Mathematical Methods for Engineers 2

Falling ladder, Swinging rod (MATLAB group project)

Late submission incurs a penalty of 10% per day, up to 40%. Work more than 4 days late will not be accepted.

Graduate Qualities:

This project is designed to help develop Graduate Qualities 1, 2 3, 4 and 5, namely Body of Knowledge, Lifelong Learning, Effective Problem Solving, Working Autonomously and Collaboratively, and Ethical Action and Social Responsibility.

Assessment:

The assessment will take into account all of your documentation of the mathematical analysis of the problem, your MATLAB or Excel file(s) (including proper parameter entry), your output, the correctness of the final solutions and the presentation of your whole report.

Ground rules for internal students

1. This is a group project, and each group should have exactly two students.
2. The two students must share the work equally between them. Each group member will be awarded individually the mark allocated to the report. If the group members have not made an equal contribution according to the details provided on the Peer Assessment form, the Course Coordinator may adjust the marks accordingly.
3. We can't have a group where one student works in Excel and the other in MATLAB. Students in the same group must be learning the same platform.
4. We will keep a list of all groups. Please advise your prac supervisor who you will partner with.
5. Each team member must participate in all aspects of the project: mathematical calculations, MATLAB/Excel work, and report writing.

   Do *not* allocate one group member the MATLAB/Excel work, and the other the task of report writing. Trust me, this never works out well.

   Do *not* allocate one person the first half of the project, and the other the second half. This doesn't work either.
6. It sometimes happens that a student will come to me close to the end of the project and say "Oh, I don't have a group yet". *Don't be that person.* It is in your interest to find a partner in good time – preferably before prac 3.
7. It sometimes happens that one partner feels as though they are carrying the group and the second partner is not contributing. If that happens, I need to be informed immediately.
8. It sometimes happens that two partners do not get along, even though both are contributing to the project. If you feel uncomfortable in your group, let me know as soon as possible.

9. If you wish, you may submit a peer assessment by using the form that can be found on the course webpage. Details for submission of the peer assessment can be found on the peer assessment sheet.

10. Under exceptional circumstances, we can allow a student to carry out the project individually. In this case, approval must be given by me. If a student elects to do the project individually, then that student must take full responsibility for the extra workload.

11. The University policy on plagiarism will apply between different groups.

Ground rules for external and OUA students

1. All of the previous rules apply. Collaboration between partners will take place online, through email, OneDrive, and Lync (a virtual classroom tool). If there is someone you wish to work with, please advise the Course Coordinator or tutor. Anyone who does not form a group will be randomly allocated a partner.

2. The University provides resources for online collaboration, found at the UniSA Anywhere site. Here is the link:

   `http://w3.unisa.edu.au/ists/new/students/software-apps/unisa-anywhere.htm`

   Internal students may also use these tools if they wish!

Presenting your work:

- Use a group coversheet.
- Your report should include:
  1. A *brief* introduction (description of the problem, objectives of your report, etc).
  2. Written worked answers to all numbered **Questions** where this is required. Some **Tasks** will also require you to write something in your report (this is clearly indicated).
  3. *Appropriately labelled* figures where required. Any necessary diagrams may be neatly hand-drawn.
  4. A brief conclusion.
- Your MATLAB or Excel files should include:
  1. Code relating to each **Task** outlined below, together with appropriate explanatory comments. Be sure to list all group members at the top of the file(s).

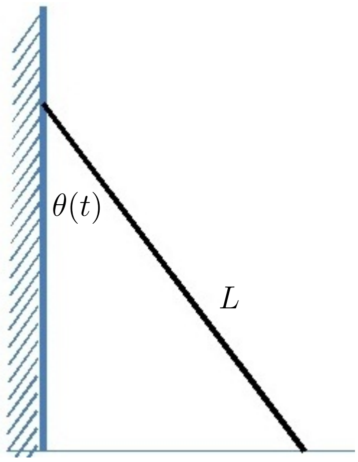
Submission:

- One member of your group should submit (1) your report, and (2) your MATLAB file(s) or Excel spreadsheet

Part 1: Motion of a ladder as it slips down a wall

Consider a uniform ladder of length $L = 5m$ and weight $W = 25kg$ which is leaning against a building. Assume the coefficient of kinetic friction between the wall and the ladder and between the floor and the ladder is $\mu_k = 0.2$.

