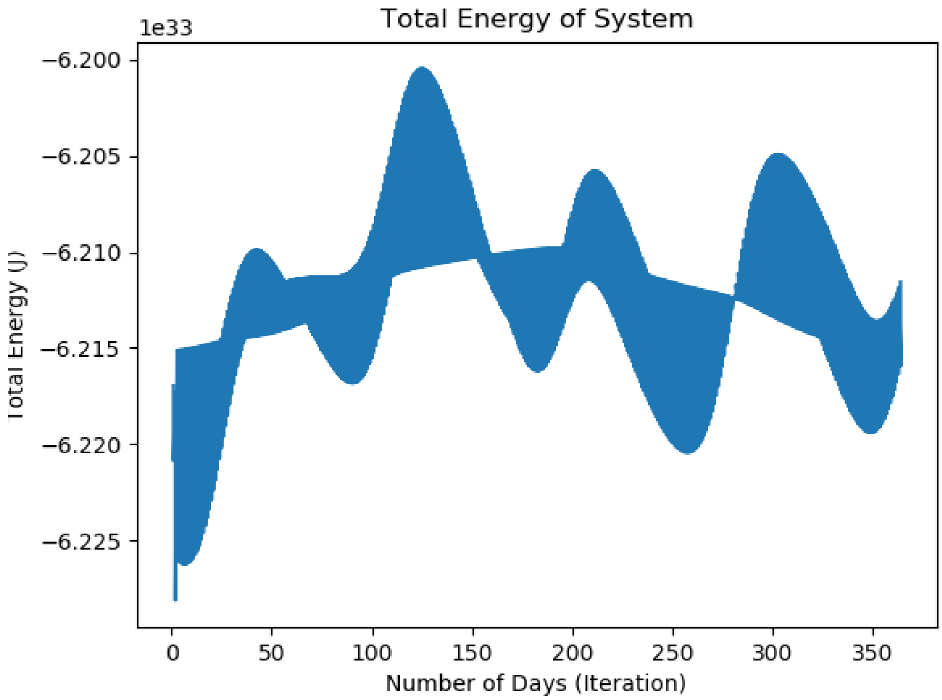
Simulating the Inner Solar System with three-step Beeman Scheme

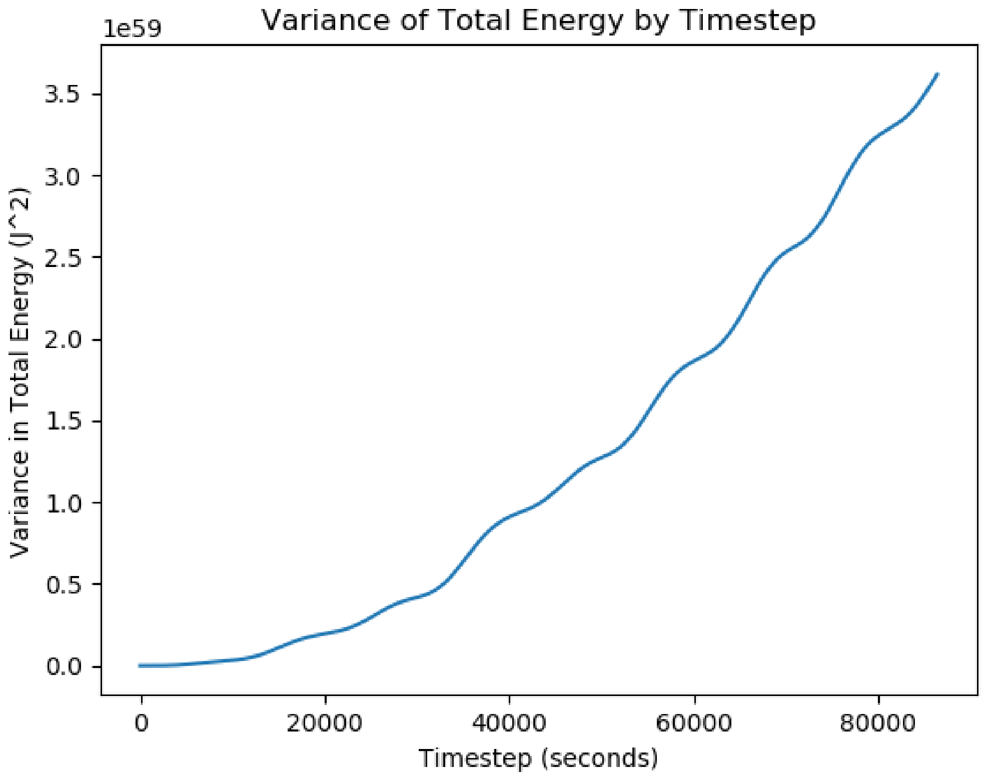
The solar system has been a topic of scientific interest for hundreds of years, and more recently we have been able to create complex simulations which give us insight into the mechanics of natural systems, and let us test ideas and make important predictions. In this simulation, I will be using a three-step Beeman algorithm to simulate the orbits of the inner planets of our solar system. I will use this simulation to test hypotheses and gain a deeper understanding of how orbits behave. I ran an experiment to determine whether total energy of the solar system is conserved in orbits as predicted by Mathematician Émilie du Châtelet, as this is one of the most important laws of nature and a necessary step in testing whether this Beeman simulation acts in accordance with natural laws. I also ran an experiment to determine the simulated orbital periods of each inner planet, and compare these to their actual periods observed by NASA. This is also an important indicator of how accurate the Beeman simulation is in predicting real systems with real parameters. I also tested some explanations for the discrepancies and deviations of the simulation.

