Lab Report

Group Project

Steven Contreras

Marwin Gonzales

Ruby Nguyen

CSULB

CECS 174

Professor Susan Tabbaa Nachawati

Source Code

```
import math
import random
import time
import string
import re
import matplotlib.pyplot as plt
# *************** constants: BEGIN ***********
N_DAYS_IN_YEAR = 365
N_DEFAULT_CLASS_SIZE = 23
N_SIM_SERIES = 6 # note that for formatting purposes, the number of series should be an even number > 0
S_ADD_SINGLETON_LIST_IDIOM = "t += [x] idiom"
S_LIST_APPEND_METHOD = "List.append() method"
TEXT_FILE_SUMMARY_TEMPLATE = """
***** SUMMARY OF TEXT FILE: {} *****
   WORD COUNT: {}
   LETTER FREQUENCY:
      ALL:
{}
{}
{}
QUIT_MESSAGE = "THAT'S ALL FOLKS! Thanks for playing. Bye bye."
# **************** constants: END ***********
# **************** functions: BEGIN ************
def is_sorted(l):
   n = len(l)
   if n == 1:
      return True
   \mbox{\#} since we compare indices i and i+1, we range from 0 to len(l)-2
   for i in range(n-1): # recall that n-1 is not inclusive when using range()
      if l[i] > l[i+1]:
          return False
```

```
return True
def test_is_sorted(l):
    b_result = is_sorted(l)
    print(f"\tTEST is_sorted(l={l}): {b_result}")
def str_to_list(s):
   This function converts a string to a list of chars (in case we want to modify the list somehow)
   return [s[i] for i in range(len(s))] if type(s) is str else s.copy()
def to_lcase(l):
   This function only convert char elements in the list to lower-case.
   Obviously, non-char elements will not be affected.
    0.00
    l_lcase = str_to_list(l)
   for i in range(len(l_lcase)):
       if type(l_lcase[i]) is str:
            l_lcase[i] = l_lcase[i].lower()
    return l_lcase
def is_anagram(l1, l2, normalize_char_case=True):
   Normally anagrams are based on words only.
    But this function supports numeric lists as anagrams, as well..
    n1 = len(l1)
    n2 = len(12)
   # we can short-circuit when the lengths are unequal
   if n1 != n2:
        return False
   # we normally disregard case when considering char anagrams
   if normalize_char_case:
        l1 = to_lcase(l1)  # but remember, the to_lcase() leaves non-char elements alone
        l2 = to_lcase(l2)
   else: # in case strings and we don't want to normalize to lcase
       l1 = str_to_list(l1)
       l2 = str_to_list(l2)
```

```
# here we sort each list
   # this greatly simplifies the problem compared to not sorting
   if not is_sorted(l1):
        l1 = sorted(l1)
   if not is_sorted(l2):
        l2 = sorted(l2)
    # because the two lists are sorted, we can now short-circuit (exit the loop) when we encounter the first mismatch
    for i in range(n1):
        if l1[i] != l2[i]:
           return False
    # if we made it this far then the two lists are necessarily the same length and have the same elements (unless case
matters, FOR STRINGS, and case differs for some element)
    return True
def test_is_anagram(l1, l2, normalize_char_case=True):
    b_result = is_anagram(l1, l2, normalize_char_case)
    print(f"\tTEST is_anagram(l1={l1}, l2={l2}, normalize_char_case={normalize_char_case}): {b_result}")
def has_duplicates(l, disregard_char_case=False):
    n = len(l)
   # a 0 or single element list is already implicitly sorted, so we can short-circuit
    if n < 2:
        return False
    if not disregard_char_case:
        l = to_lcase(l)
   else: # in case string and we don't want to normalize to lcase
       l = str_to_list(l)
   # here we sort the list
    \# this greatly simplifies the problem compared to not sorting
   if not is_sorted(l):
        l = sorted(l)
    # because the list is sorted, we can now short-circuit (exit the loop) when we encounter the first matching
adjacent pair of elements
   # range from 0 to len(l)-2
   for i in range(n-1): # recall that n-1 is not inclusive when using range()
       if l[i] == l[i+1]:
           return True
   return False
```

```
def test_has_duplicates(l, disregard_char_case=False):
    b_result = has_duplicates(l, disregard_char_case=disregard_char_case)
    print(f"\tTEST has_duplicates(l={l}, disregard_char_case={disregard_char_case}): {b_result}")
def run_bd_paradox_sim(n_sims, n_class_size=N_DEFAULT_CLASS_SIZE, is_leap_year=False):
    print(f"Running {n_sims} Birthday Paradox simulations on a class size of {n_class_size} students...")
    p = 0
   n_dups = 0
    x = []
   y = []
    for i_sim in range(n_sims):
        l_birthdays = [random.randint(1, N_DAYS_IN_YEAR + (1 if is_leap_year else 0)) for i in range(n_class_size)]
        n_dups += 1 if has_duplicates(l_birthdays) else 0
       x.append(i_sim)
        y.append(n_dups / (i_sim+1))
    p = n_dups / n_sims
    print(f"\tDONE: The probability that at least 2 students from a class size of {n_class_size} have the same birthday
converged to {p} after {n_sims} simulations.")
    return p, x, y
def run_bd_paradox_sim_series(n_powers_of_ten, n_class_size=N_DEFAULT_CLASS_SIZE, is_leap_year=False, do_plot=True):
    exponents = list(range(1,n_powers_of_ten+1))
    n_{cols} = 2
    n_rows = len(exponents) // n_cols
    if do_plot:
