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211IT058

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ASSIGNMENT - 2

1. The table can be shown as :

X\Y	1	2	3	total
1	0.1	0.2	0.1	0.4
2	0.2	0.3	0.0	0.5
3	0.0	0.1	0.0	0.1
total	0.3	0.6	0.1	

a) The marginal probability distribution of X is given by

$$p(x) = \sum_{y} P(X=x, Y=y)$$

Hence, using formula,

Marginal distribution of X :

X	1	2	3
p(x)	0.4	0.5	0.1

Marginal distribution of Y :

Y	1	2	3
p(y)	0.3	0.6	0.1

b) Conditional probability of X given Y is

$$P(X|Y) = \frac{P(X=x, Y=y)}{P(y)}$$

$$= \frac{P(x,y)}{P(y)}$$

So, we need $P(X=2 | Y=1)$

$$= \frac{P(X=2, Y=1)}{P(Y=1)}$$

$$= \frac{0.2}{0.3} = \frac{2}{3} = 0.66$$

c) Two RVs, X & Y are said to be independent if,

$$P(x,y) = P(x)\,P(y)$$

where, $P(x) \rightarrow$ marginal of x
$P(y) \rightarrow$ marginal of y
$\forall$ pairs of (x,y)

So,

say $(1,1)$

$$P(1,1) = 0.1$$

& $P(X=1) = 0.4$
$P(y=1) = 0.3$

Clearly, $P(1,1) \neq P(X=1) \cdot P(y=1)$

$$0.1 \neq (0.4) \times (0.3)$$

Hence, the two random variables are not independent.

2. Given PDF is

$$f(x) = 3x^2 \quad , \quad 0 \leq x \leq 1$$
$$f(x) = 0 \quad , \quad \text{otherwise}$$

a) The CDF is given by

$$F(x \leq X) = \int_{-\infty}^{x=X} f(x)\,dx$$

Hence, breaking it down.

when $x < 0$,

$$\cancel{F(x=0)} \quad F(x) = \int_{-\infty}^{x} f(x)\,dx$$
$$= 0$$

when $0 < x \leq 1$,

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$
$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$
$$= 0 + \int_{0}^{x} 3x^2\,dx$$
$$= (x^3]_{0}^{x}$$
$$= x^3$$

For $x > 1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{x} f(x)\,dx$$

$$= (1)^3 + 0$$

$$= 1$$

Hence,

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ x^3 & , \quad 0 \le x \le 1 \\ 1 & , \quad x > 1 \end{cases}$$

b) Mean of random variable X is given by :

$$\text{Mean} = \int_{-\infty}^{\infty} x\, f(x)\,dx$$

where $f(x)$ is PDF

So,

$$\text{mean} = \int_{-\infty}^{0} x \cdot f(x)\,dx + \int_{0}^{1} x \cdot f(x)\,dx + \int_{1}^{\infty} x \cdot f(x)\,dx$$

$$= 0 + \int_{0}^{1} x \cdot (3x^2)\,dx + 0$$

$$= 3 \int_{0}^{1} x^3$$

$$= 3 \cdot \left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{4}$$

So, $\boxed{\text{mean}(x) = \frac{3}{4}}$

c)

$$\text{Variance}(x) = E(x^2) - (E(x))^2$$

where $E(x)$ is the mean of x
$E(x^2)$ is the mean of x^2.

Now,

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_0^1 x^2 (3x^2)\, dx$$

$$= 3 \int_0^1 x^4\, dx$$

$$= \frac{3}{5} [x^5]_0^1$$

$$= \frac{3}{5}$$

and, $(E(x))^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

So, $\text{variance}(x) = E(x^2) - (E(x))^2$

$$= \frac{3}{5} - \frac{9}{16}$$

$$= \frac{16 \times 3 - 9 \times 5}{5 \times 16}$$

$$= \frac{48 - 45}{5 \times 16}$$

$$= \frac{3}{80}$$

So,

$$\boxed{var(X) = \frac{3}{80}}$$

3. Given X is uniformly distributed over $[0, 1]$, then

PDF of X is

$$f(x) = \begin{cases} 1 & ; \quad 0 \le x \le 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

a) The new random variable is Y & it is related as

$$Y = X^2$$

Since $y = x^2$ is continuously differentiable & non decreasing for $0 \le x \le 1$, we can say,

$$x = \sqrt{y} \quad , \quad y > 0$$

$$dx/dy = \frac{1}{2\sqrt{y}}$$

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and,

$$0 \le x \le 1$$
$$0 \le \sqrt{y} \le 1$$

$$0 \le y \le 1$$

and, say

$$g(y) = f(x(y)) \left| \frac{dx}{dy} \right|$$

$$= (1) \left| \frac{1}{2\sqrt{y}} \right|$$

$$= \frac{1}{2\sqrt{y}}$$

So, the pdf of Y is

$$g(y) = \begin{cases} \frac{1}{2\sqrt{y}} , & 0 \le y \le 1 \\ 0 , & \text{otherwise} \end{cases}$$

b) Mean of Y is given by :

