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Depth Control of a Submarine: An Application of Structured H_∞ Synthesis Method for Uncertain Models based on Interval Analysis

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Abstract— **This paper introduces a robust and structured controller synthesis method dealing with uncertain system dynamics. Postulating time-invariant and bounded errors on both system inner parameters and external disturbances this method provides formal proof of compliance to a given set of stability and performance criteria. This guarantee is obtained through the interval analysis of the closed-loop system which encompasses any possible time-domain and frequency-domain variations. Here this method is applied to the robust frequency attenuation, seen as H_∞ criteria, and stability for depth control of a submarine. This paper then emphasizes the synthesis method from model and regulation problem definitions to synthesis implementation and discuss its results.**

I. Introduction

Design of automatic controllers for submarines has been an active engineering and research domain since the end of the First World War and followed recent developments of automatic control theory. Nowadays, it can be viewed as a part of Autonomous Underwater Vehicles (AUV) applications which is a fast growing domain for both engineering application and research [1]. Developments of small autonomous system increased the demand for fast, flexible but still robust control synthesis methods that can offer a priori guarantees, making first experiments less hazardous.

In particular, there is an increasing use in submarine design for submarine prototyping such as [2]. It is an AUV designed to emulate and validate hydrodynamic behavior of full-scale submarines. Therefore, its control design requirements come from both small AUV and submarines.

Controllers synthesis for those systems are confronted with difficulties such as non-linearities, uncertainties and disturbances due to the environment.

This paper tackles the submarine depth control problem. The first studies on this topic, like [3], used pole placement or Linear-Quadratic synthesis methods. They did not offer a priori guarantees for disturbance rejections or robustness. Thus, controller validation required a large set of simulations and expensive experiments.

An attempt to use the H_∞ approach for submarine control was done in [4] using [5] to limit wave disturbances. Its solution presents some disadvantages: model uncertainties are not explicitly taken into account and the controller provided is non-structured causing difficulties for implementation, which is a well-known drawback of H_∞ controller [6].

The present work suggests to apply a structured H_∞ approach to solve the submarine depth control problem It uses an innovative and robust synthesis method able to offer a priori guarantees along with a formal proof of robustness, using interval analysis based on [7]. It is possible to translate all the further mentioned constraints into this method and to provide a satisfactory controller.

This paper is organized as follows. Section 2 offers a description of the dynamic model of a submarine and a presentation of the associated depth control problem. Section 3 defines the H_∞ problem and describes the H_∞ synthesis tools that will be used. Section 4 translates the control problem as a set of specifications for the synthesis tools. We then discuss the results and suggests some further improvements in Section 5.

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II. Submarine Depth Control

A. Submarine Dynamic Model

As much as we know, the first public description of a submarine dynamic model is given by [8] and was modified by [9]. They are a reference for multiple studies on submarine hydrodynamics, simulation and control, like in [10]. [1] provides a standard description for the dynamic behavior of a marine craft in which Feldman model description can be translated. Finally, [11] summarizes those works using the Fossen formalism as a template. It also provides a numerical application for a simulation software, based on a fictive submarine. For those reasons, we use [11] model in this paper as the reference model for both the equations and numerical values. Let's briefly introduce this model presented in Figure 1. A submarine can be interpreted as a 6-degree-of-freedom rigid body moving in a fluid. Its representation uses two coordinate systems: earth-fixed frame with a North-East-Down (NED) orientation and an origin at a fixed point on the sea surface and a body-fixed frame, centered on the center of buoyancy of the submarine. Let $\eta = (x, y, z, \phi, \theta, \psi)$ be the coordinates and the angle orientations of the submarine in the earth-fixed frame. Moreover, δ_s and δ_b denote the respective angles of the stern and bow planes, with angle orientation as presented on Figure 1.

Let $\nu = (u, v, w, p, q, r)$ be the speeds and rotation speeds around axis of the body-fixed frame. The general kinematic equation of the model is given by:

$$\dot{\eta} = J(\nu)\nu \tag{1}$$

where $J(\nu)$ is a transformation matrix between the two frames, as described in [11]. One can notices that this formula is very general and it is used in many AUV (see [12]).

