Assignment 2

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Download all python codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment2/codes
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and latex-tikz codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment2
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Q no. 73. Find the angle between the following pair of lines: Also find the closest points and minimum distance between them.

1)

$$L_1: \quad \mathbf{x} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \tag{0.0.1}$$

$$L_2: \quad \mathbf{x} = \begin{pmatrix} 7 \\ -6 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \tag{0.0.2}$$

2)

$$L_1: \quad \mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{0.0.3}$$

$$L_2: \quad \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ -56 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{0.0.4}$$

Explanation :

The given equations are in the form:

$$\mathbf{x} = \mathbf{p_1} + \lambda_1 \mathbf{m_1} \tag{0.0.5}$$

$$\mathbf{x} = \mathbf{p_2} + \lambda_2 \mathbf{m_2} \tag{0.0.6}$$

Finding the angle between the lines :

The angle between the lines can be found by substituting the values of the direction vectors $\mathbf{m_1}$ and $\mathbf{m_2}$ in dot product formula:

$$\cos\theta = \frac{\mathbf{m_1}^T \mathbf{m_2}}{\|\mathbf{m_1}\| \, \|\mathbf{m_2}\|} \tag{0.0.7}$$

Finding whether the lines intersect or not:

Step 1: Equate the x, y and z components of both lines and form equation in the augmented matrix form.

Step 2: Reduce the rows of the matrix to get Row echelon form.

Step 3: If Rank(A) $\neq$ Rank(A|B), the equations are inconsistent and the lines do not intersect. Else, if there is a unique solution than the point of intersection of two lines is obtained.

Finding the closest points between the skew lines:

Step 1: Find a vector $\mathbf{n}$ perpendicular to the directional vectors $\mathbf{m_1}$ and $\mathbf{m_2}$ using the dot product, forming equations and solving them in matrix form.

Step 2: When line2 is translated along the vector $\mathbf{n}$, it forms a plane. This plane intersects line1 at a point $\mathbf{c_1}$ which is nearest to line2. This point $\mathbf{c_1}$ is given by:

$$\mathbf{c_1} = \mathbf{p_1} + \frac{(\mathbf{p_2} - \mathbf{p_1})^T \mathbf{n_2}}{\mathbf{m_1}^T \mathbf{n_2}} \mathbf{m_1} \tag{0.0.8}$$

In above equation, the vector $\mathbf{n_2}$ is a vector perpendicular to $\mathbf{m_2}$ and $\mathbf{n}$. Find vector $\mathbf{n_2}$ using the dot product, forming equations and solving them in matrix form. Substitute all the values in equation 0.0.8 and find the point $\mathbf{c_1}$.

Step 3: Similarly, the point on line2 nearest to line1 is $\mathbf{c_2}$ given by:

$$\mathbf{c_2} = \mathbf{p_2} + \frac{(\mathbf{p_1} - \mathbf{p_2})^T \mathbf{n_1}}{\mathbf{m_2}^T \mathbf{n_1}} \mathbf{m_2} \tag{0.0.9}$$

In above equation, the vector $\mathbf{n_1}$ is a vector perpendicular to $\mathbf{m_1}$ and $\mathbf{n}$. Find vector $\mathbf{n_1}$ using the dot product, forming equations and solving them in matrix form. Substitute all the values in equation 0.0.9 and find the point $\mathbf{c_2}$.

The points $\mathbf{c_1}$ and $\mathbf{c_2}$ are the closest points between the given skew lines

Finding the minimum distance between two skew lines.

Step 1: Perform the Steps 1 to 3 given for finding the closest points between two skew lines.

Step 2: Using the distance formula, obtain the distance between points $\mathbf{c_1}$ and $\mathbf{c_2}$. The obtained distance is the minimum distance between the two skew lines.

Solution :
Problem 1:

The direction vectors of the lines are $\mathbf{m_1}$=$\begin{pmatrix}3\\2\\6\end{pmatrix}$ and $\mathbf{m_2}$=$\begin{pmatrix}1\\2\\2\end{pmatrix}$.
Thus, the angle θ between two vectors is given by

$$\cos\theta = \frac{\mathbf{m_1}^T\mathbf{m_2}}{\|\mathbf{m_1}\|\,\|\mathbf{m_2}\|} \tag{0.0.10}$$
$$= \frac{19}{3\times 7} \tag{0.0.11}$$
$$\implies \theta = 25.21° \tag{0.0.12}$$

Problem 2:

The direction vectors of the lines are $\mathbf{m_1}$=$\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ and $\mathbf{m_2}$=$\begin{pmatrix}3\\-5\\-4\end{pmatrix}$.
Thus, the angle θ between two vectors is given by

$$\cos\theta = \frac{\mathbf{m_1}^T\mathbf{m_2}}{\|\mathbf{m_1}\|\,\|\mathbf{m_2}\|} \tag{0.0.13}$$
$$= \frac{16}{\sqrt{6}\times\sqrt{50}} \tag{0.0.14}$$
$$\implies \theta = 22.52° \tag{0.0.15}$$

Note : In both problems, the respective pair of lines do not intersect each other (called skew lines), The obtained angle is the angle between the direction vectors of the lines. The proof that the pair of lines do not intersect in Problem 1 is as follows:

Problem 1 : Equating the x, y and z components of both lines and forming equation in the augmented matrix form. The Matrix is row reduced as follows:

$$\begin{pmatrix}3 & -1 & 5\\2 & -2 & -1\\6 & -2 & -1\end{pmatrix} \tag{0.0.16}$$
$$\xleftrightarrow{R_1\leftarrow R_1/3}\begin{pmatrix}3 & -1/3 & 5/3\\2 & -2 & -1\\6 & -2 & -1\end{pmatrix} \tag{0.0.17}$$
$$\xleftrightarrow{R_2\leftarrow R_2-2R_1}\begin{pmatrix}3 & -1/3 & 5/3\\0 & -4/3 & -13/3\\6 & -2 & -1\end{pmatrix} \tag{0.0.18}$$
$$\xleftrightarrow{R_3\leftarrow R_3-6R_1}\begin{pmatrix}3 & -1/3 & 5/3\\0 & -4/3 & -13/3\\0 & 0 & -11\end{pmatrix} \tag{0.0.19}$$

Here, Rank(A) $\neq$ Rank(A|B). Hence, these three equations are inconsistent, which proves that the two lines do not intersect in the 3D plane. (Where A is the coefficient matrix and A|B is the augmented matrix.)

Finding the closest points on the lines and minimum distance (Problem 1) :
The given equations are in the form:

$$\mathbf{x} = \mathbf{p_1} + \lambda_1\mathbf{m_1} \tag{0.0.20}$$
$$\mathbf{x} = \mathbf{p_2} + \lambda_2\mathbf{m_2} \tag{0.0.21}$$

Where, $\mathbf{p_1}$ and $\mathbf{p_2}$ are points on line1 and line2 respectively. Also, $\mathbf{m_1}$ and $\mathbf{m_2}$ are the direction vectors of respective lines.

The vector $\mathbf{n}$ is perpendicular to both lines. Thus, the dot product of the respective directional vectors and $\mathbf{n}$ is zero:

$$\mathbf{m_1}^T\mathbf{n} = 0 \tag{0.0.22}$$
$$\mathbf{m_2}^T\mathbf{n} = 0 \tag{0.0.23}$$
$$\begin{pmatrix}3 & 2 & 6\end{pmatrix}\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix} = 0 \tag{0.0.24}$$
$$\begin{pmatrix}1 & 2 & 2\end{pmatrix}\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix} = 0 \tag{0.0.25}$$

Forming augmented matrix and reducing the rows:

$$\begin{pmatrix}3 & 2 & 6 & 0\\1 & 2 & 2 & 0\end{pmatrix} \tag{0.0.26}$$

$$\xleftrightarrow{R_1\leftarrow 1/3\times R_1}\begin{pmatrix}1 & 2/3 & 2 & 0\\1 & 2 & 2 & 0\end{pmatrix} \tag{0.0.27}$$

$$\xleftrightarrow{R_2\leftarrow R_2-R1}\begin{pmatrix}1 & 2/3 & 2 & 0\\0 & 4/3 & 0 & 0\end{pmatrix} \tag{0.0.28}$$

$$\xleftrightarrow{R_2\leftarrow 3/4\times R_2}\begin{pmatrix}1 & 2/3 & 2 & 0\\0 & 1 & 0 & 0\end{pmatrix} \tag{0.0.29}$$

$$\xleftrightarrow{R_1\leftarrow R_1-2/3\times R_2}\begin{pmatrix}1 & 0 & 2 & 0\\0 & 1 & 0 & 0\end{pmatrix} \tag{0.0.30}$$