$$E(Y) = \int_{-\infty}^{\infty} y \cdot g(y) \, dy$$

$g(y) \to$ pdf

$$= \int_0^1 y \cdot \left(\frac{1}{2\sqrt{y}} \right) dy$$

$$= \frac{1}{2} \int_0^1 \sqrt{y} \, dy$$

$$= \frac{1}{2}\left[\frac{y^{3/2}}{(3/2)}\right]_0^1$$

$$= \frac{1}{3}\left(y^{3/2}\right]_0^1$$

$$= \frac{1}{3}$$

So, Mean $(Y) = \frac{1}{3}$

c) Variance of Y is $\rightarrow E(Y^2) - (E(Y))^2$

Now, $E(Y^2) = \int_{-\infty}^{\infty} y^2 \, g(y) \, dy$

$$= \int_0^1 y^2 \left(\frac{1}{2\sqrt{y}}\right) dy$$

$$= \frac{1}{2}\int_0^1 y^{3/2} \, dy$$

$$= \frac{1}{2} \frac{\left(y^{5/2}\right]_0^1}{(5/2)}$$

$$= \frac{1}{5}$$

Also, $(E(Y))^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

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So, $Var(Y) = \frac{1}{5} - \frac{1}{9}$

$= \frac{4}{45}$ //

5. Stock Price Random Variable

Given X & $P(x)$:

X (%)	-2	-1	0	1	2
P(X)	0.1	0.2	0.3	0.25	0.15

a) $E(X) = \sum_{i \in all} x_i \cdot P(x_i)$

$= (-2)(0.1) + (-1)(0.2) + 0(0.3) + 1(0.25) + 2(0.15)$

$= -0.2 - 0.2 + 0.25 + 0.3$

$= 0.55 - 0.40$

$= 0.15$

So, $E(X) = 0.15$ //

b) $V(X) = E(X^2) - (E(X))^2$

Now,

$E(X^2) = \sum_{i \in all} (x_i)^2 \, P(x_i)$

$= 4(0.1) + 1(0.2) + 0(0.3) + 1(0.25) + 4(0.15)$

$= 0.4 + 0.2 + 0.25 + 0.6$
$= 1.45$

& $(E(X))^2 = 0.0225$

So, $V(X) = 1.4275$ //

c) Standard deviation $= \sqrt{V(X)}$
$= \sqrt{1.4275}$
$= 1.194$ //

d) P(increases by atleast 1%) =

$$\sum_{x_i \geq 1} P(x_i)$$

$= 0.25 + 0.15$
$= 0.4$ //

e) P(increase or decrease by less than 2%)

$= P(-1) + P(0) + P(1)$

$= 0.2 + 0.3 + 0.25$

$= 0.75$ //

4. Given the random variables X and Y,

X/Y	1	2	3
1	0.2	0.1	0.1
2	0.1	0.2	0.1
3	0.1	0.1	0

a) Finding marginal distribution of x:

x	1	2	3
$p(x)$	0.4	0.4	0.2

Marginal distribution of y:

y	1	2	3
$P(y)$	0.4	0.4	0.2

Now, $\text{cov}(X,Y) = E(XY) - E(X) \cdot E(Y)$

So, $E(X) = \sum_i x_i \, p(x_i)$

$= 1(0.4) + 2(0.4) + 3(0.2)$

$= 0.4 + 0.8 + 0.6$

$= 1.8$

$E(Y) = \sum_i y_i \, P(y_i)$

$= 1(0.4) + 2(0.4) + 3(0.2)$

$= 0.4 + 0.8 + 0.6$

$= 1.8$

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Now, $E(XY) = \sum_{i,j} x_i y_j J_{ij}$

where J_{ij} is the joint probability distribution.

So, $E(XY) = 0.2 + 2(0.1) + 3(0.1) + 2(0.1) + 4(0.2) + 6(0.1) + 3(0.1) + 6(0.1) + 9(0)$

$= 0.2 + 0.2 + 0.3 + 0.2 + 0.8 + 0.6 + 0.3 + 0.6$

$= 3.2$

$\therefore$

$COV(X,Y) = 3.2 - (1.8)(1.8)$

$= -0.040$

b) $\because COV(X,Y) < 0$, we can conclude X and Y are negatively correlated.

c) Now,

$$\rho(X,Y) = \frac{COV(X,Y)}{\sigma_x \sigma_y}$$

$\sigma_x = \sqrt{VAR(X)}$

& $VAR(X) = E(X^2) - [E(X)]^2$

$= 1(0.4) + 4(0.4) + 9(0.2) - (1.8)^2$

$= 0.4 + 1.6 + 1.8 - 3.24$

$VAR(X) = 0.56$

$\sigma_X = \sqrt{0.56} = 0.748$

Similarly,

$\sigma_Y = \sqrt{VAR(Y)}$

$VAR(Y) = E(Y^2) - [E(Y)]^2$

$= 0.4 + 4(0.4) + 9(0.2) - (1.8)^2$

$= 3.8 - 3.24$

$= 0.56$

$\sigma_Y = \sqrt{0.56} = 0.748$

Hence,

$$\rho(X,Y) = \frac{-0.040}{(0.748)(0.748)}$$

$$= -0.071$$