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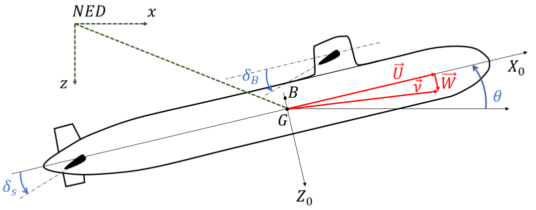

Fig. 1. Body frame and NED frame for vertical axis for a submarine vehicle

The dynamic equations of the submarine are given by the rigid-body dynamic (see equation (2)) and by a combination of hydrodynamic and hydrostatic forces (see equation (3)):

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau_{env} + \tau_{hydro} + \tau_{prop} \tag{2}$$

$$\tau_{hydro} = -M_A\dot{\nu} - C_A(\nu)\nu - D(\nu)\nu - g(\eta) \tag{3}$$

With the following parameter definitions:

- M_{RB}: (6×6) rigid-body mass and inertia matrix of the submarine.
- C_{RB}: rigid-body induced Coriolis and centripetal forces and moments.
- τ_{env}: external forces such as current forces.
- τ_{hydro}: hydrodynamic induced forces and moments.
- τ_{prop}: propulsion forces and moments.
- M_A: so called added masses and inertias, modeling water masses moved by the submarine.
- C_A: added mass induced Coriolis and centripetal forces and moments.
- D: damping forces and moments applied to the submarine body including control planes actions.
- g: hydrostatic, or restoring, forces.

Equations (1), (2) and (3) describe the dynamic behavior of the submarine state (ν, η).

B. Submarine Model for Depth Control

In the present work, we only focus on the depth control using hydroplanes. Thus, several hypothesis can be set in order to simplify the dynamic model. Due to low coupling in the model directions, it is possible to consider that both the depth and pitch do not interfere with dynamics along other axis. Obviously, depth behavior is highly correlated to the pitch. By considering the notation of [11], those previous observations allows us to only use the two non-linear equations (4) and (5) to describe the depth dynamic, as speed, heading and roll are assumed steady.

$$\begin{aligned} m(\dot{w} - uq) \quad &= \\ Z_{\dot{q}}\dot{q} + Z_{\dot{w}}\dot{w} + Z_q uq + Z_{wq} wq &+ \\ Z_\star + Z_w uw + Z_{ww} w^2 &+ \\ Z_{\delta_s} u^2 \delta_s + Z_{\delta_b} u^2 \delta_b &+ \\ (W - B)cos(\theta) & \end{aligned} \tag{4}$$

$$\begin{aligned} I_{yy}\dot{q} \quad &= \\ M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + M_q uq &+ \\ M_\star + M_w uw &+ \\ M_{\delta_s} u^2 \delta_s + M_{\delta_b} u^2 \delta_b &+ \\ B(X_B cos(\theta) + Z_B sin(\theta)) & \end{aligned} \tag{5}$$

This lets $X = (w, q, z, \theta)$ to be the degrees of freedom of the submarine and δ_s and δ_b be respectively the stern and bow hydroplane angles. Eventually, the model is linearized around a steady depth and pitch and a constant speed u_0. Its equations is given by (6). Parameter values for this linear model are given in Table I.

TABLE I
MODEL PARAMETERS OF THE SUBMARINE

Parameter	Value	Description
I_{yy}	$4,370.10^7 (kg.m^2)$	Pitch Inertia
m	$1,993.10^3 (kg)$	Submarine Mass
B	$1,955.10^7 (N)$	Buoyancy Force (Weight)
$Z_{\dot{w}}$	$-1,655.10^6$	ZZ-Added mass
$Z_{\dot{q}}$	$-1,574.10^5$	ZM-Added mass
$M_{\dot{w}}$	$-4,302.10^5$	MZ-Added mass
$M_{\dot{q}}$	$-4,062.10^8$	MM-Added mass
Z_w	$-4,746.10^4$	Damping coefficients
Z_q	$-5,024.10^5$	Damping coefficients
M_w	$5,484.10^5$	Damping coefficients
M_q	$-2,430.10^7$	Damping coefficients
Z_B	$0.1815(m)$	Center of Buoyancy on Z-Axis
$Z_{\delta s}$	$-6,850.10^3$	Hydroplane Damping coefficients
$M_{\delta s}$	$-1,922.10^5$	Hydroplane Damping coefficients