Here, Rank(A) = Rank(A|B). Hence, the system is consistent.
Also, Rank(A) < number of unknowns, Thus we have infinite number of solutions. let n_3=1, which makes n_1=-2 and n_2=0:

$$\therefore \mathbf{n} = \begin{pmatrix}-2\\0\\1\end{pmatrix} \tag{0.0.31}$$

When the line2 is translated along the vector n, it forms a plane. This plane intersects line1 at a single point $\mathbf{c_1}$ which is nearest to the line2. Point $\mathbf{c_1}$ is given by:

$$\mathbf{c_1} = \mathbf{p_1} + \frac{(\mathbf{p_2}-\mathbf{p_1})^T\mathbf{n_2}}{\mathbf{m_1}^T\mathbf{n_2}}\mathbf{m_1} \tag{0.0.32}$$

Here, $\mathbf{n_2}$ is perpendicular to both vector $\mathbf{m_2}$ and $\mathbf{n}$. Thus, the dot product of the $\mathbf{n_2}$ with $\mathbf{m_2}$ and $\mathbf{n}$ is zero:

$$\mathbf{m_2}^T\mathbf{n_2} = 0 \tag{0.0.33}$$

$$\mathbf{n}^T\mathbf{n_2} = 0 \tag{0.0.34}$$

$$\begin{pmatrix}1 & 2 & 2\end{pmatrix}\mathbf{n_2} = 0 \tag{0.0.35}$$

$$\begin{pmatrix}-2 & 0 & 1\end{pmatrix}\mathbf{n_2} = 0 \tag{0.0.36}$$

Forming augmented matrix and reducing the rows:

$$\begin{pmatrix}1 & 2 & 2 & 0\\-2 & 0 & 1 & 0\end{pmatrix} \tag{0.0.37}$$

$$\xleftrightarrow{R_2\leftarrow R_2+2R_1}\begin{pmatrix}1 & 2 & 2 & 0\\0 & 4 & 5 & 0\end{pmatrix} \tag{0.0.38}$$

$$\xleftrightarrow{R_2\leftarrow R_2/4}\begin{pmatrix}1 & 2 & 2 & 0\\0 & 1 & 5/4 & 0\end{pmatrix} \tag{0.0.39}$$

$$\xleftrightarrow{R_1\leftarrow R_1-2R_2}\begin{pmatrix}1 & 0 & -1/2 & 0\\0 & 1 & 5/4 & 0\end{pmatrix} \tag{0.0.40}$$

Here, Rank(A) = Rank(A|B). Hence, the system is consistent.
Also, Rank(A) < number of unknowns, Thus we have infinite number of solutions. let n_1=2, which makes n_3=4 and n_2=-5:

$$\therefore \mathbf{n_2} = \begin{pmatrix}2\\-5\\4\end{pmatrix} \tag{0.0.41}$$

$$\therefore \mathbf{c_1} = \begin{pmatrix}2\\-5\\1\end{pmatrix} + \frac{\begin{pmatrix}5 & -1 & -1\end{pmatrix}\begin{pmatrix}2\\-5\\4\end{pmatrix}}{\begin{pmatrix}3 & 2 & 6\end{pmatrix}\begin{pmatrix}2\\-5\\4\end{pmatrix}}\begin{pmatrix}3\\2\\6\end{pmatrix} \tag{0.0.42}$$

$$\therefore \mathbf{c_1} = \begin{pmatrix}2\\-5\\1\end{pmatrix} + \frac{11}{20}\begin{pmatrix}3\\2\\6\end{pmatrix} \tag{0.0.43}$$

$$\implies \mathbf{c_1} = \begin{pmatrix}3.65\\-3.9\\4.3\end{pmatrix} \tag{0.0.44}$$

Similarly, the point on Line2 nearest to Line 1 is $\mathbf{c_2}$ given by:

$$\mathbf{c_2} = \mathbf{p_2} + \frac{(\mathbf{p_1}-\mathbf{p_2})^T\mathbf{n_1}}{\mathbf{m_2}^T\mathbf{n_1}}\mathbf{m_2} \tag{0.0.45}$$