        fig, axes = plt.subplots(n_rows, n_cols, figsize=(8,4))
    for i, e in enumerate(exponents):
        n_sims = 10**e
       p, x, y = run_bd_paradox_sim(n_sims=n_sims, n_class_size=N_DEFAULT_CLASS_SIZE, is_leap_year=False)
       if do_plot:
            axis = axes[i//n_cols][i%n_cols]
            axis.set_title(f''# sims = \{n_sims\}, p = \{p\}")
            axis.plot(x, y)
    if do_plot:
        fig.tight_layout()
        plt.show()
def remove_duplicates(l):
    l = str_to_list(l) # in case l is a string
   n = len(l)
```

```
# a 0 or single element list is already implicitly sorted, so we can short-circuit
   if n < 2:
        return l
   # here we sort the list
    # this greatly simplifies the problem compared to not sorting
   if not is_sorted(l):
        l = sorted(l)
    l_dups_removed = []
    # iterate from index 0 to len(l)-2
    # only add the last non-repeating element, which we can do since the list has been sorted
    for i in range(n-1): # recall that n-1 is not inclusive when using range()
        if l[i] != l[i+1]:
            l_dups_removed.append(l[i])
   # but we still have the very last index to add
   # this works since if there was a dup at the n-2 index, it will not have been added to l_dups_removed
    l_dups_removed.append(l[n-1])
    return l_dups_removed
def test_remove_duplicates(l):
    b_result = remove_duplicates(l)
    print(f"\tTEST remove_duplicates(l={l}): {b_result}")
def words_file_to_list(fname, use_list_append=True):
    l_words = []
    try:
       with open(fname, 'r') as f_words:
            for words_line in f_words:
                for word in words_line.split():
                    if use_list_append:
                        l_words.append(word.strip()) # dynamically resizes
                        l_words += [word] # adding to separate lists (which exist in two different places in memory)
            f_words.close()
    except Exception as e:
        print(f"words_file_to_list: ***RUNTIME ERROR caught***: {e}")
    return l_words
def benchmark_words_file_to_list(fname, use_list_append, debug=False):
   if debug:
```

```
s_append_list_mechanic = ("t += [x] idiom" if not use_list_append else S_LIST_APPEND_METHOD)
       print(f"Benchmarking '{fname}' file to words list (using {s_append_list_mechanic})")
   # timestamp for start of the execution of words_file_to_list()
    t0 = time.time()
   # execute words_file_to_list()
   l_words = words_file_to_list(fname)
   # timestamp for end of the execution of words_file_to_list()
   t1 = time.time()
   # the delta is just the elapsed time
   t_delta = t1 - t0
   if debug:
       print(f"\ttime elapsed (using {s_append_list_mechanic}): {t_delta} seconds") # CPU seconds elapsed (floating
point)
   return t_delta, l_words
def run_benchmark_words_file_to_list_series(fname, n_sims, debug=False):
   n_list_append_more_efficient = 0
   n_add_singleton_list_idiom_more_efficient = 0
   print(f"Running {n_sims} words_file_to_list() iterations...")
   for i_sim in range(n_sims):
       # benchmark using List.append()
       if debug:
           print()
       # benchmark using the t += [x] idiom
       tdelta\_add\_list\_idiom, \ l\_words = benchmark\_words\_file\_to\_list(fname, \ use\_list\_append=False, \ debug=debug)
       # the rest of this code is just for formatting the summary output when debugging
       s_mech = ""
       eff_factor = 0
       # update summary and count of times the particular mechanic is more efficient
       if tdelta__add_list_idiom < tdelta__list_append:</pre>
           s_mech = S_ADD_SINGLETON_LIST_IDIOM
           eff_factor = tdelta__list_append / tdelta__add_list_idiom
           n_add_singleton_list_idiom_more_efficient += 1
       else:
           s_mech = S_LIST_APPEND_METHOD
```

```
eff_factor = tdelta__add_list_idiom / tdelta__list_append
            n_list_append_more_efficient += 1
        s_efficiency = f"{s_mech} is {eff_factor} more efficient!"
        if debug:
            \# print(f"\n{s_efficiency}\n\n{l_words}") \# uncomment this to see l_words
            print(f"\n{s_efficiency}")
    # always display summary after all simulations are complete
    eff_ratio__list_append = n_list_append_more_efficient/n_sims
    eff_ratio__add_singleton_list = 1 - eff_ratio__list_append
    print(f"\tDONE: Out of {n_sims} iterations, {S_LIST_APPEND_METHOD} was more efficient
{round(eff_ratio_list_append,2)*100}% of the time, while {S_ADD_SINGLETON_LIST_IDIOM} was more efficient
{round(eff_ratio__add_singleton_list,2)*100}% of the time.")
def bisect(l, i_lb, i_ub, target_value, debug=False):
    This function implements what is also known as 'binary search'.
    Adapted from https://www.geeksforgeeks.org/python-program-for-binary-search/
    Even though the spec we are given idicates we can assume l is already sorted, we will double-check and do the
sorting just in case.
    parameters:
              the list (elementas should all be of the same type and comparable)
        i_lb: the index lower-bound of l to search
        i_ub: the index upper-bound of l to search
    From our spec:
        'to check whether a <value> is in the list'
        'returns the index of the value in the list, if it's there, or None if it's not'
    # short-circuit for 0-length and singleton lists, this is also the "base case" when recursion is used (but we will
go the iteration route instead of recursion)
    n = len(l)
    if n == 0:
       return None
    if n == 1:
        return 0 if l[0] == target_value else None
    # is_sorted check: avoid performance hit (checking if sorted only at top level) - i.e. only when i_lb==0 AND
i_ub==len(l)-1
    if i_lb==0 and i_ub==len(l)-1:
      if not is_sorted(l):
```

```
if debug:
                                    print("\tl is not sorted! sorting...")
                           l = sorted(l)
                           if debug:
                                    # print(f"\t\tsorted l: {l}")
                                    print(f"\t\tDONE")
                  print(f"\tbisecting l for target value -->{target_value}<-- ...")</pre>
         # if we are here, we are guaranteed that l is sorted... now we can implement proper binary search logic
         # first step is to validate that i_ub >= i_lb
        if i_ub >= i_lb:
                 # since we are here, we can proceed with "bisecting"
                  # so the first thing we need to do is find the midpoint between i_ub and i_lb: this is the basis of
"bisection"
                  i_midpoint = (i_ub + i_lb) // 2  # integer division
                  val_at_midpoint = l[i_midpoint]
                  if debug:
                           print(f"\t\mbox{ti_lb}) \ and \ \{i\_b\} \ and \ \{i\_b\} \ is: \{i\_midpoint\} \ and \ \{i\_b\} \ and \ and \ \{i\_b\} \ and \ and \ \{i\_b\} \ and \
l[{i_midpoint}]=={val_at_midpoint}")
                  # if target_value is at index i_midpoint, return i_midpoint
                  if val_at_midpoint == target_value:
                                    print(f"\t\target value -->{target_value}<-- found at midpoint index {i_midpoint}")</pre>
                           return i_midpoint
                  else: # we have already excluded the equality case
                           # now we use the fact that elements in l are comparable... this happens recursively
                           if target_value < val_at_midpoint: # then we look in the left half... this is the binary split</pre>
                                    return bisect(l, i_lb, i_midpoint, target_value, debug)
                           else: # otherwise we look in the right half... this happens recursively
                                    return bisect(l, i_midpoint, i_ub, target_value, debug)
         else: # i_ub < i_lb (which is illogical, therefore return None)</pre>
                  return None
def test_bisect(l, i_lb, i_ub, target_value, debug=True):
         result = bisect(l, i_lb, i_ub, target_value, debug=debug)
         print(f"\tTEST bisect(l={l if len(l)<50 else '<l contents SUPRESSED due to length>'}, i_lb={i_lb}, i_ub={i_ub},
target_value={target_value}): {result}")
def process_token_to_word(tkn):
```

```
.....
    This function's sole purpose is to "clean" a token and return a word (or None if the token is not actually a word).
    For instance, we want to strip preceding and trailing whitespace if any exists.
    We also want to remove punctuation characters.
    if tkn is None:
       return None
    tkn = tkn.strip()
    # strip punctuation
    tkn = re.sub(r"[^\w\s]", "", tkn)
    if len(tkn) == 0:
        return None
    return tkn
def tokens_list_to_inverted_index(l_tokens, fn_process_token_to_word=process_token_to_word, debug=False):
    This function converts a list of tokens into two dictionaries:
        1. the first dictionary is keyed by each unquie word and the corresponding value is the count of that word
       2. the second dictionary is keyed by each unquie character and the corresponding value is the count of that
character
    The above happens AFTER the token is cleaned by the function specified by the fn_process_token_to_word argument
    d_w_index = {}
    d_c_index = {}
    for tkn in l_tokens:
        w = fn_process_token_to_word(tkn)
        if w is not None:
            w_lower = w.lower()
            d_w_index[w_lower] = d_w_index.get(w_lower, 0) + 1
            for c in w:
                d_c_index[c] = d_c_index.get(c, 0) + 1
    return d_w_index, d_c_index
def summarize_text_file(fname):
```

```
.....
         This function opens a text file and summarizes its word count and letter count.
         Note that case matters!
         arguments:
         returns:
                   1. the summary string, which is formatted, containg the summary statistics:
                              1. the frequency count that each (case-sensitive) letter occurs (out of the total number of letters), as
well as the frequency ratio (as a percentage)
                             2. the frequency (count and ratio) of upper-case letters as a group
                             3. the frequency (count and ratio) of lower-case letters as a group
                   2. a dictionary keyed by words, containing the count of each unique word
