A Calculation of Cricket Ball Trajectories

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Introduction

This article shows:

- Constant force coefficient in sub-critical and super-critical Reynold number region.
- The transition between this two region with variable gradient.
- Approximate analysis of the trajectory equation which results very simple forms of trajectories.
- From the approximate analysis, the governing parameters are also observed.
- The effect of "No wind" and "Cross wind" on the trajectories of the cricket balls.

Literature Survey

- In [1-3], the authors have shown various forces on stationary balls with/without spin in wind tunnels of different types. So, **drag** and **side** forces are determined.
- In [4], some major simplifications are done in trajectory measurement. The trajectories of balls are studied in presence of different aerodynamic force profile.
- In [5], the trajectory of flying debris during extreme windstrom is calculated.

Trajectory Equation

- The trajectory equation is derived from [5], where the author sets an equation for both compact debris and sheet debris.
- For cricket balls, compact debris equation is used.

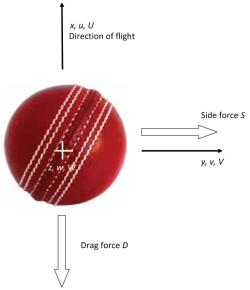

Figure: Axis system, velocities, and forces

Trajectory Equation

The basic trajectory equation for cricket balls:

$$\frac{du}{dt} = -[(\mathbf{u}-\mathbf{U})^2 + (\mathbf{v}-\mathbf{V})^2 + (\mathbf{w}-\mathbf{W})^2]^{0.5}(\mathbf{u}-\mathbf{U})C_D T \tag{1}$$

$$\frac{dv}{dt} = -[(\mathbf{u}-\mathbf{U})^2 + (\mathbf{v}-\mathbf{V})^2 + (\mathbf{w}-\mathbf{W})^2]^{0.5}[-(\mathbf{v}-\mathbf{V})C_D T + (\mathbf{u}-\mathbf{U})C_S T] \tag{2}$$

$$\frac{dw}{dt} = -[(\mathbf{u}-\mathbf{U})^2 + (\mathbf{v}-\mathbf{V})^2 + (\mathbf{w}-\mathbf{W})^2]^{0.5}(\mathbf{w}-\mathbf{W})C_D T - 1 \tag{3}$$

Trajectory Equation

Parameter symbol	Parameter name	Expression
C_D	Drag Force Coeff.	$\frac{D}{0.5A\rho Q^2}$
C_S	Side Force Coeff.	$\frac{S}{0.5A\rho Q^2}$
T	Tachikawa no.	$\frac{\rho A q^2}{2mq}$
Re	Reynold no.	$\frac{q\rho d}{\mu}$

Parameter symbol	Parameter name
D	Drag Force
S	Side Force
A	Ball cross-section area
ρ	Density of air
q	Initial speed of the ball
m	Mass of the ball

Trajectory Equation

Parameter symbol	Parameter name
d	Diameter of the ball
μ	Dynamic Viscosity of Air
U	longitudinal wind speed
V	lateral wind speed
W	vertical wind speed
u	longitudinal ball velocity
v	lateral ball velocity
w	vertical ball velocity

Reynolds number

- The drag and side force coefficients depends on the Reynolds number of the ball denoted by Re:

$$Re = \frac{q\rho d}{\mu} \tag{4}$$

Where,

- q is the initial speed at which the ball is bowled,
- ρ is the density of air,
- d is the diameter of the cricket ball and
- μ is the dynamic viscosity of air.

Significance of Reynolds number in cricket ball trajectory

- When the cricket ball travel through the air towards the batsman, the air flow around the ball can be **laminar** or **turbulent**.
- **Laminar** air flow occurs at lower velocity, where the air flow follows a smooth path. Also, laminar air flow is frequently observed for balls with smooth surfaces (new balls). At higher velocity the air flow around the ball is chaotic and referred to as **turbulent** flow (frequently observed in old balls with rough surface)
- The **Reynolds number** is a parameter with determines the transition between from the laminar flow to turbulent flow. Thus, the Reynolds number plays a huge role in determining the drag and side force coefficients acting on the ball as the ball travels through air.

