Assignment 2

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Download all python codes from

https://github.com/sachinomdubey/Matrix−theory/Assignment2/codes

and latex-tikz codes from

https://github.com/sachinomdubey/Matrix−theory/Assignment2

0.1 Problem

Q no. 73. Find the angle between the following pair of lines.

1)

$$L_1: \quad \mathbf{x} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \tag{0.1.1}$$

$$L_2: \quad \mathbf{x} = \begin{pmatrix} 7 \\ -6 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \tag{0.1.2}$$

2)

$$L_1: \quad \mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{0.1.3}$$

$$L_2: \quad \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ -56 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{0.1.4}$$

0.2 Explanation :

The given equations are in the form:

$$\mathbf{x} = \mathbf{p_1} + \lambda_1 \mathbf{m_1} \tag{0.2.1}$$

$$\mathbf{x} = \mathbf{p_2} + \lambda_2 \mathbf{m_2} \tag{0.2.2}$$

The angle between the lines can be found by substituting the values of the direction vectors $\mathbf{m_1}$ and $\mathbf{m_2}$ in dot product formula:

$$\cos\theta = \frac{\mathbf{m_1}^T \mathbf{m_2}}{\|\mathbf{m_1}\| \, \|\mathbf{m_2}\|} \tag{0.2.3}$$

0.3 Solution :

1) The direction vectors of the lines are :

$$\mathbf{m_1} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \tag{0.3.1}$$

$$\mathbf{m_2} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \tag{0.3.2}$$

Thus, the angle θ between two vectors is given by

$$\cos\theta = \frac{\mathbf{m_1}^T \mathbf{m_2}}{\|\mathbf{m_1}\| \, \|\mathbf{m_2}\|} \tag{0.3.3}$$

$$= \frac{19}{3 \times 7} \tag{0.3.4}$$

$$\implies \theta = 25.21^\circ \tag{0.3.5}$$

2) The direction vectors of the lines are:

$$\mathbf{m_1} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{0.3.6}$$

$$\mathbf{m_2} = \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix}. \tag{0.3.7}$$

Thus, the angle θ between two vectors is given by

$$\cos\theta = \frac{\mathbf{m_1}^T \mathbf{m_2}}{\|\mathbf{m_1}\| \, \|\mathbf{m_2}\|} \tag{0.3.8}$$

$$= \frac{16}{\sqrt{6} \times \sqrt{50}} \tag{0.3.9}$$

$$\implies \theta = 22.52^\circ \tag{0.3.10}$$