                   3. a dictionary keyed by letter, containing the count of each unique letter
         0.00
         l_words = words_file_to_list(fname, use_list_append=False)
         d_w_index, d_c_index = tokens_list_to_inverted_index(l_words)
         n_words = 0
         for k in d_w_index.keys():
                   n_words += d_w_index[k]
         \hbox{\# re-arrange d\_c\_index so that keys are in alphabetical order (based on sorted() order)}
         d_c_index = {k:d_c_index[k] for k in sorted(d_c_index.keys())}
         # count all letters (so that we can provide frequency of each letter as a ratio or percentage)
         n_c_all = sum([n_c for _, n_c in d_c_index.items()])
         # now create separate counts of upper and lower case letter (and create formatted individual letter freq summary)
         s_letter_freq__all = ""
         n_c_uc = 0
         n_c_lc = 0
         for c, n_c in d_c_index.items():
                   s_{\text{tht}} + \text{witht} + \text{LETTER\_FREQ\_TEMPLATE.format}(c, n_c, n_c_all, (n_c/n_c_all)*100) + \text{whrese substitution} + \text
                   if c.isalpha():
                             if c.isupper():
                                       n_c_uc += n_c
                             else:
                                       n_c_lc += n_c
```

```
# now create summary strings of upper and lower case freqs
   s\_letter\_freq\_\_ucase = "\t" + LETTER\_FREQ\_TEMPLATE.format("UPPER-CASE", n\_c\_uc, n\_c\_all, (n\_c\_uc/n\_c\_all)*100)
   s_letter_freq__lcase = "\t" + LETTER_FREQ_TEMPLATE.format("LOWER-CASE", n_c_lc, n_c_all, (n_c_lc/n_c_all)*100)
   # return entire formatted summary string as well as the dictionaries (in case we want to use them later)
   return TEXT_FILE_SUMMARY_TEMPLATE.format(
       fname,
       n_words,
       s_letter_freq__all,
       s_letter_freq__ucase,
       s_letter_freq__lcase
   ), d_w_index, d_c_index
def words_file_to_toggle_case(fname_in, fname_out):
   try:
       with open(fname_in, 'r') as f_words_in:
          with open(fname_out, 'w') as f_words_out:
              for words_line_in in f_words_in:
                  words_line_out = ""
                  for c_in in words_line_in:
                      if c_in.isalpha():
                         if c_in.isupper():
                             words_line_out += c_in.lower()
                         else:
                             words_line_out += c_in.upper()
                     else:
                         words_line_out += c_in
                  f_words_out.write(words_line_out)
              f_words_out.close()
              print(f"{fname_out} file written")
           f_words_in.close()
   except Exception as e:
       print(f"words_file_to_toggle_case: ***RUNTIME ERROR caught***: {e}")
# **************** functions: END **********
BEGIN ************************
if __name__ == '__main__':
   print("Testing is_sorted()...")
   test_is_sorted([1,2,2])
   test_is_sorted(['b','a'])
   test_is_sorted(['b','a','b'])
   print()
```

```
print("Testing is_anagram()...")
    test_is_anagram("never", "REven")
    test_is_anagram("steve", "STEVEN")
    print()
    print("Testing has_duplicates()...")
    test_has_duplicates([1,2,3])
    test_has_duplicates([1,2,1])
    test_has_duplicates(['a','b','c'])
    test_has_duplicates(['a','b','a'])
    test_has_duplicates(["steven", "steve"])
    test_has_duplicates(["steven", "steven"])
    test_has_duplicates([0])
    test_has_duplicates([])
    print()
    run_bd_paradox_sim_series(N_SIM_SERIES) # note that for formatting purposes, the number of series should be an even
number > 0
    print()
    print("Testing remove_duplicates()...")
    test_remove_duplicates("steven")
    test_remove_duplicates([1,2,3,4,2,1])
    test_remove_duplicates(['s','t','e','v','e','n'])
    print()
    fname = "mobysmall.txt"
   N_ITERATIONS = 100000
    run_benchmark_words_file_to_list_series(fname, N_ITERATIONS, debug=False) # set debug to True will produce A LOT of
output... use it only if you think something is broken
    print()
    print("Testing bisect()...")
    print(f"\tloading word list from {fname}...")
    l_words = words_file_to_list(fname, use_list_append=False)
    print(f"\t\tDONE")
    test_bisect(l_words, 0, len(l_words)-1, "a", debug=True) # set debug to False for less output
    print()
    s_text_file_summary, _, d_c_index = summarize_text_file(fname)
    print(s_text_file_summary)
```

```
words_file_to_toggle_case("mobysmall.txt", "mobysmall-case-toggled.txt")
   try:
      with open("mobysmall-summary.txt", "w") as f_summary_out:
         f_summary_out.write(s_text_file_summary)
         f_summary_out.close()
         print(f"mobysmall-summary.txt file written")
   except Exception as e:
      print(f"write textfile summary: ***RUNTIME ERROR caught***: {e}")
   # histogram (bar chart) creation
   fig = plt.figure(figsize=(8,4))
   x = d_c_index.keys()
   y = [d_c_index[c] for c in x]
   plt.bar(x, y)
   plt.title(f"Letter frequency for {fname}")
   plt.show()
   print(f"\n{QUIT\_MESSAGE}\n\n")
```

