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ASSIGNMENT — I

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Q1 Find the Value of k such that rank of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{pmatrix}$$ is 2

Soln To make the rank (A) < No of rows i.e 2

$\det |A| = 0$

$$\Rightarrow \det \begin{vmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{vmatrix} = 0$$

$$\Rightarrow 1(10k-42) - 2(20-21) + (12-3k) = 0$$

$$\Rightarrow 10k - 42 + 2 + 12 - 3k = 0$$

$$\Rightarrow 7k - 28 = 0 \quad 7(k-4) = 0$$

$$7 \neq 0 \text{ So, } k - 4 = 0$$

$$\boxed{k = 4}$$

Q2 use row reduction Method to find the inverse of matrix

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

Sol

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right]$$

Applying RREF on $[A \mid I]$

$R_2 \leftarrow R_2 + 3R_1$

$R_3 \leftarrow R_3 + (-2)R_1$

$$\cong \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right)$$

$R_3 \leftarrow R_3 + (3)R_2$

$$\cong \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right)$$

$R_3 \leftarrow \left(\frac{1}{2}\right) R_3$

$$\cong \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right)$$

$R_2 \leftarrow R_2 + (2) R_3$

$R_1 \leftarrow R_1 + (2) R_3$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right)$$

clearly A is invertible

$$\boxed{A^{-1} = \begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{pmatrix}}$$

Q3 Find the basis for the row space, column space and the null space of the matrix given

$$A = \begin{bmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \end{bmatrix}$$

Sol

Transforming A into RREF form using elementary row operations. where

$$A = \begin{pmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} 1 & -5 & 2 & -2 \\ 3 & 4 & 0 & 7 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{pmatrix} \end{array} \begin{array}{l} r_2 \\ r_1 \\ r_3 \\ r_4 \end{array}$$

$R_2 \leftarrow R_2 + (-3)R_1$

$R_3 \leftarrow R_3 + R_1$

$R_4 \leftarrow R_4 - R_1$

$$\cong \begin{pmatrix} 1 & -5 & 2 & -2 \\ 0 & 19 & -6 & 13 \\ 0 & -1 & 2 & 1 \\ 0 & 4 & 0 & 4 \end{pmatrix} \cong \begin{pmatrix} 1 & -5 & 2 & -2 \\ 0 & -1 & 2 & 1 \\ 0 & 4 & 0 & 4 \\ 0 & 19 & -6 & 13 \end{pmatrix} \begin{array}{l} r_2 \\ r_3 \\ r_4 \\ r_1 \end{array}$$

$R_2 \leftarrow R_2 \cdot (-1)$

$$\cong \begin{pmatrix} 1 & -5 & 2 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 4 & 0 & 4 \\ 0 & 19 & -6 & 13 \end{pmatrix}$$

$R_3 \leftarrow R_3 - 4R_2$

$R_4 \leftarrow R_4 - 19R_2$

$$\simeq \begin{pmatrix} 1 & -5 & 2 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 8 & 8 \\ 0 & 0 & 32 & 32 \end{pmatrix}.$$

$R_1 \leftarrow R_1 + (5)R_2$

$R_3 \leftarrow R_3 \cdot \left(\frac{1}{8}\right)$

$R_4 \leftarrow R_4 \left(\frac{1}{32}\right)$

$$\simeq \begin{pmatrix} 1 & 0 & -8 & -7 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

$R_4 \leftarrow R_4 + (-1)R_3$

$R_2 \leftarrow R_2 + 2R_3$

$R_1 \leftarrow R_1 + 8R_3$

$$\simeq \begin{pmatrix} 1 & 0 & \cancel{0} & 1 \\ 0 & 1 & \cancel{0} & +1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$R_1 \leftarrow R_1 + (-1)R_3$

$R_2 \leftarrow R_2 + (-1)R_3$

$$\simeq \begin{array}{c} \\ r_1 \\ r_2 \\ r_3 \\ r_4 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{pmatrix} \boxed{1} & 0 & 0 & 1 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \rightarrow \text{No zero rows}$$

$$\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array}$$
$$\begin{pmatrix} \boxed{1} & 0 & 0 & 1 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Clearly we obtain pivot element in A

