BCS / CSC 229: Computer Models of Human Perception and Cognition

<u>Homework Assignment #4</u>

Instructions: Answer all questions below. Include all requested calculations and graphs. Also include all Matlab code that you used to answer these questions.

(0) (Part A) At the top of the document that you turn in, place your name and the date. (Part B) Next, please take the honor pledge. That is, write (by hand using a pen): "I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own." Then sign your name.

(1) (Problem 3.7 from the draft of the textbook by Ma, Kording, and Goldreich) In Chapters 2 and 3 (of the Ma, Kording, and Goldreich textbook), we were able to derive analytical expressions for the posterior distribution. For more complex psychophysical tasks, however, analytical solutions often do not exist. In such a case, we can use numerical methods to approximate the distribution of interest. To get some familiarity with this method, we will reconsider the cue combination experiment described in this chapter, but we will now compute the distribution of MAP estimates using numerical methods. We assume that the experimenter introduces a cue conflict between the auditory and the visual stimulus: $s_A = 5$ and $s_V = 10$. The standard deviation of the auditory and of the visual noise is $\sigma_A = 2$ and $\sigma_V = 1$, respectively. We assume a flat (uniform) prior over s.

(a) Randomly draw an auditory measurement x_A and a visual measurement x_V from their respective distributions. (It's okay if a measurement has a negative value.)

(b) Plot the corresponding elementary likelihood functions, $p(x_A|s)$ and $p(x_V|s)$, in one figure.

(c) Calculate the combined likelihood function, $p(x_A, x_V|s)$, by numerically multiplying the elementary likelihood functions in Matlab. Plot this function.

(d) Calculate the posterior distribution by normalizing the combined likelihood function. Plot this distribution in the same figure as the likelihood functions.

(e) Use Matlab to find the MAP estimate of s, i.e., the value of s at which the posterior distribution is maximal.

(f) Compare with the MAP estimate of s computed from Eq. (3.3) using the measurements drawn in (a). For convenience, here is Eq. (3.3):

$$\hat{s}_{\mathrm{MAP}} = \frac{\frac{x_A}{\sigma_A^2} + \frac{x_V}{\sigma_V^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_V^2}}$$

(g) In the above, we simulated a single trial and computed the observer's MAP estimate of s, given the noisy measurements on that trial. If an analytical solution does not exist for the distribution of MAP estimates, we can repeat the above procedure many times to approximate this distribution. Here, we practice this method even though an analytical solution is available in this case. Draw 100 pairs (x_A, x_V) and numerically compute the observer's MAP estimate for each pair as in (e).

(h) Compute the mean of the MAP estimates obtained in (g) and compare with the mean estimate predicted using Eq. (3.5). For convenience, here is Eq. (3.5):

$$\begin{aligned}
w_A &= \frac{\frac{1}{\sigma_A^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_V^2}} \\
w_V &= \frac{\frac{1}{\sigma_V^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_V^2}} \\
\langle \hat{s} \rangle &= w_A s_A + w_V s_V
\end{aligned}$$

(i) Make a histogram of the MAP estimate (in Matlab, use the "hist" function).

(j) *Relative auditory bias* is defined as the mean MAP estimate minus the true auditory stimulus, divided by the true visual stimulus minus the true auditory stimulus. Compute relative auditory bias for your set of estimates.