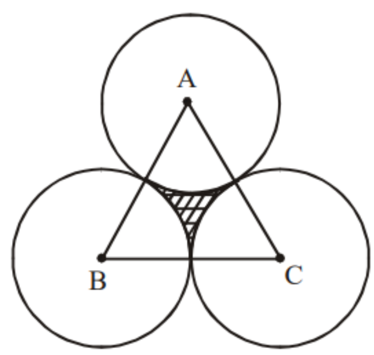
(iii) If ABC is an equilateral triangle with side a, then area of circle = $\frac{\pi a^2}{3}$.



(iv) If ΔABC is an equilateral triangle, and two circles with radius r and R, then $\frac{r}{R} = \frac{1}{2}$ and $\frac{\pi r^2}{\pi R^2} = \frac{1}{4}$

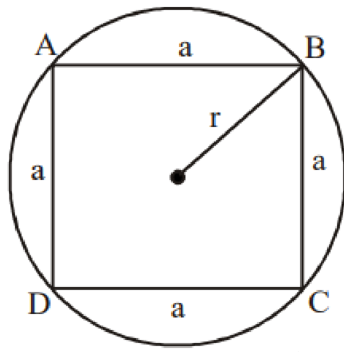


(v) Three equal circle with radius r and an equilateral triangle ABC, then area of shaded region = $\left(2\sqrt{3} - \pi\right).\frac{r^2}{2}$



10.

ABCD is a square placed inside a circle with side a and radius of circle r, then $\frac{\text{area of square}}{\text{area of circle}} = \frac{7}{11}$



11.

Diagonal of a cube = $\sqrt{3}$ × side

Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$; where $\ell \to$ Length, $b \to$ breadth, $h \to$ height

12.

For two cubes

$$\boxed{\sqrt{\frac{A_1}{A_2}} = \sqrt[3]{\frac{v_1}{v_2}} = \frac{a_1}{a_2} = \frac{d_1}{d_2}}$$

where $A_1, A_2 \to$ Area of cubes
$v_1, v_2 \to$ Volume
$a_1, a_2 \to$ Sides
$d_1, d_2 \to$ Diagonals

13.

The longest diagonal $= \sqrt{a^2 + b^2 + c^2}$

(i) If the height of a cuboid is zero it becomes a rectangle.

(ii) If "a" be the edge of a cube, then

(iii) The longest diagonal $= a\sqrt{3}$

14.

Volume of pyramid $= \frac{1}{3} \times \text{Base Area} \times \text{height (H)}$

15.

(i) If A_1 & A_2 denote the areas of two similar figures and l_1 & l_2 denote their corresponding linear measures, then $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

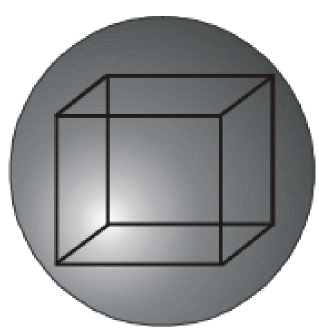
(ii) If V_1 & V_2 denote the volumes of two similar solids and l_1, l_2 denote their corresponding linear measures, then $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$

(iii) The rise or fall of liquid level in a container $= \frac{\text{Total volume of objects submerged or taken out}}{\text{Cross sectional area of container}}$

16.

If a largest possible cube is inscribed in a sphere of radius 'a' cm, then

(i) the edge of the cube $= \frac{2a}{\sqrt{3}}$.



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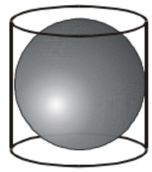
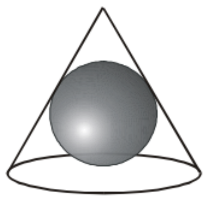
(ii) If a largest possible sphere is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then for h > a,

- the radius of the sphere = a and
- the radius $= \frac{h}{2}$ (for a > h)

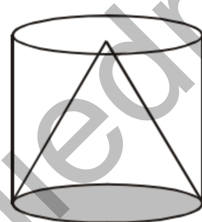


(iii) If a largest possible sphere is inscribed in a cone of radius 'a' cm and slant height equal to the diameter of the base, then

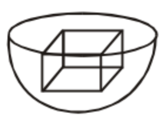
- the radius of the sphere $= \frac{a}{\sqrt{3}}$.



(iv) If a largest possible cone is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then the radius of the cone = a and height = h.



(v) If a largest possible cube is inscribed in a hemisphere of radius 'a' cm, then the edge of the cube $= a\sqrt{\frac{2}{3}}$.



17.

In any quadrilateral

(i) Area $= \frac{1}{2} \times$ one diagonal × (sum of perpendiculars to it from opposite vertices) $= \frac{1}{2} \times d\,(d_1 + d_2)$

(ii) Area of a cyclic quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

where a, b, c, d are sides of quadrilateral and

s = semi perimeter $= \frac{a+b+c+d}{2}$

E-Mail: gatecse@appliedroots.com

18.