$$M\dot{X} = A_z X + B_z \delta \tag{6}$$

with:

$$M = \begin{bmatrix} m - Z_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 \\ -M_{\dot{w}} & Iy - M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; X = \begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix}$$

$$A_Z = \begin{bmatrix} Z_w u_0 & (m + Z_q)u_0 & 0 & 0 \\ M_w u_0 & M_q u_0 & 0 & BZ_B \\ 1 & 0 & 0 & -U_0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B_Z = \begin{bmatrix} Z_{ds} u_0^2 & Z_{db} u_0^2 \\ M_{ds} u_0^2 & M_{db} u_0^2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \delta = \begin{bmatrix} \delta_s \\ \delta_b \end{bmatrix}$$

For further use, let's define the submarine depth model as a complete LTI system with its (A, B, C, D) matrices. It is assumed for convenience that all the states of X are directly observable with no delay, or are provided by an observer which is not considered in this work. One can notice that M, as an inertia matrix is obviously invertible:

$$A = M^{-1} A_Z \quad ; \quad B = M^{-1} B_Z;$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and let's call $G(s)$ the transfer matrix from the command δ to the observed submarine state $Y = X$, defined in the Laplace domain. Its formula is given by equation (7) and its representation by Figure 2.

$$G(s) = C(sI - A)^{-1}B \tag{7}$$

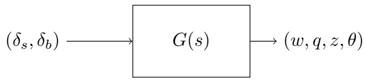

Fig. 2. Scheme for the transfer function of the submarine linearized depth model

Reliability of parameter values can be discussed. For example, hydrodynamic coefficients are generally obtained by semi-empirical methods. [10] shows a significant dispersion for obtained values between different approved methods even for first order or linear coefficients. On the other hand, even if inertia and mass terms can be known with a good precision, they can change between or during a mission due to a different vessel loading.

Let us consider the case scenario used in [2], the aim of this submarine drone is to identify hydrodynamic coefficients from its behavior. It means there is a high risk of uncertainty on their values for the first tests. On the other hand, masses and inertias can be precisely controlled and known. Those model uncertainties must be taken into account during the design control.

The case discussed in this scenario includes an uncertainty of 30% below or above the nominal value from Table I for the coefficients Z_w and M_q which are assumed to be the most significant. Figure 3 shows a set of the submarine transfer functions from the stern plane to depth for different values inside the investigated uncertainties.

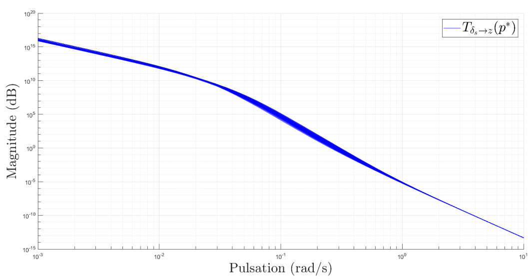

Fig. 3. Transfer Functions from the Stern Plane to the Depth of the Submarine with 30% Parameter Uncertainties

C. Depth Control Strategy

The depth controller is a critical part of a submarine control system. For high speed cruises, an inaccurate design can make the submarine emerge as well as cross the depth maximum limit resulting in a vital emergency. Most submarines use both bow and stern hydroplanes, making them able to independently control depth and pitch.

The use-case in which the current controller is applied is the same as in [13]. The submarine depth is supposed deep and wave-perturbation free. Hence, this work only involves the stern plane as a functional actuator, the bow plane is set to zero angle or retracted, which is a use case for high speed cruise.