Let $\theta(t)$ be the angle between the ladder and the vertical wall, and $\omega(t) = \dfrac{d\theta}{dt}$ be the angular velocity (in rad/sec).



Suppose, at time $t = 0$ sec, the ladder starts to slip from rest, so that $\theta(0) = \theta_0$ and $\omega(0) = 0$. Then, at that instant, a balance of forces generates the following matrix equation to describe the system behaviour:

$$\begin{bmatrix} -\mu_k & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\mu_k & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -\dfrac{W}{g} & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -\dfrac{W}{g} & 0 \\ \cos\theta_0 & -\sin\theta_0 & \sin\theta_0 & \cos\theta_0 & 0 & 0 & \dfrac{WL}{6g} \\ 0 & 0 & 0 & 0 & 2 & 0 & -L\cos\theta_0 \\ 0 & 0 & 0 & 0 & 0 & 2 & L\sin\theta_0 \end{bmatrix} \begin{bmatrix} N_t \\ N_b \\ f_t \\ f_b \\ a_x \\ a_y \\ a_\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ W \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where gravity, $g = 9.81$ m s^{-2}, and the unknown quantities can be described as follows:

- N_t is the normal force acting between the top of the ladder and the wall
- N_b is the normal force acting between the bottom of the ladder and the floor
- f_t is the friction force between the ladder and the wall
- f_b is the friction force between the ladder and the floor
- a_x is the component of the acceleration at the centre of the ladder in the horizontal direction (ms^{-2})

- a_y is the component of the acceleration at the centre of the ladder in the vertical direction (ms^{-2})
- a_θ is the angular acceleration of the ladder, $a_\theta = \dfrac{d^2\theta}{dt^2} = \dfrac{d\omega}{dt}$ (rad s^{-2})

Task 1

(a) *Assuming the ladder is initially at rest ($\omega(0) = 0$), solve for the seven output values in the matrix equation if the ladder is positioned at an angle of $\theta_0 = \pi/6$ to the wall.* **Write your answers in your report** *(to 4 decimal places).*

(b) *Set up an array of θ_0 values, $0 \le \theta_0 \le \pi/2$. Use a for loop to find the corresponding three accelerations a_x, a_y, a_θ at all of these initial values by repeatedly solving the matrix equation, and so create three arrays* `a_x`, `a_y`, `a_theta` *of initial accelerations depending upon θ_0.*

(c) *Create a plot (Figure 1) (with legend, labels on the axes, title, etc) that has θ_0 on the horizontal axis and the three initial accelerations on the vertical axis (make a_x blue, a_y red and a_θ black). You will see that the three curves all pass through the same point on the horizontal axis, corresponding to a critical value of θ_0.*

Question 1

(a) *Why are the sections to the left of the critical value of θ_0 meaningless? What should they be?*

(b) *Using the first, second, third and fifth of the linear equations (in the matrix equation) prove that if all three accelerations are initially zero, then $\tan\theta_0 = \dfrac{2\mu_k}{1-\mu_k^2}$.*

(c) *Evaluate the critical value of θ_0 (when the ladder starts from rest) (2 decimal places), giving your answer in radians. Write your answer in your report.*

Assume that the top of the ladder slips down the wall for $t > 0$ until it hits the floor. Then, at any instant, the motion of the ladder is governed by

$$\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{3}{(2-\mu_k^2)L}\left[(1-\mu_k^2)g\sin\theta - 2\mu_k g\cos\theta + \mu_k L\omega^2\right] \tag{1}$$

This nonlinear second order differential equation does not have a solution in terms of standard mathematical functions. However, MATLAB does provide a function `ode45` that solves a system of first order ODEs. Equation (1) can be rewritten as

$$\begin{aligned} \frac{d\theta}{dt} &= \omega \qquad \text{and} \\ \frac{d\omega}{dt} &= \frac{3}{(2-\mu_k^2)L}\left[(1-\mu_k^2)g\sin\theta - 2\mu_k g\cos\theta + \mu_k L\omega^2\right] \end{aligned}$$

The MATLAB function `ode45` will solve this system of first order ODEs, and produce arrays for the time, t, the angle $\theta(t)$ and the angular velocity $\omega(t)$ over some interval $0 \le t \le t_{end}$.

