27/11/2024

Module - 5

ANOVA → Analysis of variance

$H_0 = \mu_1 = \mu_2 = \dots = \mu_n$

One way classification

completely ~~Rabomized~~ Radomized design (CRD)

A	B	C	.....
a_1	b_1	c_1	
a_2	b_2	c_2	
⋮	⋮	⋮	
a_{n_1}	b_{n_2}	c_{n_2}	
ΣA	ΣB	ΣC	

N = total no. of data given

Correction factor → $C.F = \frac{(\Sigma A + \Sigma B + \Sigma C + \dots)^2}{N}$

Sum of square of column → $SSC = \frac{(\Sigma A)^2}{n_1} + \frac{(\Sigma B)^2}{n_2} + \dots - C.F$

Sum of square total → $SST = a_1^2 + a_2^2 + \dots a_{n_1}^2 + b_1^2 + \dots b_{n_2}^2 + c_1^2 + \dots c_{n_3}^2 + \dots - CF$

Sum of square of error → $SSE = SST - SSC$

Source of variation	Degree of freedom	SS	MSS	F
Column	No. of column − 1	SSC	$MSSC = \frac{SSC}{Dof}$	$F = \frac{MSSC}{MSSE} > 1$
Error	N − 1 (No. of Comm − 1)	SSE	$MSSE = \frac{SSE}{Dof}$	
Total	N − 1			$F = \frac{MSSE}{MSSC} > 1$

Q A random sample is selected from each of the 3 makes of ropes and their breaking strength are measured with following results

Test whether the breaking strength of the ropes differ significantly

I	II	III
70	100	60
72	110	65
75	108	57
80	112	84
83	113	87
	120	78
380	107	431
	770	

$N = 18$

$n_1 = 5$

$n_2 = 7$

$n_3 = 6$

$$C.F = \frac{(380 + 770 + 431)^2}{18} = 138864.5$$

$$SSC = \frac{(380)^2}{5} + \frac{(770)^2}{7} + \frac{(431)^2}{6} - 138864.5$$

$$SSC = 28880 + 84700 + 30960.17 - 138864.5$$

$$SSC = 5675.67$$

$$SST = (70)^2 + (72)^2 + (75)^2 + \dots + (78)^2 - 138864.5$$

$$SST = 145707 - 138864.5$$

$$SST = 6842.5$$

$$SSE = SST - SSC$$

$$SSE = 6842.5 - 5675.67$$

$$SSE = 1166.83$$

Source of Variation	Dof	SS	MSS	F
column	$3-1=2$	5675.67	$\frac{5675.67}{2} = 2837.83$	$\frac{2837.83}{77.79} = 36.48$
Error	$17-2=15$	1166.88	$\frac{1166.88}{15} = 77.79$	
Total	$18-1=17$	6842.5		
table value	$F_{0.05}(2,15) = 3.68$.		$F > F_{0.05}(2,15)$	
			Reject H_0.	

Q The following of the number mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test whether the difference among the 4 sample mean can be attributed to chance.

I	II	III	IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11
49	59	55	50

$N = 20$

$n_1 = 5$

$n_2 = 5$

$n_3 = 5$

$n_4 = 5$

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$$CF = \frac{(49+59+55+50)^2}{20} = \frac{45369}{20} = 2268.45.$$

$$SSC = \frac{(49)^2}{5} + \frac{(59)^2}{5} + \frac{(55)^2}{5} + \frac{(50)^2}{5} - 2268.45$$

$$= 2281.4 - 2268.45$$

$$= 12.95$$

$$SST = 6^2 + 14^2 + 10^2 + 8^2 + \ldots\ldots + 11^2 - 2268.45.$$

$$SST = 2383 - 2268.45$$

$$SST = 114.55$$

$$SSE = SST - SSC$$

$$SSE = 114.55 - 12.95$$

$$SSE = 101.6$$

Source of variation	Dof	SS	MSS	F
Column	3	12.95	$\frac{12.95}{3} = 4.31$	$F = \frac{6.35}{4.31} = 1.47$
Error	16	101.6	$\frac{101.6}{16} = 6.35$	
Total	19	114.35		

$F_{0.01}(16,3) = 29.83$

$F < F_{0.01}(16,3)$

Accept H_0.