Here, $\mathbf{n_1}$ is perpendicular to both vector $\mathbf{m_1}$ and $\mathbf{n}$. Thus, the dot product of the $\mathbf{n_1}$ with $\mathbf{m_1}$ and $\mathbf{n}$ is zero:

$$\mathbf{m_1}^T\mathbf{n_1} = 0 \tag{0.0.46}$$

$$\mathbf{n}^T\mathbf{n_1} = 0 \tag{0.0.47}$$

$$\begin{pmatrix}3 & 2 & 6\end{pmatrix}\mathbf{n_2} = 0 \tag{0.0.48}$$

$$\begin{pmatrix}-2 & 0 & 1\end{pmatrix}\mathbf{n_2} = 0 \tag{0.0.49}$$

Forming augmented matrix and reducing the rows:

$$\begin{pmatrix} 3 & 2 & 6 & 0 \\ -2 & 0 & 1 & 0 \end{pmatrix} \tag{0.0.50}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 \times 1/3} \begin{pmatrix} 1 & 2/3 & 2 & 0 \\ -2 & 0 & 1 & 0 \end{pmatrix} \tag{0.0.51}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{pmatrix} 1 & 2/3 & 2 & 0 \\ 0 & 4/3 & 5 & 0 \end{pmatrix} \tag{0.0.52}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 \times 3/4} \begin{pmatrix} 1 & 2/3 & 2 & 0 \\ 0 & 1 & 15/4 & 0 \end{pmatrix} \tag{0.0.53}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 2/3 \times R_2} \begin{pmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 15/4 & 0 \end{pmatrix} \tag{0.0.54}$$

$$\tag{0.0.55}$$

Here, Rank(A) = Rank(A|B). Hence, the system is consistent.
Also, Rank(A) < number of unknowns, Thus we have infinite number of solutions. let n_1=2, which makes n_3=4 and n_1=-15:

$$\therefore \mathbf{n_1} = \begin{pmatrix} 2 \\ -15 \\ 4 \end{pmatrix} \tag{0.0.56}$$

$$\therefore \mathbf{c_2} = \begin{pmatrix} 7 \\ -6 \\ 0 \end{pmatrix} + \frac{\begin{pmatrix} -5 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -15 \\ 4 \end{pmatrix}}{\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -15 \\ 4 \end{pmatrix}} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \tag{0.0.57}$$

$$\therefore \mathbf{c_2} = \begin{pmatrix} 7 \\ -6 \\ 0 \end{pmatrix} + \frac{21}{20}\begin{pmatrix} 1 & 2 & 2 \end{pmatrix} \tag{0.0.58}$$

$$\implies \mathbf{c_2} = \begin{pmatrix} 8.05 \\ -3.9 \\ 2.1 \end{pmatrix} \tag{0.0.59}$$

The minimum distance can be found using points $\mathbf{c_1}$ and $\mathbf{c_2}$ as :

$$d_{min} = \sqrt{(3.65 - 8.05)^2 + (-3.9 + 3.9)^2 + (4.3 - 2.1)^2} \tag{0.0.60}$$

$$d_{min} = \sqrt{24.2} \implies d_{min} = \mathbf{4.92} \tag{0.0.61}$$

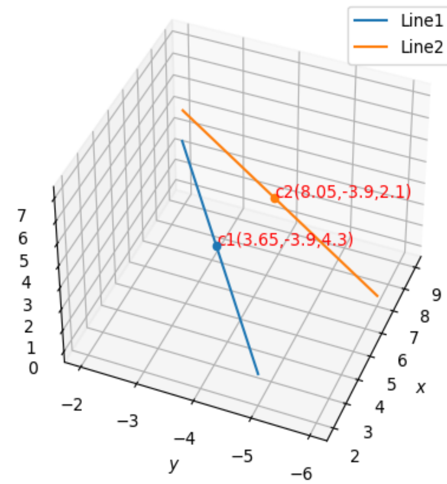

Fig. 2: Problem 1 : Lines crossing each other, but not intersecting

The proof that the pair of lines do not intersect in Problem 2 is as follows:

Problem 2 : Equating the x, y and z components of both lines and forming equation in the augmented matrix forms. The Matrix is row reduced as follows:

$$\begin{pmatrix} 1 & -3 & -1 \\ -1 & 5 & -2 \\ -2 & 4 & -54 \end{pmatrix} \tag{0.0.62}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 2 & -3 \\ -2 & 4 & -54 \end{pmatrix} \tag{0.0.63}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 2R_1} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 2 & -3 \\ 0 & -2 & -56 \end{pmatrix} \tag{0.0.64}$$

$$\xleftrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 1 & -3/2 \\ 0 & -2 & -56 \end{pmatrix} \tag{0.0.65}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 2R_2} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 1 & -3/2 \\ 0 & 0 & -59 \end{pmatrix} \tag{0.0.66}$$

Here, Rank(A) ≠ Rank(A|B).
Hence, the equations are inconsistent, which proves that the two lines do not intersect in the 3D plane. (Where A is the coefficient matrix and A|B is the augmented matrix.)