Performance goals can be adapted to control strategies. The regulator can be specifically designed to reduce the impact of low frequency waves in the case of a low depth configuration like in [4], or in order to preserve the acoustic discretion of the hydroplanes.

Let's define the following requirements for the controller performances:

- R1 - Stabilize both depth and pitch states
- R2 - Cancel depth tracking error
- R3 - Minimize depth tracking error cancellation time
- R4 - Minimize both sensitivity of depth and stern plane moves to depth perturbations from the environment
- R5 - Minimize the overshoot during a depth transition

Requirements 1 and 2 are hard constraints. Requirements 3, 4 and 5 can be seen as possibly conflicting performance criteria.

It is assumed that the longitudinal speed of the submarine varies more slowly than the depth dynamics. Moreover, as seen in the previous section, the behavior of the submarine may be very different between slow and fast speed regime. Thus, it seems adequate to use a Linear-Parameter-Varying (LPV) control strategy, with the speed as the varying parameter. The controller will be designed for several specific functioning speeds and, as the controller is structured, it is suggested to make the gain scheduling as a simple affine interpolation through the speed.

For the sake of understanding, lets assume the desired regulator to be a full-state feedback controller. As buoyancy regulation inaccuracies can occur, we add an integral action on the depth error to counter the resulting static gap error. Controller transfer function is given by equation (8):

$$K(s) = \left[k_{dz}, \quad k_{d\theta}, \quad k_{pz} + \frac{k_{iz}}{s}, \quad k_{p\theta}\right] \tag{8}$$

with $k = (k_{dz}, k_{d\theta}, k_{pz}, k_{iz}, k_{p\theta})$ as the set of its tunable parameters.

As for the submarine dynamic model, a state-space representation of the controller is given by the system (A_k, B_k, C_k, D_k) with:

$$A_k = 0 \quad ; \quad B_k = \begin{bmatrix} 0 & 0 & k_{iz} & 0 \end{bmatrix},$$

$$C_k = 1 \quad ; \quad D_k = \begin{bmatrix} k_{dz} & k_{d\theta} & k_{pz} & k_{p\theta} \end{bmatrix},$$

and

$$K(s) = C_k(sI - A_k)^{-1}B_k + D_k \tag{9}$$

.

Figure 4 summarizes the control scheme of the problem analyzed in this work. r is the reference signal $(0,0,r_z,0)$ and K, G, X and δ_s. keep the same definitions as before. p symbolizes uncertain parameters of the submarine model.

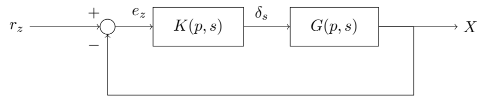

Fig. 4. Closed loop of the regulated submarine's depth

For further use, we define respectively the depth and stern plane sensitivity to depth perturbation as $T_{r_z \to e_z}$ with the equation (10) and $T_{r_z \to \delta}$ with the equation (11):

$$T_{r_z \to e_z}(k,p,s) = (I + G(p,s)K(k,s))^{-1}(3,3) \qquad (10)$$

$$T_{r_z \to \delta}(k,p,s) = K(k,s)(I+G(p,s)K(k,s))^{-1}(1,3) \qquad (11)$$

III. H_∞ Robust Control Problem

A. H_∞ Problem Formulation

H_∞ synthesis is a method to design controllers from frequency specifications. Its classical formulation involves the regulation scheme in the figure (5) where K is the the controller to compute and P is the plant to control, seen as Linear Time Invariant (LTI) systems. w represents the vector of inputs perturbations, z the regulated outputs, u is the control signal and y are the measured outputs.

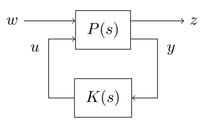

Fig. 5. Standard regulation scheme.