C_1, C_2, C_3

Col space of $A = Col(A) = L\{C_1, C_2, C_3\}$

where $C_1 = \begin{pmatrix} 3 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ $C_2 = \begin{pmatrix} 4 \\ -5 \\ 4 \\ 1 \end{pmatrix}$ $C_3 = \begin{pmatrix} 6 \\ 2 \\ 0 \\ 2 \end{pmatrix}$

& Basis for $Col(A) = \{C_1, C_2, C_3\}$

Now, since non-zero rows in row echelon form of A are 1st, 2nd & 3rd row

row space of $A = Row(A) = L\{R_2^T, R_3^T, R_4^T\}$

where $R_2 = (1, -5, 2, -2)$

$R_3 = (-1, 4, 0, 3)$

$R_4 = (1, -1, 2, 2)$

& Bases for row space is $\{R_2^T, R_3^T, R_4^T\}$

$\{(1,-5,2,-2),(-1,4,0,3),(1,-1,2,2)\}$

or

$\{(1,0,0,1),(0,1,0,1),(0,0,1,1).\}$

& Null space of A is $N(A)=\{x \,/\, A\bar{x}=\bar{0}\}$

Solution set of $A\bar{x}=\bar{0}$ is null space

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & \boxed{0} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

free variable

$x_3+x_4=0$ $\quad$ $x_2+x_4=0$ $\quad$ $x_1+x_4=0$

$\boxed{x_3=-x_4}$ $\quad$ $\boxed{x_2=-x_4}$ $\quad$ $\boxed{x_1=-x_4}$

$$N(A)=\left\{x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \; Ax=\bar{0}\right\}$$

$$=\left\{\begin{pmatrix} -x_4 \\ -x_4 \\ -x_4 \\ 1 \end{pmatrix} \; x_4 \in \mathbb{R}\right\}$$

$$= \left\{ x_4 \begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \; x_4 \in \mathbb{R} \right\}$$

Basis for Null Space $= \left\{ \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix} \right\}$ or $\begin{matrix} 1 \\ 1 \\ 1 \\ -1 \end{matrix} \}$

Q4 Suppose that the following matrix A is the augmented matrix for a system of linear equations where 'a' is a real number. Determine all the values of a so that the system is consistent.

Sol

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & -1 & -2 & a^2 \\ -1 & -7 & -11 & a \end{bmatrix}$$

Let's do some row elementary row operation to given above matrix A.

$$R_3 \leftarrow R_1 + R_3$$
$$R_2 \leftarrow R_2 + (-2) R_1$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -5 & -8 & a^2-8 \\ 0 & -5 & -8 & a+4 \end{bmatrix}$$

$$R_2 = R_2 \cdot (-1)$$
$$R_3 = R_3 \cdot (-1)$$

$$\mathring{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 8 & 8-a^2 \\ 0 & 5 & 8 & -(a+4) \end{bmatrix}$$

$$R_3 \leftarrow R_3 + (-1) R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 5 & 8 & 8-a^2 \\ 0 & 0 & 0 & -a-4+a^2-8 \end{array}\right]$$

Above matrix will be CONSISTENT only and only if

rank of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 8 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 8 & 8-a^2 \\ 0 & 0 & 0 & -a-4+a^2-8 \end{bmatrix}$

is same.

~~Since~~ $\therefore$

Clearly rank of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 8 \\ 0 & 0 & 0 \end{bmatrix} = 2$

$$\text{rank of } \begin{array}{c} \\ \\ r_3 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 8 & 8-a^2 \\ 0 & 0 & 0 & -a-4+a^2-8 \end{bmatrix} = 2$$

