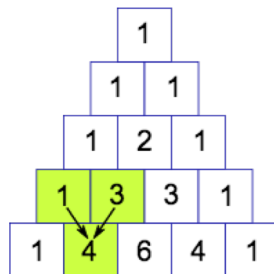# Pascal's Triangle

What is Pascal's triangle?

Pascal's triangle is a triangular array constructed by summing adjacent elements in preceding rows. The Pascal's triangle contains the values of the binomial coefficient. It is named after French mathematician Blaise Pascal.

All the extreme left and right boxes contain the number 1. The centre boxes are a sum of the two numbers above it. For example 1 + 3 = 4.

What are the uses of the Pascal's Triangle?

The Pascal's Triangle can be used to find combinations, list number patterns (triangular, tetrahedral and pentalobe) numbers, the possibility of heads and tails with a number of coins or even to show coefficients through binomial expansion.

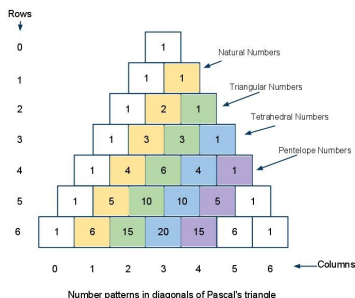| Exponent	Pascal's Triangle	Binomial Expansion
0	1	$(a+b)^0 = 1$
1	1 1	$(a+b)^1 = 1a + 1b$
2	1 2 1	$(a+b)^2 = 1a^2 + 2ab + 1b^2$
3	1 3 3 1	$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$
4	1 4 6 4 1	$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$
5	1 5 10 10 5 1	$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

Binomial expansion in the Pascal's Triangle
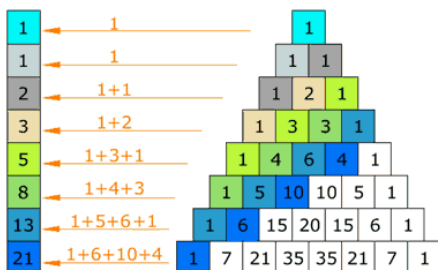
Who was Blaise Pascal?

Blaise Pascal (1623 - 1622) made many contributions and inventions in mathematics and physics. His contributions to math include Pascal's triangle and probability theory. Blaise Pascal was also one of the first few contributors to the early digital calculator.


Blaise Pascal

Patterns in the Pascal's Triangle:

Number patterns in diagonals of Pascal's triangle

Some patterns that can be seen in the pascal's triangle include the numbers found in each diagonal. The leftmost diagonal has only ones. The second diagonal has triangular numbers followed by tetrahedral numbers and then pentalobe numbers.

If you colour the Odd and Even numbers, you end up

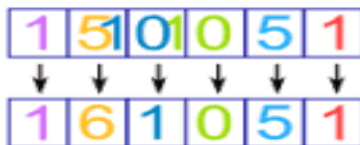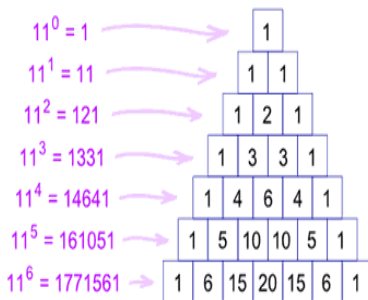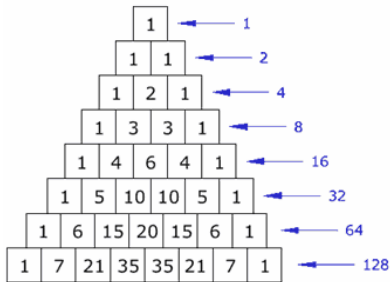with a pattern same as the Sierpinski Triangle.

If you make a pattern by going up and then along, then add up the values you will be able to form the Fibonacci sequence. (The Fibonacci Sequence starts "0, 1" and then continues by adding the two previous numbers, for example 3+5=8, then 5+8=13, etc).

For the second diagonal, the square of a number is equal to the sum of the numbers next to it and below both of those. Examples: $3^2 = 3 + 6 = 9$, $4^2 = 6 + 10 = 16$, $5^2 = 10 + 15 = 25$.
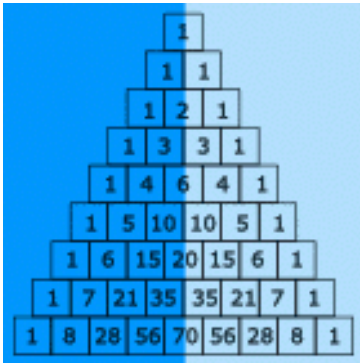

$4^2 = 6 + 10$
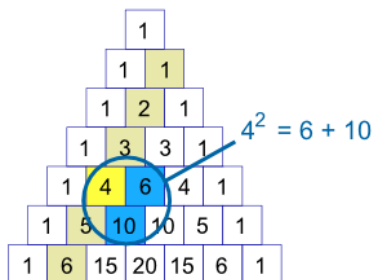
Each line is also a exponent of 11. For example the first line is 11 to the power 0 which is one, followed by 11 to the power 1 or 11. The third line is 11 to the power 2 or 121 and so on. When it comes to 11 to the power 5, the digits just overlap as shown in the second image. The same happens with 11 to the power 6.

$11^0 = 1$
$11^1 = 11$
$11^2 = 121$
$11^3 = 1331$
$11^4 = 14641$
$11^5 = 161051$
$11^6 = 1771561$

```
            1      ← 1
          1   1    ← 2
        1   2   1    ← 4
      1   3   3   1    ← 8
    1   4   6   4   1    ← 16
  1   5  10  10   5   1    ← 32
1   6  15  20  15   6   1    ← 64
1   7  21  35  35  21   7   1    ← 128
```

Another pattern can be seen in the sum of all the numbers in a row. The sums of each row double as they move to the next row (powers of 2).

```
              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5  10  10   5   1
  1   6  15  20  15   6   1
1   7  21  35  35  21   7   1
1  8  28  56  70  56  28  8  1
```

The triangle is also symmetrical. The numbers on the right occur in a mirror image on the left.

Citations:

- https://www.mathsisfun.com/pascals-triangle.html#:~:text=One%20of%20the%20most%20interesting,directly%20above%20it%20added%20together.
- https://sites.google.com/site/funproblemsolving/topics-1/pascal-s-triangle
- https://www.britannica.com/biography/Blaise-Pascal
- https://www.onlinemathlearning.com/binomial-theorem-3.html
- https://brilliant.org/wiki/pascals-triangle/#:~:text=Pascal's%20triangle%20is%20a%20triangular,Pascal%20(1623%20%2D%201662).
- https://study.com/academy/lesson/blaise-pascal-contributions-inventions-facts.html