Let $F(P,K)$ be the Linear Fractional Transform of P and K. It can be described as a matrix of transfer functions that maps the inputs w to the outputs z, such as,

$$F(G,K) = \begin{pmatrix} T_{w \to z_1}(s) \\ \vdots \\ T_{w \to z_q}(s) \end{pmatrix}$$

where s is the Laplace variable and $T_{w \to z_i}(s) = (T_{w_1 \to z_i}(s), ..., T_{w_n \to z_i}(s))$ is a row vector that maps w to z_i.

Practically, the goal of the H_∞ synthesis is to compute a controller that minimizes the maximal response of the outputs z from inputs w over the frequencies. The maximal responses can be quantified by the H_∞ norm. With the H_∞ norm defined as $||T_{w \to z_i}||_\infty = \sup\limits_{\omega \neq 0} \frac{||z_i(j\omega)||_2}{||w(j\omega)||_2}$, the H_∞ synthesis can be defined as the following optimization problem:

$$\begin{cases} \min\limits_{K}(\max\limits_{i \in \{1,...,q\}} ||T_{w \to z_i}||_\infty) \\ \text{subject to } K \text{stabilizes } F(P,K) \end{cases} \qquad (12)$$

The plant P is built from G the plant to control, augmented with weighting filters that amplify non-desired behaviors of the objective outputs. With $T_{w \to z_i}(s)$ the ith channel of G, and W an associated weighting filter, the corresponding transfer of the plant P will be $T_{w \to \tilde{z}_i}(j\omega) = W(j\omega)T_{w \to z_i}(j\omega)$. $\tilde{z}_i$ is the weighted objective output. W can be chosen as a frequency template. If $||WT_{w \to z}||_\infty \leq 1$, then W bounds the frequency response of $T_{w \to z}$.

A adequate choice for the problem formulation and the weightings W allows the H_∞ synthesis to be a powerful tool with multiple objectives such as disturbance rejection or the minimization of tracking error.

B. Solving the H_∞ synthesis problem

The solution of the H_∞ synthesis problem can be computed by solving Riccati equations as in [5] or by using Linear Matrix Inequalities as in [14]. However those methods fail to provide structured controllers that make LPV implementation difficult.

In this paper it is suggested to use an innovative optimization algorithm to provide a structured and therefore low order solution to the H_∞ synthesis problem with a guaranteed robustness to explicit model uncertainties. This method is fully described in [7]. It suggests to formulate the H_∞ problem as a minimax global optimization problem under non-convex constraints and to solve it with a branch and bound algorithm based on interval analysis.

First we translate the standard H_∞ problem into a minimax optimization problem. The structured controller K has the parameter gains $k \in \mathbb{R}^n$ and the model uncertain parameters are defined as:

$$p \in \{[p1_{min} \quad p1_{max}], ..., [p_{m_{min}} \quad p_{m_{max}}]\}.$$

The optimization goal is to find the appropriate controller gain values k that minimize the maximal values of the H_∞ norms set through uncertain parameters. As the H_∞ norm is also the maximal gain value through pulsation, the two maximum are merged in the expression given in equation (13):

$$\begin{cases} \min\limits_{k \in \mathbb{R}^n}(\max\limits_{i \in \{1,...,q\}, p \in P, \omega \in \mathbb{R}} (|T_{w \to z_i}(j\omega,p,k)|^2)) \\ \text{subject to } K(k) \text{ stabilizes } F(P,K) \end{cases} \qquad (13)$$

There are several methods to translate the stabilization problem into a constraint satisfaction problem. The one chosen in this work will be described in the following section that is about the application of the synthesis method.

IV. Application to Submarine Depth Control

A. Translation of control goals into H_∞ criteria

In this section, the requirements from Section II-C are translated as H_∞ criteria. Requirement R3 can be explicitly interpreted as the minimization of the H_∞ norm of depth and stern plane sensitivity. Their equations are already given in equations (10) and (11). Requirement R2 and R3, dealing with the depth tracking error are related to the integral gain of the controller. An adequate use of this gain parameter can be forced by bounding depth sensitivity (10) on low frequencies. It can be suggested to bound the sensitivity with a first order ramp W_{z1}, increasing with pulsation. Its cut-off frequency will be correlated to the depth tracking error dynamics and can be chosen as a tuning parameter.