* 2 way classification randomised block design

Q. Following data represents no. of units of production per day turned out by different workers using 4 diff types of machines. Test whether the 5 men differ with respect to the mean productivity and also test whether the mean productivity is same for 4 different machines.

Workers	Machine Type A	B	C	D	
I	44	38	47	36	165
II	46	40	52	43	181
III	34	36	44	32	146
IV	43	38	46	33	160
V	38	42	49	39	168
	205	194	238	183	820

112,93

→ $H_0 : \mu_A = \mu_B = \mu_C = \mu_D$

i.e. $\mu_I = \mu_{II} = \mu_{III} = \mu_{IV}$

$N = 20$

$$CF = \frac{(820)^2}{20} = 33620$$

$$SSC = \underbrace{\frac{(205)^2}{5} + \frac{(194)^2}{5} + \frac{(238)^2}{5} + \frac{(183)^2}{5}}_{33958.8} - 33620$$

$$SSC = 338.8$$

$$SSR = \underbrace{\frac{(165)^2}{4} + \frac{(181)^2}{4} + \frac{(146)^2}{4} + \frac{(160)^2}{4} + \frac{(168)^2}{4}}_{33781.5} - 33620$$

$$SSR = 161.5$$

$$SST = \underbrace{44^2 + 38^2 + 47^2 + 36^2 + \dots + 39^2}_{34194} - 33620$$

$$SST = 574$$

$$SSE = SST - SSC - SSR$$

$$SSE = 574 - 338.8 - 161.5$$

$$SSE = 73.7$$

Source of variation	DOF	SS	MSS	F
Column	$4-1=3$	338.8	$\frac{338.8}{3}=112.9$	$F_c=\frac{112.9}{6.1}=18.5$
Row	$5-1=4$	161.5	$\frac{161.5}{4}=40.3$	$F_R=\frac{40.3}{6.1}=6.1$
Error	12	73.7	$\frac{73.7}{12}=6.1$	
Total	19	574		

$F_{0.05}(3,12)=3.49$

$F_{0.05}(4,12)=3.26$

$F_c > F_{0.05}(3,12)$

Reject H_0

$F_R > F_{0.05}(4,12)$

Reject H_0.

Q A company appoints 4 sales men A, B, C, & D and observes their sales in 3 seasons Summer, winter & monsoon.

Season \ Salesmen	A	B	C	D	
Summer	45	40	38	37	160
Winter	43	41	45	38	167
Monison	39	39	41	41	160
	127	120	124	116	487

Carry out and analysis of variance

$H_0: \mu_A=\mu_B=\mu_C=\mu_D$

$\mu_1=\mu_2=\mu_3$

$N=12$

$CF=\frac{(487)^2}{12}=19764.08$

$SSC=\frac{127^2}{3}+\frac{120^2}{3}+\frac{124^2}{3}+\frac{116^2}{3}-19764.08=$

$SSC=19787-19764.08$

$SSC=22.92$

$SSR=\frac{160^2}{4}+\frac{167^2}{4}+\frac{160^2}{4}-19764.08$

$=19772.25-19764.08$

$=8.17$

$$SST = 45^2 + 43^2 + 39^2 + 40^2 + \ldots\ldots 41^2 - 19764.08$$

$$= 19841 - 19764.08$$

$$SST = 76.92$$

$$SSE = SST - SSC - SSR$$

$$SSE = 76.92 - 22.92 - 8.17$$

$$SSE = 45.83$$

Source of Variation	Dof	SS	MSS	F
Column	$4-1=3$	22.92	$\frac{22.92}{3} = 7.64$	$\frac{7.64}{7.63} = 1.001$
ROW	$3-1=2$	8.17	$\frac{8.17}{2} = 4.08$	
Error	6	45.83	$\frac{45.83}{6} = 7.63$	$\frac{4.08}{7.63} = 0.53$
Total	11	76.92		