$\Rightarrow$ row r_3 will have all entry $=0$

$\therefore \; -a-4+a^2-8=0$

or, $a^2-a-12=0$

~~or,~~ or, $a^2-4a+3a-12=0$

or, $a(a-4)+3(a-4)=0$

either $a-4=0$ or, $a+3=0$

$a=4$ $\qquad a=-3$

$$\boxed{a=-3,4}$$

Q 5 Find the least-square solution of the inconsistent system $Ax=b$ for

$$A=\begin{bmatrix}1&1&0\\1&1&0\\1&0&1\\1&0&1\end{bmatrix} \quad b=\begin{bmatrix}1\\3\\8\\2\end{bmatrix}$$

Sol Given $A=\begin{bmatrix}1&1&0\\1&1&0\\1&0&1\\1&0&1\end{bmatrix}$. $b=\begin{bmatrix}1\\3\\8\\2\end{bmatrix}$

$$A^T=\begin{bmatrix}1&1&1&1\\1&1&0&0\\0&0&1&1\end{bmatrix}$$

$$A^T.A=\begin{pmatrix}1&1&1&1\\1&1&0&0\\0&0&1&1\end{pmatrix}\begin{pmatrix}1&1&0\\1&1&0\\1&0&1\\1&0&1\end{pmatrix}=\begin{pmatrix}4&2&2\\2&2&0\\2&0&2\end{pmatrix}$$

$$A^T.b=\begin{pmatrix}1&1&1&1\\1&1&0&0\\0&0&1&1\end{pmatrix}\begin{pmatrix}1\\3\\8\\2\end{pmatrix}=\begin{pmatrix}14\\4\\6\end{pmatrix}.$$

Now, the Agumented matrix for. $A^T.A.x=A^Tb$

is
$$\left(\begin{array}{ccc|c}4&2&2&14\\2&2&0&4\\2&0&2&6\end{array}\right)$$

$$\left(\begin{array}{ccc|c} 4 & 2 & 2 & 14 \\ 2 & 2 & 0 & 4 \\ 2 & 0 & 2 & 6 \end{array}\right)$$

$$R_1 \leftarrow \left(\frac{1}{4}\right) R_1$$

$$= \left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 7/2 \\ 2 & 2 & 0 & 4 \\ 2 & 0 & 2 & 0 \end{array}\right)$$

$$R_3 \leftarrow R_2 + (-2) R_1$$
$$R_3 \leftarrow R_3 + (-2) R_1$$

$$= \left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 7/2 \\ 0 & 1 & -1 & -3 \\ 0 & -1 & 1 & -7 \end{array}\right)$$

$$R_3 \leftarrow R_3 + R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 7/2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -10 \end{array}\right)$$

$$R_1 \leftarrow R_1 + \left(\frac{-1}{2}\right) R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & \boxed{0} & -10 \end{array}\right)$$

→ free variable

$x_3 = t$

$x_2 - x_3 = -3$

$x_2 = x_3 - 3$

x_1

$x_1 + x_3 = 5$

$x_1 = 5 - x_3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

Hence, the general least square solution of $Ax=b$ has the form

$$\hat{x} = \left\{ \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \; ; \; x_3 \in \mathbb{R} \right\}$$

for $x_3 = 1$

$$\hat{x} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

Q6 Reduce to Echelon form and find the Rank of given matrix

$$A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$$

Sol for the above given matrix A, we will do elementary row operation to reduce to Echelon form:—

$R_4 \leftarrow R_4 + (-1) R_1$
$R_5 \leftarrow R_5 + (-2) R_1$

$$\sim \begin{pmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & -2 & -4 & -1 & 0 \\ 0 & -1 & -2 & -2 & -3 \end{pmatrix}$$

$R_4 \leftarrow R_4 + 2R_2$
$R_5 \leftarrow R_5 + R_2$

$$\sim \begin{pmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & +2 \\ 0 & 0 & 0 & -1 & -2 \end{pmatrix}$$

~~$R_1 \leftarrow R_1 + (-1) R_2$~~

$R_4 \leftarrow R_4 + (-1) R_3$

$$\sim \begin{pmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -2 \end{pmatrix}$$

$\cancel{R_4 \leftarrow R_4 + (-1) R_3}$

$R_5 \leftarrow R_5 + R_3$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Number of non-zero rows = 3

So, $\rho(A) = 3$

$$\boxed{\rho(A) = 3}$$

End