Problems (with associated Source Code)

Α

Write a function called **is_sorted** that takes a **list** as a parameter and returns **True** if the list is **sorted in ascending** order and **False** otherwise.

You can assume that the elements of the list can be compared with the relational operators <, >, etc. For example, is_sorted(['b','a']) should return Frue and is_sorted(['b','a']) should return False.

Answer/Source Code

```
def is_sorted(l):
    n = len(l)
    if n == 1:
        return True
```

```
# since we compare indices i and i+1, we range from 0 to len(l)-2
for i in range(n-1): # recall that n-1 is not inclusive when using range()
   if l[i] > l[i+1]:
       return False

return True
```

If we let sequence (list) L be defined by elements e_1, \ldots, e_k , where L has length k, then, by definition, L, is *sorted* if an only if, for ALL all adjacent pairs of elements, we have $e_i \le e_{i+1}$.

Thus, L is not *sorted*, or fails the condition, when we have $e_i > e_{i+1}$ for ANY pair of adjacent elements. Otherwise, L is *sorted*.

В

Two words are anagrams if you can rearrange the letters from one to spell the other. Write a function called **is_anagram** that takes two strings and returns True if they are anagrams.

Answer/Source Code Primary Function

```
def is_anagram(l1, l2, normalize_char_case=True):
    """
    Normally anagrams are based on words only.
    But this function supports numeric lists as anagrams, as well.
    """
    n1 = len(l1)
    n2 = len(l2)

# we can short-circuit when the lengths are unequal
    if n1 != n2:
        return False

# we normally disregard case when considering char anagrams
    if normalize_char_case:
        l1 = to_lcase(l1)  # but remember, the to_lcase() leaves non-char elements alone
        l2 = to_lcase(l2)
    else: # in case strings and we don't want to normalize to lcase
```

```
l1 = str_to_list(l1)
    l2 = str_to_list(l2)

# here we sort each list
# this greatly simplifies the problem compared to not sorting
if not is_sorted(l1):
    l1 = sorted(l1)
    if not is_sorted(l2):
        l2 = sorted(l2)

# because the two lists are sorted, we can now short-circuit (exit the loop) when we encounter the first mismatch
for i in range(n1):
    if l1[i] != l2[i]:
        return False

# if we made it this far then the two lists are necessarily the same length and have the same elements (unless case
matters, FOR STRINGS, and case differs for some element)
    return True
```

Helper Functions

Discussion

Given sequence (list) L_1 , defined by elements e_1, \ldots, e_k , where L_1 has length k, and L_2 , defined by elements f_1, \ldots, f_k , where L_2 has length l, then, by definition, L_1 and L_2 are said to be anagrams of one another, if an only if:

- 1. k = l, i.e. L_1 and L_2 are the same length
- 2. upon sorting L_1 and L_2 in non-decreasing order, for EVERY index i, we have $e_i = f_i$, i.e. every corresponding element in each list, after sorting, is identically equal to the other.

Thus, L_1 is not an anagram of L_2 (and vice versa) if not both conditions (1) and (2) hold, i.e. if L_1 and L_2 are different lengths, OR there is any index, i, where $e_i \neq f_i$.

C

The Birthday Paradox:

Write a function called has_duplicates that takes a list and returns True if there is any element that appears more than once. It should not modify the original list.

If there are 23 students in your class, what are the chances that two of you have the same birthday? You can estimate this probability by generating random samples of 23 birthdays and checking for matches. Hint: you can generate random birthdays with the randint function in the random module.

Answer/Source Code

Primary Function

```
def run_bd_paradox_sim(n_sims, n_class_size=N_DEFAULT_CLASS_SIZE, is_leap_year=False):
    print(f"Running {n_sims} Birthday Paradox simulations on a class size of {n_class_size} students...")
    p = 0
    n_dups = 0
    x = []
    y = []
    for i_sim in range(n_sims):
        l_birthdays = [random.randint(1, N_DAYS_IN_YEAR + (1 if is_leap_year else 0)) for i in range(n_class_size)]
        n_dups += 1 if has_duplicates(l_birthdays) else 0
        x.append(i_sim)
        y.append(n_dups / (i_sim+1))
    p = n_dups / n_sims
    print(f"\tDONE: The probability that at least 2 students from a class size of {n_class_size} have the same birthday
converged to {p} after {n_sims} simulations.")
```

Helper Functions

```
def run_bd_paradox_sim_series(n_powers_of_ten, n_class_size=N_DEFAULT_CLASS_SIZE, is_leap_year=False, do_plot=True):
    exponents = list(range(1,n_powers_of_ten+1))
```

```
n_cols = 2
n_rows = len(exponents) // n_cols
if do_plot:
    fig, axes = plt.subplots(n_rows, n_cols, figsize=(8,4))

for i, e in enumerate(exponents):
    n_sims = 10**e
    p, x, y = run_bd_paradox_sim(n_sims=n_sims, n_class_size=N_DEFAULT_CLASS_SIZE, is_leap_year=False)
    if do_plot:
        axis = axes[i//n_cols][i%n_cols]
        axis.set_title(f"# sims = {n_sims}, p = {p}")
        axis.plot(x, y)

if do_plot:
    fig.tight_layout()
    plt.show()
```

```
def has_duplicates(l, disregard_char_case=False):
    n = len(l)
   # a 0 or single element list is already implicitly sorted, so we can short-circuit
    if n < 2:
        return False
   if not disregard_char_case:
        l = to_lcase(l)
   else: # in case string and we don't want to normalize to lcase
        l = str_to_list(l)
   # here we sort the list
   # this greatly simplifies the problem compared to not sorting
   if not is_sorted(l):
        l = sorted(l)
    # because the list is sorted, we can now short-circuit (exit the loop) when we encounter the first matching
adjacent pair of elements
   # range from 0 to len(l)-2
   for i in range(n-1): # recall that n-1 is not inclusive when using range()
       if l[i] == l[i+1]:
           return True
   return False
```

The core logic of the *Birthday Paradox* is housed in the $run_bd_paradox_sim()$ function. This function simply implements the logic specified in the reference document https://en.wikipedia.org/wiki/Birthday problem. As specified in the reference document, the function executes some fixed number of trials (tracked in n_sims) wherein each trial:

- 1. simulates a list of 23 random birthdays in a given year, i.e. creates a list of 23 random integers in the range [1, 365] (we assume non leap-year)
- 2. checks if there are any duplicate values via the $n_{as_duplicates()}$ function and increments n_dups by 1 if true

After all n_sims are executed, $run_bd_paradox_sim()$ computes the convergent probability (over n_sims trials) that at least 2 out of 23 students will have the same birthday as $\frac{n_dups}{n\ sims}$.

run_bd_paradox_sim() executes a single "batch" of n_sims where $n_sims = 10^i$. But run_bd_paradox_sim_series() wraps run_bd_paradox_sim() to execute from 1 to k batches of simulations where, if a batch is indexed by $1 \le i \le k$, then each batch executes $n_sims = 10^i$ simulations (as defined above). For example, if k = 6, then batch 1 will execute $10^1 = 10$ simulations, batch 2 will execute $10^2 = 100$, ..., (up to) batch 6 will execute $10^6 = 1000000$ (one million) simulations.

