

NA variation processes

Error on slide 21 of lecture 3 (and also on slide 5 of lecture 4)

Original:

$$\langle \int H dM \rangle = \int H d\langle M \rangle$$

and

$$[\int H dM] = \int H d[M].$$

Correct version (I omitted the H^2)

$$\langle \int H dM \rangle = \int H^2 d\langle M \rangle$$

and

$$[\int H dM] = \int H^2 d[M].$$

So for M that is the martingale of a counting process, we know that $\langle M \rangle(t) = \Lambda(t) = \int_0^t \lambda(u) du$ and $[M](t) = N(t)$.

Now for the Nelson-Aalen estimator

$$\hat{A}(t) - A^*(t) = \int_0^t \frac{J(u)}{Y(u)} dM(u).$$

Hence,

$$[\hat{A} - A^*](t) = \int_0^t \left(\frac{J(u)}{Y(u)} \right)^2 dN(u) = \int_0^t \frac{J(u)}{Y(u)^2} dN(u),$$

since $J(u)^2 = J(u)$ because it is an indicator function that can only be 0 or 1.

And

$$\langle \hat{A} - A^* \rangle(t) = \int_0^t \left(\frac{J(u)}{Y(u)} \right)^2 d\Lambda(u) = \int_0^t \frac{J(u)}{Y(u)^2} \alpha(u) Y(u) du = \int_0^t \frac{J(u)}{Y(u)} \alpha(u) du.$$