Additive regression confidence bands

Michael C Sachs

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Question

1. Look at section 4.2.3, on martingale tests for the additive model. Can this test be inverted to obtain a confidence band procedure for the cumulative regression functions?

Solution

The test statistic (omitting the subscript q) is

$$\frac{\int_0^t L(u)\, d\hat{B}(u)}{\sqrt{\int_0^t L^2(u)\, d\hat{\sigma}(u)}},$$

where L is a predictable function, and $\hat{\sigma}$ is the qth diagonal element of the estimated covariance process. Under the null hypothesis that $B(t) = 0$ for all t, this is asymptotically standard normal.

To construct a confidence band, lets assume that $B(t) = B^*(t)$ for all t. Then with $M(t) = \hat{B}(t) - B^*(t)$ we know that

$$\frac{\int_0^t L(u)\, dM(u)}{\sqrt{\int_0^t L^2(u)\, d\hat{\sigma}(u)}},$$

is asymptotically a mean 0 Gaussian process. Hence we could find functions $l(t), u(t)$ such that

$$P\left(l(t) < \frac{\int_0^t L(u)\, dM(u)}{\sqrt{\int_0^t L^2(u)\, d\hat{\sigma}(u)}} < u(t) : t \in [0, t_0]\right) \leq 0.05.$$

Let's see if we can invert this to get an expression for a confidence band for $B^*(t)$. One easy way to do this is to assume that $L(t) = 1$, in which case we have

$$P\left(l(t)\sqrt{\hat{\sigma}(t)} < M(t) < u(t)\sqrt{\hat{\sigma}(t)} : t \in [0, t_0]\right) =$$
$$P\left(l(t)\sqrt{\hat{\sigma}(t)} < \hat{B}(t) - B^*(t) < u(t)\sqrt{\hat{\sigma}(t)} : t \in [0, t_0]\right) =$$
$$P\left(\hat{B}(t) - u(t)\sqrt{\hat{\sigma}(t)} < B^*(t) < \hat{B}(t) - l(t)\sqrt{\hat{\sigma}(t)} : t \in [0, t_0]\right).$$

Now the hard part is to find the functions $l(t), u(t)$, which it turns out is not trivial. In practice, the same procedure used to construct the Gill bands or the Hall-Wellner bands are used for the cumulative coefficients. Alternatively, in the `timereg` package, a similar procedure to this is used, but with $l(t), u(t) =$ a fixed constant that is determined based on Monte Carlo techniques.