If length, breadth & height of a three dimensional figure increase/decrease by x%, y% and z%, then

$$\text{Change in area} = \left[\left(\frac{100\pm x}{100}\right)\left(\frac{100\pm y}{100}\right)-1\right]\times 100\%$$

$$\text{Change in Volume} = \left[\left(\frac{100\pm x}{100}\right)\left(\frac{100\pm y}{100}\right)\left(\frac{100\pm z}{100}\right)-1\right]\times 100\%$$

19.

	Volume	Total Surface Area	Lateral / Curved Surface Area
Cube	Side^3	$6 \times \text{Side}^2$	$4 \times \text{Side}^2$
Cuboid	L x B x H	2(LB + LH + BH)	2 (LH + BH)
Cylinder	$\pi r^2 h$	$2\pi r\,(r + h)$	$2\pi rh$
Cone	$(1/3)\ \pi r^2 h$	$\pi r\,(r + L)$	πrl {where $L = \sqrt{r^2 + h^2}$}
Sphere	$(4/3)\ \pi r^3$	$4\ \pi r^2$	$4\ \pi r^2$
Hemisphere	$(2/3)\ \pi r^3$	$3\ \pi r^2$	$2\ \pi r^2$

PERMUTATION AND COMBINATION

1.

When two tasks are performed in succession, i.e., they are connected by an **'AND'**, to find the total number of ways of performing the two tasks, you have to **MULTIPLY** the individual number of ways. When only one of the two tasks is performed, i.e. the tasks are connected by an '**OR**', to find the total number of ways of performing the two tasks you have to **ADD** the individual number of ways.

Eg: In a shop there are 'd' doors and 'w' windows.

*Case1: If a thief wants to enter via a door or window, he can do it in – **(d+w) ways**.*

*Case2: If a thief enters via a door and leaves via a window, he can do it in – **(d x w) ways**.*

Linear arrangement of 'r' out of 'n' distinct items (^nP_r):

The first item in the line can be selected in 'n' ways AND the second in (n — 1) ways AND the third in (n — 2) ways AND so on. So, the total number of ways of arranging 'r' items out of 'n' is

$$(n)(n - 1)(n - 2)\ldots(n - r + 1) = \frac{n!}{(n-r)!}$$

Circular arrangement of 'n' distinct items: Fix the first item and then arrange all the other items linearly with respect to the first item. This can be done in **(n — 1)!** ways.

In a necklace, it can be done in $\frac{(n-1)!}{2}$ ways.

2.

Selection of r items out of 'n' distinct items (^nCr): Arrange of r items out of n = Select r items out of n and then arrange those r items on r linear positions.

$^nP_r = {^nC_r} \times r! \rightarrow {^nC_r} = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

Dearrangement If 'n' things are arranged in a row, the number of ways in which they can, be deranged so that none of them occupies its original place is

$$n!\left(\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}- \ldots +\frac{(-1)^n}{n!}\right)$$

Number of ways of arranging 'n' items out of which `p' are alike, 'q' are alike, 'r' are alike in a line is given by = $\frac{n\,!}{p!q!r!}$

3. Partitioning

'n' similar items in **'r' distinct** groups	No restrictions	$^{n+r-1}C_{r-1}$
	No group empty	$^{n-1}C_{r-1}$
'n' distinct items in **'r' distinct** groups	No restrictions	r^n
	Arrangement in a group important	$\frac{(n+r-1)!}{(r-1)!}$
'n' similar items in **'r' similar** groups	List the cases and then find out in how many ways is each case possible	
'n' similar items in **'r' similar** groups	List the cases and then find out in how many ways is each case possible	

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PROBABILITY

P(A) = $\frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ outcomes}$

For **Complimentary Events:** *P(A) + P(A') = 1*

For **Exhaustive Events:** *P(A) + P(B) +P(C)... = 1*

Addition Rule:

P (A U B) = P(A) + P(B) – P(A ∩ B)

For **Mutually Exclusive Events** P(A ∩ B) = 0

⇨ *P (A U B) = P(A) + P(B)*

Multiplication Rule :*P(A ∩ B) = P(A) P(B/A) = P(B) P(A/B)*

For **Independent Events** P(A/B) = P(B) and P(B/A) = P(B)

⇨ *P(A ∩ B) = P(A).P(B)*

P (A U B) = P(A) + P(B) – P(A).P(B) **Funda**: If the probability of an event occurring is P, then the probability of that event occurring 'r' times in 'n' trials is = $^nC_r \times P^r \times (1-P)^{n-r}$

Odds

Odds in favor = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ unfavorable\ outcomes}$

Odds against = $\frac{Number\ of\ unfavorable\ outcomes}{Number\ of\ favorable\ outcomes}$

E-Mail: gatecse@appliedroots.com