Finding the closest points on the lines and minimum distance (Problem 2) :

The given equations are in the form:

$$\mathbf{x} = \mathbf{p_1} + \lambda_1 \mathbf{m_1} \tag{0.0.67}$$
$$\mathbf{x} = \mathbf{p_2} + \lambda_2 \mathbf{m_2} \tag{0.0.68}$$

Where, $\mathbf{p_1}$ and $\mathbf{p_2}$ are points on line1 and line2 respectively. Also, $\mathbf{m_1}$ and $\mathbf{m_2}$ are the direction vectors of respective lines.
The vector $\mathbf{n}$ is perpendicular to both lines. Thus, the dot product of the respective directional vectors and n is zero:

$$\mathbf{m_1}^T \mathbf{n} = 0 \tag{0.0.69}$$
$$\mathbf{m_2}^T \mathbf{n} = 0 \tag{0.0.70}$$

$$\begin{pmatrix} 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 0 \tag{0.0.71}$$

$$\begin{pmatrix} 3 & -5 & -4 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 0 \tag{0.0.72}$$

Forming augmented matrix and reducing the rows:

$$\begin{pmatrix} 1 & -1 & -2 & 0 \\ 3 & -5 & -4 & 0 \end{pmatrix} \tag{0.0.73}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & -2 & 2 & 0 \end{pmatrix} \tag{0.0.74}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 \times -1/2} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \tag{0.0.75}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \tag{0.0.76}$$

Here, Rank(A) = Rank(A|B). Hence, the system is consistent.
Also, Rank(A) < number of unknowns, Thus we have infinite number of solutions. let n_2=1, which makes n_3=1 and n_1=3:

$$\therefore \mathbf{n} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \tag{0.0.77}$$

When the line2 is translated along the vector n, it forms a plane. This plane intersects line1 at a single point $\mathbf{c_1}$ which is nearest to the line2. Point $\mathbf{c_1}$ is given by:

$$\mathbf{c_1} = \mathbf{p_1} + \frac{(\mathbf{p_2} - \mathbf{p_1})^T \mathbf{n_2}}{\mathbf{m_1}^T \mathbf{n_2}} \mathbf{m_1} \tag{0.0.78}$$

Here, $\mathbf{n_2}$ is perpendicular to both vector $\mathbf{m_2}$ and $\mathbf{n}$. Thus, the dot product of the $\mathbf{n_2}$ with $\mathbf{m_2}$ and $\mathbf{n}$ is zero:

$$\mathbf{m_2}^T \mathbf{n_2} = 0 \tag{0.0.79}$$
$$\mathbf{n}^T \mathbf{n_2} = 0 \tag{0.0.80}$$
$$\begin{pmatrix} 3 & -5 & -4 \end{pmatrix} \mathbf{n_2} = 0 \tag{0.0.81}$$
$$\begin{pmatrix} 3 & 1 & 1 \end{pmatrix} \mathbf{n_2} = 0 \tag{0.0.82}$$

Forming augmented matrix and reducing the rows:

$$\begin{pmatrix} 3 & -5 & -4 & 0 \\ 3 & 1 & 1 & 0 \end{pmatrix} \tag{0.0.83}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 1 & -5/3 & -4/3 & 0 \\ 3 & 1 & 1 & 0 \end{pmatrix} \tag{0.0.84}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & -5/3 & -4/3 & 0 \\ 0 & 6 & 5 & 0 \end{pmatrix} \tag{0.0.85}$$
$$\xleftrightarrow{R_2 \leftarrow R_2/6} \begin{pmatrix} 1 & -5/3 & -4/3 & 0 \\ 0 & 1 & 5/6 & 0 \end{pmatrix} \tag{0.0.86}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 + 5/3_2} \begin{pmatrix} 1 & 0 & 1/18 & 0 \\ 0 & 1 & 5/6 & 0 \end{pmatrix} \tag{0.0.87}$$

Here, Rank(A) = Rank(A|B). Hence, the system is consistent.
Also, Rank(A) < number of unknowns, Thus we have infinite number of solutions. let n_2=15, which makes n_3=-18 and n_1=1:

$$\therefore \mathbf{n_2} = \begin{pmatrix} 1 \\ 15 \\ -18 \end{pmatrix} \tag{0.0.88}$$

$$\mathbf{c_1} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \frac{\begin{pmatrix} -1 & -2 & -54 \end{pmatrix} \begin{pmatrix} 1 \\ 15 \\ -18 \end{pmatrix}}{\begin{pmatrix} 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 15 \\ -18 \end{pmatrix}} \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{0.0.89}$$

$$\mathbf{c_1} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \frac{941}{22} \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{0.0.90}$$

$$\implies \mathbf{c_1} = \begin{pmatrix} 45.77 \\ -41.77 \\ -87.54 \end{pmatrix} \tag{0.0.91}$$

