Multidimensional Scaling

Sophie Sackstein

March 2, 2018

Abstract

The purpose of multidimensional scaling (MDS) is to provide a visual representation of the pattern of proximities (i.e. similarities or distances) among a set of objects in this case A, B, C, D. Given pairwise dissimilarities we reconstruct a map that preserves distances given for each assignment

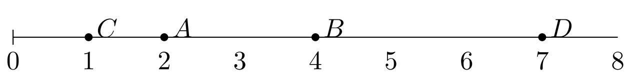
1 Assignment

Imagine four novel objects, A, B, C, D. Make up two different similarity matrices such that the conditions in each hold

1.1 MDS puts these four items in a line

Alternatively: they can be put into one dimension using MDS

	DISTANCES			
A	-			
B	2	-		
C	1	3	-	
D	5	3	6	-



1.2 MDS can put these four items into a 2D arrangement

ABCD such that that would not fit in one dimension (1D)

DISTANCES

A	-			
B	5.6	-		
C	7.0	5.2	-	
D	8.9	7.3	2.6	6.8

1.3 MDS in 3D arrangement

Suppose we found an example of a similarity matrix that did not fit well even in 2D. Is that possible in principle or not? What would that tell us about psychological space?

DISTANCES

A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0

Gram matrix B(44) with eigenvalues (.5, .5, .5, 0).In retrieving the coordinate matrix X, we cannot take a squareroot of it since it gives complex numbers. A three dimensional plot reveals a tetrahedron. Classical MDS seeks to find an optimal configuration, x_i that gives

$$d_{ij} \approx \hat{d_{ij}} = ||x_i - x_j||_2 \tag{1}$$

as close as possible.

2 Conclusion

Distance and similarity satisify a set of conditions (Similar to Tverksy's Contrast Model)
d(x, y) e 0,
d(x, y) = 0 if and only if x = y,
d(x, y) = d(y, x),
d(x, z) d d(x, y) + d(y, z) z)