Turbulent and Laminar Flow

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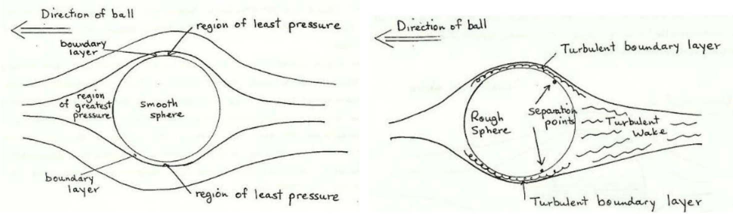


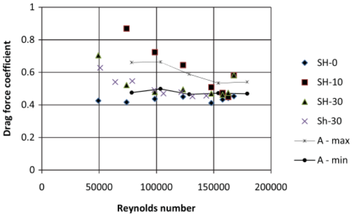
(a) Laminar Flow (b) Turbulent Flow

Figure: Different types of Flows

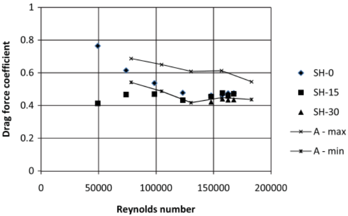
Various cricket parameters which affect the Reynolds number of the ball.

- The Reynolds number is directly proportional to **initial speed** of the ball bowled (denoted by q). A typical fast delivery is in the range of 85–95 mph (miles per hour), while most spin bowlers bowl at 45 to 55 mph.
- **Diameter** of the ball: The larger the ball's diameter, higher is the Reynolds number.
- **Altitude** (from sea level): The density of air decreases with increasing altitude and thus affects the Reynolds number.
- **Humidity**: The addition of water vapor to air (making the air humid) reduces the density of the air (as molar mass of water is less than that of dry air), which in turn reduces the Reynolds number.
- **Temperature**: High atmospheric temperature causes lower air density and viscosity of air. Hence, temperature also affects the Reynolds number.

Results of Past Experiments



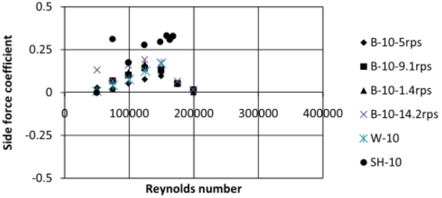
(a) Smooth sphere/New Ball



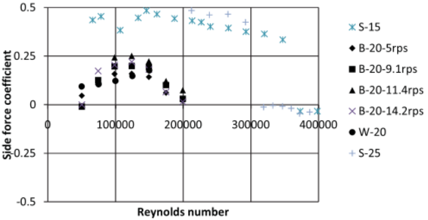
(b) Rough Sphere/Old Ball

Figure: Compilation of cricket ball drag coefficient data

Results of Past Experiments



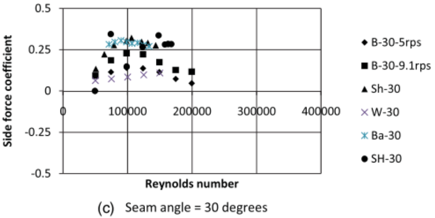
(a) Seam angle = 10 degrees



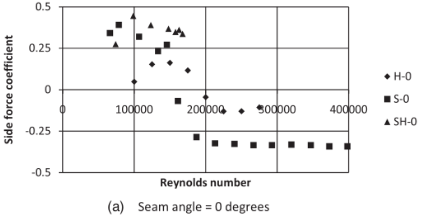
(b) Seam angle = 15 to 25 degrees

Figure: Compilation of side force coefficient data for smooth spheres/new balls.