Task 2

(a) *Use the* `ode45` *solver to solve for the motion of the falling ladder, assuming that the initial angle of the ladder is* $\theta_0 = \pi/6$ *(which is more than the critical value).*

The MATLAB *command to use* `ode45` *is*

```
[T Z] = ode45(@fladder,[0,t_end],[theta_0,omega_0]);
```

The system of first order ODEs must be be defined in a function. To do this, you should rename the functions $\theta(t)$ *and* $\omega(t)$ *and place them in an array* `Z`*, say, where* `Z(1)`=$\theta(t)$ *and* `Z(2)`=$\omega(t)$.

The corresponding function would be of the form

```
function derivs = fladder(t,z)

global mu g L W

dz = zeros(2,1);

dz(1) =

dz(2) =

derivs =

end
```

Your parameters (μ, g, L, W) *should be defined in your main function using the* `global` *command.*

The output from `ode45` *yields a column array* `T` *of time values (chosen to maintain high accuracy) between* $t = 0$ *and* $t = t_{end}$ *, and a matrix* `Z` *of two columns, where the first column contains the corresponding* $\theta(t)$ *values and the second column contains the corresponding* $\omega(t)$ *values.*

(b) *The ladder will slip down the wall, with the motion described by the differential equations, until it hits the floor. The problem is that you do not know the time* t_{end} *when this occurs. You can assume it is at most 2.5 s. Search (by varying* t_{end}*) until you find* t_{end} *correct to 4 decimal places (***write this number in your report***), and then produce two separate graphs of* θ *against* t*, and of* ω *against* t *for* $0 \le t \le t_{\mathrm{end}}$ *(Figures 2 and 3).*

Find the angular velocity at the instant the ladder hits the floor. **Write this number in your report** *(with correct units, 4 decimal places).*

Part 2: Swinging rod

You should start a new `.m`-file for Part 2.

By now you're experts at using `ode45`. Let's look at another problem from engineering dynamics.

Consider a rod of length $L = 2\text{m}$ and mass $m = 1.2$ kg which is initially held at rest in a horizontal position. The left end is pinned, enabling the rod to swing about the pin. Assume the coefficient of friction at the pin is $c_f = 0.3$.

Let $\theta(t)$ be the angle that the rod makes with the horizontal, and let $\omega(t) = \dfrac{d\theta}{dt}$ be the angular velocity.

The dynamic equations of motion give rise to the differential equation

$$\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{3g}{2L}\cos\theta - \frac{3c_f}{mL^2}\,\text{sgn}(\omega) \tag{2}$$

$\text{sgn}(x)$ is the mathematical sign (or signum) function defined by

$$\text{sgn}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(ie it extracts the sign of the variable in the brackets) and in MATLAB is denoted `sign(x)`. The reason for the $\text{sgn}(\omega)$ term in equation (2) is that the moment of friction should always oppose the motion and therefore it will be in the opposite direction to that of the angular velocity $\omega(t)$.

Question 2

(a) *Give a physical interpretation of the initial conditions ($\theta(0) = \theta_0 = 0$ and $\omega(0) = 0$).*

(b) *Once again, the equation of motion can be rewritten as two first order equations. In your report, rewrite equation (2) as two first order equations.*

Task 3

(a) *Use* `ode45` *to produce a graph of $\theta(t)$ for the first 20 seconds (Figure 4). You will need a separate function to enter the equations to describe a swinging rod.*

(b) *Use* `max` *to estimate the maximum angle the rod reaches on its first swing, and* `min` *to find the angle the rod reaches on its first return. Give these two angles in radians, to 4 decimal places (***write these two numbers in your report***).*

(c) *Create an animation of the swinging rod for times $0 \le t \le 20$. You will need to fill in the code below to calculate the position of the swinging end of the rod. Assume that the fixed end of the rod is at (0,0) and* **provide handwritten workings (in your report)** *that show how to calculate the position of the swinging end (r_x, r_y). Include a print out of the* *final position of the rod in your report.* **Explain in your report** *precisely what the lines marked with (***) do.*

```
% animation of the swinging rod

r_x =

r_y =

figure(5)

for ii=1:length(T)                                    % (***)

  clf                                                  % (***)

  axis([-2 2 -2 0.05]), axis equal, axis manual, hold on

  plot([0 r_x(ii)],[0 r_y(ii)],'b')                    % (***)

  title(sprintf('t=%.1f seconds',T(ii)))               % (***)

  pause(0.005)                                         % (***)

end

hold off
```