I decided to use a binary class, using only two classes, as object oriented thinking pointed me to the simplest option of representing each celestial body as an object with attributes, and the simulation itself as an object with parameters and attributes. I chose to set the time step to the number of seconds in a day, as all of the computations and parameters in the simulation are measured by the second, and updating the planet positions once every day resulted in the best animation speed. All of the planet parameters are from NASA[1], and I decided to set the initial radii and velocities to the real values, to get a simulation as close to reality as possible. I chose to test the orbital periods of each planet by recording how many time steps it took for each planet to pass its starting position. Although this may include slight inaccuracies as the 1 day time step is discrete and the planets may slightly overshoot, it is accurate to $1/365 = 0.0027$, which is adequate for this simulation. I also chose to run a new simulation for each experiment, as running all of the experiments on the same simulation made the program design unnecessarily complex and disorganized. I also wanted to give as much customization as possible to the user, and included full control over the input (bodies, mass, velocity, color, diameter, name, timestep…etc).

The first experiment was to determine the total energy of the system over time.



Although it seems as if there is a major violation of the Conservation of Energy, the total energy only changes by less than 0.07% during the orbit. One possible explanation for this small variation in energy is the fact that unlike a real orbit, we use discrete time steps instead of continuous position and velocity adjustment. To test my hypothesis, I plotted the variance of the total energy by the timestep (seconds).



As was hypothesized, the discrete and relatively large jumps in time while updating the positions of the planets may be the cause of the small variations in energy. Another interesting observation is that the pattern seen in the total energy over time seems to be the superposition of the four planet frequencies. To test whether more planets resulted in higher variation in total energy, I computed the results in a table:

N-Bodies	Variance (J^2)
2.0	3.426497455691629e+60
3.0	7.163596856863361e+60
4.0	1.3946114262629492e+61
5.0	1.8036332079262805e+61

As we can see, the variance does significantly increase with each added body. This may be because of the increased frequency of interactions between the planets, and is a very interesting observation. Other than the very small changes in total energy, this experiment indicates that the Beeman simulation abides to the laws of nature.

Another experiment was to record the orbital periods of each planet.

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Simulated Earth Orbital Period (yrs): 1.0054794520547945
Simulated Earth Orbital Period (days): 367
Simulated Mercury Orbital Period (yrs): 0.23835616438356164
Simulated Mercury Orbital Period (days): 87
Simulated Venus Orbital Period (yrs): 0.6191780821917808
Simulated Venus Orbital Period (days): 226
Simulated Mars Orbital Period (yrs): 1.8767123287671232
Simulated Mars Orbital Period (days): 685
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As we can see there seems to be a slight error in the orbital periods, as Earth should have a period of 365 days, Mercury should be 88 days, Venus should be 225 days, and Mars should be 687 days. This slight error could also be explained by a few limitations of the simulation. First, the discretization of the orbital motion may result in slight differences in the orbital periods, because the orbit is not as exact as in reality. Second, although the initial positions and velocities are real values, they may not be perfectly correct, as real elliptical orbits change velocities and distances from the sun, and we do not know where in the cycle these parameters come from. These initial parameters also result in circular orbits, which indicate that in reality they are different.

The final implementation of the code creates a simulation based on a three-step Beeman scheme, and clearly visualizes the solar system with the given parameters. The experiments show that the simulation comes close to accurately representing the solar systems and the laws of nature. The errors can be explained by the limitations of such a simple discrete model and the uncertainty of the initial parameters. This project has taught me lots about testing and experimenting with simulations, and if I could do it again I would add more features and run more interesting additional experiments. I would also further explore the idea of continuously updating the position, in order to address the timestep problems.

909 words.