With this approach, we show that the probability that at least 2 students out of a class of 23 converges to approximately 0.507311. We note that the reference document has "exact" probability (computed combinatorically) as 0.50730. But since we are dealing with a discrete computational problem, our estimate (after one million simulations) is arguably "pretty darn good".

In order to make it abundantly clear how the number of simulations play out to produce the convergent probability, we end this problem by generating a visual plot of each series of simulations (see output section).

D

Write a function called **remove_duplicates** that takes a list and returns a new list with only the unique elements from the original. Hint: they don't have to be in the same order.

Answer/Source Code

Primary Function

```
def remove_duplicates(l):
   l = str_to_list(l) # in case l is a string
    n = len(l)
    # a 0 or single element list is already implicitly sorted, so we can short-circuit
   if n < 2:
        return l
    # here we sort the list
    # this greatly simplifies the problem compared to not sorting
    if not is_sorted(l):
        l = sorted(l)
    l_dups_removed = []
    # iterate from index 0 to len(l)-2
    # only add the last non-repeating element, which we can do since the list has been sorted
    for i in range(n-1): # recall that n-1 is not inclusive when using range()
        if l[i] != l[i+1]:
            l_dups_removed.append(l[i])
    # but we still have the very last index to add
    # this works since if there was a dup at the n-2 index, it will not have been added to l_dups_removed
    l_dups_removed.append(l[n-1])
    return l_dups_removed
```

Helper Functions

```
def str_to_list(s):
    """

This function converts a string to a list of chars (in case we want to modify the list somehow)
    """

return [s[i] for i in range(len(s))] if type(s) is str else s.copy()
```

```
def is_sorted(l):
    n = len(l)

if n == 1:
    return True

# since we compare indices i and i+1, we range from 0 to len(l)-2

for i in range(n-1): # recall that n-1 is not inclusive when using range()
    if l[i] > l[i+1]:
        return False

return True
```

The $_{\text{remove_duplicates}()}$ function depends on the core concept of sorting. So, if the list is not sorted, we sort it (in non-decreasing order). This allows us to exploit the ordering (comparability) of any two adjacent elements e_i and e_{i+1} in list L. L will have duplicates if and only if there exists any index, i, such that $e_i = e_{i+1}$. We have thus described the logic housed within the $_{\text{has_duplicates}()}$ function. But $_{\text{remove_duplicates}()}$ uses the complement of the $_{\text{has_duplicates}()}$ function to identify when a repeating sequence of duplicates terminates and then adds only the terminal element in that (repeating) sequence to the list of distinct/unique elements. Finally, since the very last element in the original list will always be considered unique by following this logic – note that we must index only up to the length minus 2 – we can always safely consider the last element unique.

Ε

Write a function that reads the file words.txt (You create your own file) and builds a list with one element per word. Write two versions of this function, one using the **append method** and the other using the idiom $\mathbf{t} = \mathbf{t} + [\mathbf{x}]$. Which one takes longer to run? Why?

use the time module to measure elapsed time check the following example.

```
import time
t0= time.time()
print("Hello")
t1 = time.time() - t0
print("Time elapsed: ", t1) # CPU seconds elapsed (floating point)
```

Answer/Source Code

Primary Function

def words_file_to_list(fname, use_list_append=True):