The requirement R5 more difficult to formalize and turn into frequency-domain equations. We may nevertheless assume, as it has been empirically observed, that the overshoot is tightly correlated with the maximum value of the sensitivity, or its H_∞ norm. It is suggested to bound the depth sensitivity with a constant W_{z2}, close to 0 dB. Its value can also be seen as a tuning parameter. To conclude, the depth sensitivity will be bounded by the weighting function $W_z(s) = \min(W_{z1}(s), W_{z2}(s))$. Eventually, the requirement R1 is obtained by the internal stabilization of the closed-loop system seen in Figure 4.

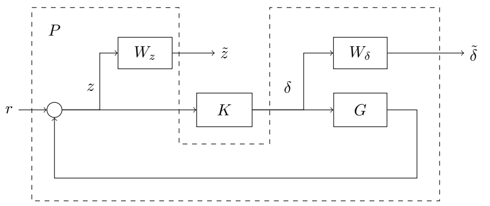

Fig. 6. Control scheme with weighting functions

Numerical values for weighting functions are given by equations (14) to (17):

$$W_{z1}(s) = 30\ s \tag{14}$$
$$W_{z2}(s) = 1.6\ (4\ \text{dB}) \tag{15}$$
$$W_z(s) = \min(W_{z1}(s), W_{z2}(s)) \tag{16}$$
$$W_\delta(s) = 1.6\ (4\ \text{dB}) \tag{17}$$

One can notice that aforementioned methods for H_∞ synthesis implied to uses a stable inverse weighting functions. It is not the case with our synthesis method which gives more freedom to the designer. We take this opportunity to make criteria formal expression as simple as possible in order to make it faster to compute using the optimization algorithm.

The control scheme presented on Figure 6 shows the location of weighting functions and the regulated outputs. It follows the standard regulation scheme on Figure 5.

Robust internal stability can be validated by the computation of an analytic criterion such as the Lienard-Chipart Criterion from [15]. It is derived from the Routh-Hurwitz Criterion but it has a reduced computation complexity. [16] shows how the validation of this criterion can be translated into a constraint satisfaction problem.

Using previous LTI formulations of controller (equation (9)) and controlled (equation (7)) systems, let $D(s)$ by the characteristic polynomial of the closed-loop system, given by equation (18):

$$D(k, p, s) = det(sI - A_c), \tag{18}$$

with

$$A_c = \begin{pmatrix} A + BD_kC & BC_k \\ B_kC & A_k \end{pmatrix}.$$

Lienard-Chipart stability criterion provides a set of coefficients built from a combination of coefficients of $D(s)$. Stability is guaranteed if all of them are strictly positive. A problem with this criterion when used with uncertain parameters is that the size of its formal expression increases dramatically with the order of D which is the order of the closed-loop system.

Let $\{T_{stab}(p, k)\}$ be the set of polynomials given by the Lienard-Chipart Criterion from $D(s)$.

Eventually, the synthesis problem is summarized by equation(19):

$$\begin{cases} \min_k(\max_p ||W_z T_{r_z \to e_z}(k, p)||_\infty, ||W_\delta T_{r_z \to \delta}(k, p)||_\infty), \\ \text{such as } T_{stab}(p, k) > 0. \end{cases} \tag{19}$$

B. Problem Implementation

The main inputs of the synthesis algorithm are formal expressions of the criteria. Their complexity increases with the number of uncertain parameters, controller gains and order of the system which increase their evaluation computation time. Moreover, as illustrated in [17], this complexity also increases evaluation pessimism, dramatically slowing the convergence of the optimization algorithm. It is possible to attenuate this phenomenon by preserving factorized forms in the expression with the polynomial Horner form. We used the Symbolic Matlab Toolbox in order to compute formal expression faster with reliability. During the implementation of the specific control problem described in this work, the stability criteria evaluation appeared to be the most critical in term of computation time and convergence.