Results of Past Experiments



(a) for smooth spheres/new balls



(b) for semi-roughened spheres

Figure: side force coefficient data

Results of Past Experiments

Symbol	Significance
$SH - n$	Experiments done by Sayer and Hill, n is the seam angle
$Sh - n$	Experiments done by Sherwin, n is the seam angle
$B - n - m$	Experiments done by Bentley, n is the seam angle, m is spin rate
$Ba - n$	Experiments done by Bentley, n is the seam angle
$S - n$	Experiments done by Sayer, n is the seam angle
$W - n$	Experiments done by Ward, n is the seam angle
$H - n$	Experiments done by Hunt, n is the seam angle

Results of Past Experiments

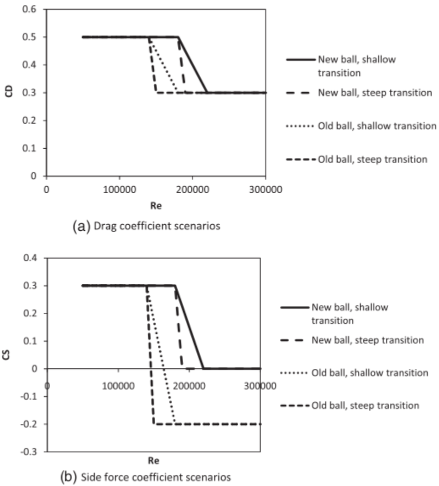


Figure: Force coefficient scenarios for trajectory calculations

Observations

From the plots **??**, it can be observed that:

- In sub-critical region the drag force coeffiecient is 0.5 for new ball as well as old ball.
- In super-critical region the drag force coeffiecient is 0.3 for new ball as well as old ball.
- In sub-critical region the side force coeffiecient is 0.3 for new ball as well as old ball.
- In super-critical region the side force coeffiecient is 0 for new ball where as for old ball it is -2.5.
- These plots with different values of C_D and C_S are experimentally done, there no particular expression of C_D and C_S are derived from the experimental data yet.

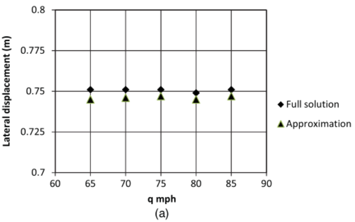
Steep and Shallow Transition

Type of Transition	Range of Bowling speed
Shallow Transition	81 mile/hr to 100 mile/hr for new ball
Steep Transition	81 mile/hr to 86 mile/hr for new ball
Shallow Transition	63 mile/hr to 81 mile/hr for Old ball
Steep Transition	63 mile/hr to 68 mile/hr for Old ball

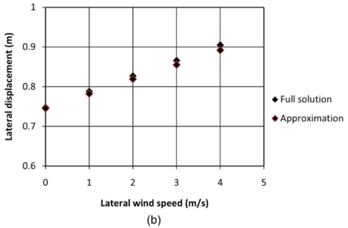
Significance:

- For both old and new ball, it is observed that in case of shallow transition the velocity of the ball is more than the velocity of the ball for steep transition at the end of the transition.

Approximate Solution



(a) No wind case

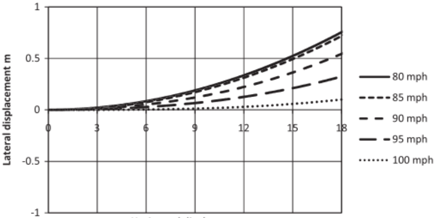


(b) Cross wind case

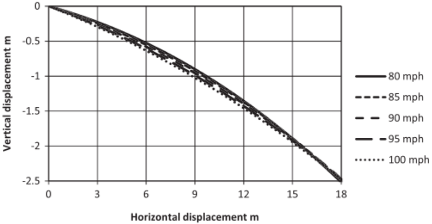
Figure: Accuracy of approximate methods

- **No wind case:** $\mathbf{U} = \mathbf{V} = \mathbf{W} = 0$ and $C_D, C_S =$ constant, $\mathbf{y} = \frac{C_S T}{2}\mathbf{x}^2$
- **Cross wind case:** $\mathbf{U} = \mathbf{W} = 0$ and $\mathbf{V} \leq \mathbf{u}$ and $\mathbf{y} = \frac{(C_S + C_D \mathbf{V})T}{2}\mathbf{x}^2$

Full Solution Trajectory Equation



(a) Lateral displacements



(b) Vertical displacements

Figure: Trajectories for new ball/shallow transition sce- nario.