Helper Functions

```
def benchmark_words_file_to_list(fname, use_list_append, debug=False):
    if debug:
        s_append_list_mechanic = ("t += [x] idiom" if not use_list_append else S_LIST_APPEND_METHOD)
        print(f"Benchmarking '{fname}' file to words list (using {s_append_list_mechanic})")

# timestamp for start of the execution of words_file_to_list()
    t0 = time.time()

# execute words_file_to_list()
    l_words = words_file_to_list(fname)

# timestamp for end of the execution of words_file_to_list()
    t1 = time.time()

# the delta is just the elapsed time
    t_delta = t1 - t0

if debug:
    print(f"\ttime elapsed (using {s_append_list_mechanic}): {t_delta} seconds") # CPU seconds elapsed (floating point)

return t_delta, l_words
```

```
def run_benchmark_words_file_to_list_series(fname, n_sims, debug=False):
    n_list_append_more_efficient = 0
    n_add_singleton_list_idiom_more_efficient = 0
    print(f"Running {n_sims} words_file_to_list() iterations...")
```

```
for i_sim in range(n_sims):
        # benchmark using List.append()
        {\tt tdelta\_list\_append}, \ l\_words = benchmark\_words\_file\_to\_list(fname, \ use\_list\_append={\tt True}, \ debug=debug)
        if debug:
            print()
        # benchmark using the t += [x] idiom
        tdelta__add_list_idiom, l_words = benchmark_words_file_to_list(fname, use_list_append=False, debug=debug)
        # the rest of this code is just for formatting the summary output when debugging
        s_mech = ""
        eff factor = 0
        # update summary and count of times the particular mechanic is more efficient
        if tdelta__add_list_idiom < tdelta__list_append:</pre>
            s_mech = S_ADD_SINGLETON_LIST_IDIOM
            eff_factor = tdelta__list_append / tdelta__add_list_idiom
            n\_add\_singleton\_list\_idiom\_more\_efficient \; += \; 1
        else:
            s_mech = S_LIST_APPEND_METHOD
            eff_factor = tdelta__add_list_idiom / tdelta__list_append
            n_list_append_more_efficient += 1
        s_efficiency = f"{s_mech} is {eff_factor} more efficient!"
        if debug:
             \begin{tabular}{ll} \# \ print(f"\n\{s\_efficiency\}\n\n\{l\_words\}") \ \# \ uncomment \ this \ to \ see \ l\_words \end{tabular} 
            print(f"\n{s_efficiency}")
    # always display summary after all simulations are complete
    eff_ratio__list_append = n_list_append_more_efficient/n_sims
    eff_ratio__add_singleton_list = 1 - eff_ratio__list_append
    print(f"\tDONE: Out of {n_sims} iterations, {S_LIST_APPEND_METHOD} was more efficient
{round(eff_ratio_list_append,2)*100}% of the time, while {S_ADD_SINGLETON_LIST_IDIOM} was more efficient
{round(eff_ratio__add_singleton_list,2)*100}% of the time.")
```

Instead of writing two separate functions for this problem, we decided to write a single function, words_file_to_list(), and make it behave differently depending on the value of the use_list_append (boolean) argument. If True, the function will obviously use the List.append()

Method. Otherwise, the function will use the l_words += [word] idiom. But the words_file_to_list() function simply opens the file and then inserts each word into a list. It is the benchmark_words_file_to_list() function that does the benchmarking, by wrapping timestamps around the call(s) to words_file_to_list(), and then taking the difference to get the elapsed time.

The behavior seems to be inconsistent from one single run of $words_{file_{to_{list}()}}$ to the next, however. So, we took a *simulation aggregate* approach to this problem, similar to the approach to the Birthday Paradox problem. That is, we run n_{sims} simulations and compute the percentage of times out of n_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other. The m_{sims} times that one of the two mechanics is more efficient (faster) than the other.

F

To check whether a word is in the word list, you could use the **in** operator, but it would be slow because it searches through the words in order.

Because the words are in alphabetical order, we can speed things up with a bisection search (also known as binary search), which is similar to what you do when you look a word up in the dictionary.

You start in the middle and check to see whether the word you are looking for comes before the word in the middle of the list. If so, then you search the first half of the list the same way. Otherwise you search the second half. Either way, you cut the remaining search space in half. If the word list has 113,809 words, it will take about 17 steps to find the word or conclude that it's not there.

Write a function called **bisect** that takes a sorted list and a target value and returns the index of the value in the list, if it's there, or None if it's not.

Answer/Source Code

Primary Function

```
def bisect(l, i_lb, i_ub, target_value, debug=False):
    """
    This function implements what is also known as 'binary search'.
    This implementation is based on RECURSION and is adapted from https://www.geeksforgeeks.org/python-program-for-binary-search/
    Even though the spec we are given indicates we can assume l is already sorted, we will double-check and do the sorting just in case.

parameters:
    l: the list (elements should all be of the same type and comparable)
    i_lb: the index lower-bound of l to search
    i_ub: the index upper-bound of l to search

From our spec:
    'to check whether a <value> is in the list'
    'returns the index of the value in the list, if it's there, or None if it's not'
    """
```

```
# short-circuit for 0-length and singleton lists, this is also the "base case" when recursion is used (but we will
go the iteration route instead of recursion)
   n = len(l)
   if n == 0:
       return None
   if n == 1:
       return 0 if l[0] == target_value else None
    # is_sorted check: avoid performance hit (checking if sorted only at top level) - i.e. only when i_lb==0 AND
i_ub==len(l)-1
   if i_lb==0 and i_ub==len(l)-1:
       if not is_sorted(l):
           if debug:
                print("\tl is not sorted! sorting...")
            l = sorted(l)
           if debug:
                # print(f"\t\tsorted l: {l}")
                print(f"\t\tDONE")
       print(f"\tbisecting l for target value -->{target_value}<-- ...")</pre>
   # if we are here, we are guaranteed that l is sorted... now we can implement proper binary search logic
   # first step is to validate that i_ub >= i_lb
   if i_ub >= i_lb:
       # since we are here, we can proceed with "bisecting"
       # so the first thing we need to do is find the midpoint between i_ub and i_lb: this is the basis of
"bisection"
       i_midpoint = (i_ub + i_lb) // 2  # integer division
       val_at_midpoint = l[i_midpoint]
        if debug:
            print(f"\t\tmidpoint (index) of l between index {i_lb} and {i_ub} is: {i_midpoint} and
l[{i_midpoint}]=={val_at_midpoint}")
       # if target_value is at index i_midpoint, return i_midpoint
       if val_at_midpoint == target_value:
            if debug:
                print(f"\t\ttarget value -->{target_value}<-- found at midpoint index {i_midpoint}")</pre>
           return i_midpoint
       else: # we have already excluded the equality case
            # now we use the fact that elements in l are comparable... this happens recursively
            if target_value < val_at_midpoint: # then we look in the left half... this is the binary split</pre>
                return bisect(l, i_lb, i_midpoint, target_value, debug)
            else: # otherwise we look in the right half... this happens recursively
```

```
return bisect(l, i_midpoint, i_ub, target_value, debug)
else: # i_ub < i_lb (which is illogical, therefore return None)
return None</pre>
```