Similarly, the point on Line2 nearest to Line 1 is $\mathbf{c_2}$ given by:

$$\mathbf{c_2} = \mathbf{p_2} + \frac{(\mathbf{p_1} - \mathbf{p_2})^T \mathbf{n_1}}{\mathbf{m_2}^T \mathbf{n_1}} \mathbf{m_2} \tag{0.0.92}$$

Here, $\mathbf{n_1}$ is perpendicular to both vector $\mathbf{m_1}$ and $\mathbf{n}$. Thus, the dot product of the $\mathbf{n_1}$ with $\mathbf{m_1}$ and $\mathbf{n}$ is zero:

$$\mathbf{m_1}^T \mathbf{n_1} = 0 \tag{0.0.93}$$

$$\mathbf{n}^T \mathbf{n_1} = 0 \tag{0.0.94}$$

$$\begin{pmatrix} 1 & -1 & -2 \end{pmatrix} \mathbf{n_2} = 0 \tag{0.0.95}$$

$$\begin{pmatrix} 3 & 1 & 1 \end{pmatrix} \mathbf{n_2} = 0 \tag{0.0.96}$$

Forming augmented matrix and reducing the rows:

$$\begin{pmatrix} 1 & -1 & -2 & 0 \\ 3 & 1 & 1 & 0 \end{pmatrix} \tag{0.0.97}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 4 & 7 & 0 \end{pmatrix} \tag{0.0.98}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 \times 1/4} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 1 & 7/4 & 0 \end{pmatrix} \tag{0.0.99}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & -1/4 & 0 \\ 0 & 1 & 7/4 & 0 \end{pmatrix} \tag{0.0.100}$$

Here, Rank(A) = Rank(A|B). Hence, the system is consistent.
Also, Rank(A) < number of unknowns, Thus we have infinite number of solutions. let n_1=1, which makes n_3=4 and n_1=-7:

$$\therefore \mathbf{n_1} = \begin{pmatrix} 1 \\ -7 \\ 4 \end{pmatrix} \tag{0.0.101}$$

$$\therefore \mathbf{c_2} = \begin{pmatrix} 2 \\ -1 \\ -56 \end{pmatrix} + \frac{\begin{pmatrix} 5 & -1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -7 \\ 4 \end{pmatrix}}{\begin{pmatrix} 3 & -5 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ -7 \\ 4 \end{pmatrix}} \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{0.0.102}$$

$$\therefore \mathbf{c_2} = \begin{pmatrix} 2 \\ -1 \\ -56 \end{pmatrix} + \frac{8}{22} \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{0.0.103}$$

$$\implies \mathbf{c_2} = \begin{pmatrix} 29.69 \\ -47.15 \\ -92.92 \end{pmatrix} \tag{0.0.104}$$

The minimum distance can be found using points $\mathbf{c_1}$ and $\mathbf{c_2}$ as:

$$d_{min} = \sqrt{(16.08)^2 + (5.38)^2 + (5.38)^2} \tag{0.0.105}$$

$$d_{min} = \sqrt{316.8} \implies d_{min} = \mathbf{17.79} \tag{0.0.106}$$

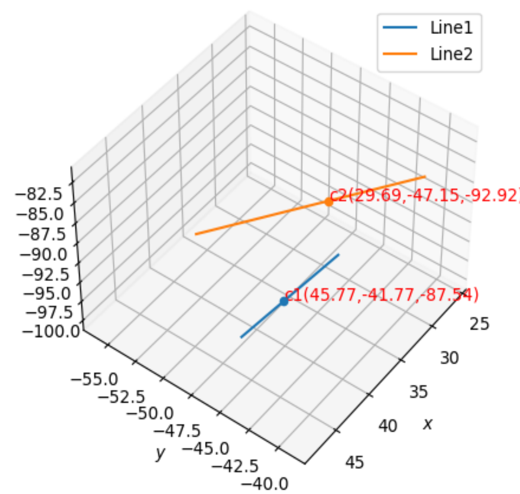

Fig. 2: Problem 2 : Lines crossing each other, but not intersecting