V. Results

Here, the control problem formulated in the previous section is solved by the algorithm presented in [7]. The starting box for the gain values is $[-30, 30]$ except for the integral gain, k_{i_z} which is $[-1, 1]$. Once gives an uncertainty of 30% for the model uncertain parameters Z_w and M_q. In this configuration, the control gains found by the optimizer are given in Table II. The H_∞ norm is included in the interval $[0.3812\ 0.9388]$ which was found in about 10 minutes on a standard computer.

TABLE II
CONTROLLER GAINS

Name	Gain value	Name	Gain value
k_{p_z}	0.1763	k_{p_θ}	-23.631
k_{d_z}	0.5322	k_{d_θ}	-88.610
k_{i_z}	0.0001913		

Figure 7 shows the sensitivity function of the depth reference error and the associated weighting function. We can see that the integral action is efficient for low frequencies and that the maximal gain was successfully contained by the weighting function. We can notice that the spread due to parameter's variations is condensed, meaning that the regulated system is robust for a considered model parameter error.

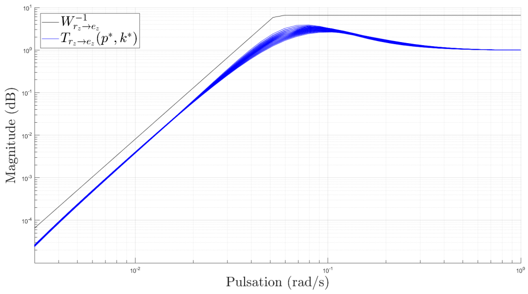

Fig. 7. Sensitivity of depth subject to depth perturbations in the closed loop system for a speed of $U = 5m/s$

Figure 8 shows the sensitivity function of the depth reference error and the associated weighting function. We notice the absence of a local maximal gain, meaning that the optimizer managed to find a controller without resonance peak for this channel, despite the lack of filters in the controller.

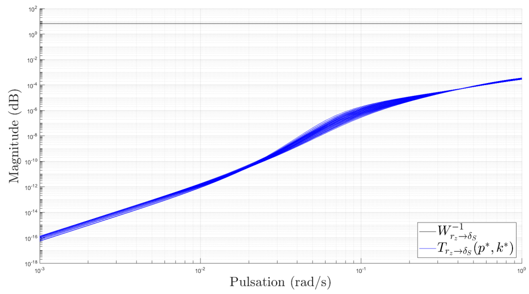

Fig. 8. Sensitivity of stern plane angle subject to depth perturbations in the closed loop system for a speed of $U = 5m/s$

Eventually, Figure 9 confirms the conclusions of the analysis of Figure 7 such as the small variation of behavior towards parameters and the mitigation of the overshoot for the depth response.

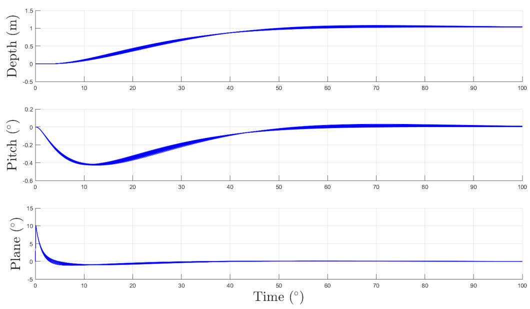

Fig. 9. Step Responses of submarine states for a speed of $U = 5m/s$

Considering those figures, the controller built by the presented method is valid and meets all specifications defined in Section II-C, and is guaranteed as optimal towards the criterion established in Section IV.

VI. Conclusions

This work presented a formulation for the regulation problem of the submarine depth using H_∞ methods, taking into account parametric model uncertainties. It achieved to find an efficient controller using an innovative synthesis algorithm. The structured nature of the controller makes it implementable into a submarine systems. Further improvements of the computation efficiency could make it possible to tackle higher order problems. As an example, the multivariate depth control at periscope depth problem using both stern and bow planes described in [18] could be a good application.

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