Table value:
- $F_{0.05}(3,6) = 4.76$
- $F_{0.05}(2,6) = 5.14$

$F_c < F_{0.05}(3,6)$

Accept H_0

$F_R < F_{0.05}(2,6)$

Accept H_0

Q Analyse the following RBD and draw your conclusion

	Treatments				
	T_1	T_2	T_3	T_4	
B_1	12	14	20	22	68
B_2	17	27	19	15	78
B_3	15	14	17	12	58
B_4	18	16	22	12	68
B_5	19	15	20	14	68
	81	86	98	75	340

$H_0: \mu_{T_1} = \mu_{T_2} = \mu_{T_3} = \mu_{T_4}$

$\mu_{B_1} = \mu_{B_2} = \mu_{B_3} = \mu_{B_4} = \mu_{B_5}$

$N = 20$

$$CF = \frac{(340)^2}{20} = 5780$$

$$SSC = \frac{81^2}{5} + \frac{86^2}{5} + \frac{98^2}{5} + \frac{75^2}{5} - 5780$$

$$SSC = 5837.2 - 5780$$

$$SSC = 57.2$$

$$SSR = \frac{68^2}{4} + \frac{78^2}{4} + \frac{58^2}{4} + \frac{68^2}{4} + \frac{68^2}{4} - 5780$$

$$= 5830 - 5780 = 50$$

$$SST = 12^2 + 17^2 + 15^2 + 18^2 + \ldots\ldots 12^2 + 14^2$$

$$= 6072 - 5780$$

$$= 292$$

$$SSE = SST - SSC - SSR$$

$$= 292 - 57.2 - 50$$

$$= 184.8$$

Source of variation	DOF	SS	MSS	F
Column	$4-1=3$	57.2	$\frac{57.2}{3} = 19.06$	$\frac{19.06}{15.4} = 1.237$
ROW	$5-1=4$	50	$\frac{50}{4} = 12.5$	$\frac{12.5}{15.4} = 0.812$
error	12	184.8	$\frac{184.8}{12} = 15.4$	
error	19	292		

- $F_c < F_{0.05}(3,4)$.
  Accept H_0
- $F_c < F_{0.05}(4,12)$.
  Accept H_0

3 way classification, Latin square design

Q Set of analysis of variance for the following ~~variance~~ results of a latin square design use 0.01 level of significance.

A 12	C 19	B 10	D 8	49
C 18	B 12	D 6	A 7	43
B 22	D 10	A 5	C 21	58
D 12	A 7	C 27	B 17	63
64	48	48	53	213

2878.25

42.69

Ho: There is no significant difference b/w means of rows, columns and treatments

A	B	C	D
12	10	19	8
7	12	18	6
5	22	21	10
7	17	27	12
31	61	85	36

N = 16

3483

$$CF = \frac{(213)^2}{16} = 2835.56$$

$$SSC = \frac{(61)^2}{4} + \frac{(48)^2}{4} + \frac{(48)^2}{4} + \frac{(53)^2}{4} - 2835.56$$

$$= 2878.25 - 2835.56$$

$$= 42.69$$

$$SSR = \frac{(49)^2}{4} + \frac{(43)^2}{4} + \frac{(58)^2}{4} + \frac{(63)^2}{4} - 2835.56$$

$$= 2895.75 - 2835.56$$

$$= 60.19$$

$$SST_r = \frac{(31)^2}{4} + \frac{61^2}{4} + \frac{85^2}{4} + \frac{36^2}{4} - 2835.56$$