Full Solution Trajectory Equation

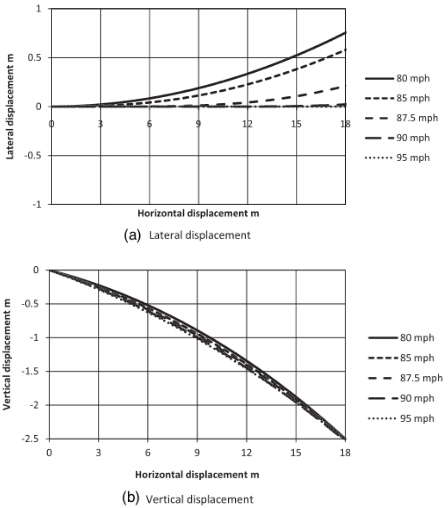


Figure: Trajectories for new ball/steep transition scenario.

Full Solution Trajectory Equation

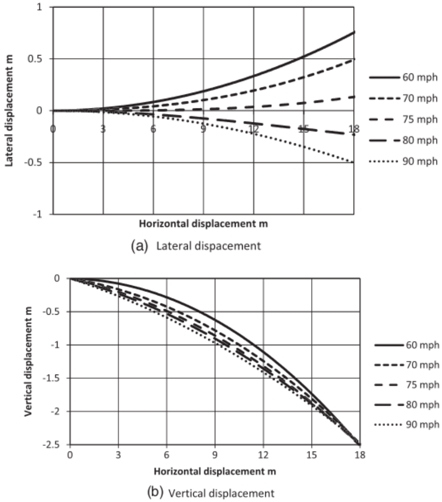


Figure: Trajectories for old ball/shallow transition sce- nario.

Full Solution Trajectory Equation

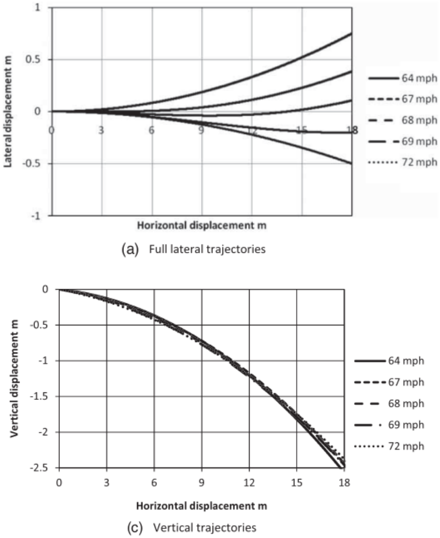


Figure: Trajectories for old ball/steep transition sce- nario.

Conclusion

- In trajectory equation, the main governing parameters are C_D, C_S and T.
- The drag and side forces are reasonable constant in the sub and super crical Re number region.
- The supercritical values of the side force coefficient are in general zero for new balls, and less than zero for old balls.
- The approximate analysis of the trajectory equations shows that, for constant drag and side force coefficients, the trajectories take on a simple parabolic form.
- A full solution of the trajectory equations enables the trajectories to be calculated for all bowling speeds for different types of ball.

Reference

1 Mehta, R. D. Aerodynamics of sports balls. Annu. Rev. Fluid Mech., 1985, 17, 151–189.

2 Mehta, R. D. Cricket ball aerodynamics: myth versus sci- ence. In The engineering of sport – research, development and innovation (Eds A. J. Subic and S. J. Haake), 2000, pp. 153–167 (Blackwell Science, London).

3 Mehta, R. D. A review of cricket ball swing. Sports Eng., 2005, 8, 181–192.

4 Bentley, K., Varty, P., Proudlove, M., and Mehta, R. D. An experimental study of cricket ball swing. Imperial College Aero Technical Note, 1982, pp. 82–106.

5 Baker, C. J. The debris flight equations. J. Wind Eng. Ind. Aerodyn., 2007, 95(5), 329–353.

Back up

$$\bar{x} = \frac{1}{C_D T} log(1 + C_D T \bar{t}) \tag{5}$$

$$\bar{y} = (\bar{t} - (\frac{1}{C_D T} log(1 + C_D T \bar{t}))) \frac{C_S}{C_D} \tag{6}$$

$$\bar{z} = \bar{t} \sin \alpha - \frac{\bar{t}^2}{2} \tag{7}$$

where

$$\bar{t} = \frac{tg}{q} \tag{8}$$

t is the time, g is acceleration due to gravity.