$$= 3300.75 - 2835.56$$

$$= 465.19$$

$$SST = (12)^2 + 19^2 + 10^2 + 8^2 + \ldots 17^2 - 2835.56$$

$$= 3483 - 2835.56$$

$$= 647.44$$

$$SSE = SST - SSC - SSR - SSTr$$
$$= 667.44 - 42.67 - 60.19 - 465.19$$
$$= 79.37$$

	DOf	SS (Sum of sq)	MSS (Mean sum of squares)	F
Columns	3	42.69	$\frac{42.69}{3} = 14.23$	$F_C = \frac{14.23}{13.23} = 1.07$
Row	3	60.19	$\frac{60.19}{3} = 20.06$	$F_R = \frac{20.06}{13.23} = 1.51$
Treatment	3	465.19	$\frac{465.19}{3} = 155.06$	$F_{TR} = \frac{155.06}{13.23} = 11.73$
Error	6 $(n-1)(n-2)$	79.37	$\frac{79.37}{6} = 13.23$	
Total	15	647.44		

Table value $F_{0.01}(3,6) = 4.76$

$F_C < F_{0.01}(3,6) \rightarrow$ Accept H_0

$F_R < F_{0.01}(3,6) \rightarrow$ Accept H_0

$F_{Tr} > F_{0.01}(3,6) \rightarrow$ Reject H_0.

There is no significance difference b/w means of rows and columns and there is significant difference b/w means of treatment.

Q The figures in the following 5×5 latin square of the numbers of minutes, engines e_1, e_2, e_3, e_4, e_5 tuned up by mechanics M_1, M_2, M_3, M_4, M_5 ran with a gallon of fuel A, B, C, D & E

	E_1	E_2	E_3	E_4	E_5	
M_1	A 31	B 24	C 20	D 20	E 18	113
M_2	B 21	C 27	D 23	E 25	A 31	127
M_3	C 21	D 27	E 25	A 29	B 21	123
M_4	D 21	E 25	A 33	B 25	C 22	126
M_5	E 21	A 37	B 24	C 24	D 20	126
	115	140	125	123	112	615

Do Anonova test to check if any significance difference b/w engines, mechanics, gallon of fuel

A	B	C	D	E
31	24	20	20	18
31	21	27	23	25
29	21	21	27	25
33	25	22	21	25
37	24	24	20	21
161	115	114	111	114

$N = 25$

$$CR = \frac{(615)^2}{25} = 15129$$

$$SSC = \frac{(115)^2}{5} + \frac{140^2}{5} + \frac{125^2}{5} + \frac{123^2}{5} + \frac{112^2}{5} - 15129$$

$$= 15224.6 - 15129$$

$$= 95.6$$

$$SSR = \frac{(113)^2}{5} + \frac{(127)^2}{5} + \frac{123^2}{5} + \frac{126^2}{5} + \frac{126^2}{5} - 15129$$

$$= 15155.8 - 15129$$

$$= 26.8$$

$$SST_r = \frac{161^2}{5} + \frac{115^2}{5} + \frac{114^2}{5} + \frac{111^2}{5} + \frac{114^2}{5} - 15129$$

$$= 15491.8 - 15129$$

$$= 362.8$$

$$SST = 31^2 + 24^2 + \dots + 20^2 - 15129$$

$$= 15649 - 15129$$

$$= 520$$

$$SSE = SST - SSC - SSR - SST_r$$

$$= 520 - 95.6 - 26.8 - 362.8$$

$$= 34.8$$

	Dof	SS	MSS	F
Column	5-1 = 4	95.6	$\frac{95.6}{4} = 23.9$	$\frac{23.9}{2.9} = 8.24$
Row	5-1 = 4	26.8	$\frac{26.8}{4} = 6.7$	$\frac{6.7}{2.9} = 2.31$
Treatment	4	362.8	$\frac{362.8}{4} = 90.7$	$\frac{90.7}{2.9} = 31.27$
error	12	34.8	$\frac{34.8}{12} = 2.9$	
Total	24	520		

$F_C > F_{0.01}(4,12)$ → Reject H_0

$F_R < F_{0.01}(4,12)$ → accept H_0

$F_{Tr} > F_{0.01}(4,12)$ → Reject H_0

there is significance difference in engines, no significance difference in mechanics, there is significance difference in fuel.