Helper Functions

```
def is_sorted(l):
    n = len(l)

if n == 1:
    return True

# since we compare indices i and i+1, we range from 0 to len(l)-2

for i in range(n-1): # recall that n-1 is not inclusive when using range()
    if l[i] > l[i+1]:
        return False

return True
```

Discussion

The bisect() function is a straightforward *recursive* implementation of the classic *binary search algorithm* (see https://www.khanacademy.org/computing/computer-science/algorithms/binary-search/a/binary-search). We adapted our specific implementation from https://www.geeksforgeeks.org/python-program-for-binary-search/. Even though the reference documentation provides a description of how the binary search algorithm works, we would like to provide our own summary, as follows:

- 1. specify the inclusive range of indices (i_lb and i_ub) to search for target_value within list l
- 2. if the list is empty, return None
- 3. if the list has only a single element:
 - a. if that element is target_value return 0
 - b. otherwise, return None
- 4. the core logic:
 - a. if i_ub >= i_lb

 i. find i_midpoint = (i_ub + i_lb) // 2 # integer division

 ii. set val_at_midpoint = l[i_midpoint]

 iii. compare target_value to val_at_midpoint

- 1. if val_at_midpoint equal to target_value, then we have found the target_value in list 1 at index i_midpoint ... return the index i_midpoint
- otherwise, we need to check in one of the two "halves" of bisected at index i_midpoint
 - a. if target_value is strictly less than val_at_midpoint then we search the <u>left</u> "half" by (recursively) calling bisect() with arguments:
 - i. i_lb unchanged
 - ii. set i_ub = i_midpoint
 - b. otherwise, we search the <u>right</u> "half" by (recursively) calling <code>bisect()</code> with arguments:
 - i. set i_lb = i_midpoint
 - ii. i_ub unchanged
- b. otherwise i_ub < i_lb and we have non-sensical range for the upper and lower bounds... return None

This logic guarantees that recursively calling <code>bisect()</code> will always find <code>target_value</code> if it exists in list <code>l</code>, or will return <code>None</code> if it doesn't. But either way, <code>bisect()</code> will not recurse infinitely. Note that this implementation of binary search will return the index of the sorted list (assuming <code>target_value</code> was in the list), not where it was in the original (unsorted list).

G

Write a function that counts the number of times each unique letter (character) occurs in a sentence entered by the user or in words in a list, and then output the result for each character in the sentence.

H- Read the Moby Deck text file. Find out the:

- 1. Number of words in the file.
- 2. The frequency of each letter in the file
- 3. The frequency of upper case letters file
- 4. The frequency of lower case letters in the file

- 5. Convert all uppercase to lower case and vise versa, write in a new file.
- 6. Towards the end of the file (or in a new file) write the results of 1 4 in an output text file
- 7. Plot the letter frequency.

Answer/Source Code

Primary Function

```
def summarize_text_file(fname):
    This function opens a text file and summarizes its word count and letter count.
   Note that case matters!
    arguments:
        fname
    returns:
        1. the summary string, which is formatted, containing the summary statistics:
            1. the frequency count that each (case-sensitive) letter occurs (out of the total number of letters), as
well as the frequency ratio (as a percentage)
            2. the frequency (count and ratio) of upper-case letters as a group
            3. the frequency (count and ratio) of lower-case letters as a group
       2. a dictionary keyed by words, containing the count of each unique word
       3. a dictionary keyed by letter, containing the count of each unique letter
    .....
    l_words = words_file_to_list(fname, use_list_append=False)
    d_w_index, d_c_index = tokens_list_to_inverted_index(l_words)
    n words = 0
    for k in d_w_index.keys():
       n_words += d_w_index[k]
    # re-arrange d_c_index so that keys are in alphabetical order (based on sorted() order)
    d_c_index = {k:d_c_index[k] for k in sorted(d_c_index.keys())}
    # count all letters (so that we can provide frequency of each letter as a ratio or percentage)
    n_c_all = sum([n_c for _, n_c in d_c_index.items()])
    # now create separate counts of upper and lower case letter (and create formatted individual letter freq summary)
    s_letter_freq__all = ""
    n_c_uc = 0
```

```
n_c_lc = 0
for c, n_c in d_c_index.items():
                     s_{\text{tht}_{\text{freq}}} = \text{ltht}_{\text{freq}} = \text{ltht}_{\text{freq}} + \text{letter}_{\text{freq}} = \text{ltht}_{\text{freq}} = \text{ltht}_{\text{freq}} + \text{ltht}_{\text{freq}} = \text{ltht}_{\text{freq}} = \text{ltht}_{\text{freq}} + \text{ltht}_{\text{freq}} = \text{ltht}_{\text{freq}} = \text{ltht}_{\text{freq}} + \text{ltht}_{\text{freq}} = \text{ltht}_{\text{freq}} =
                     if c.isalpha():
                                          if c.isupper():
                                                                 n_c_uc += n_c
                                                                n_c_lc += n_c
# now create summary strings of upper and lower case freqs
 s_letter_freq__ucase = "\t" + LETTER_FREQ_TEMPLATE.format("UPPER-CASE", n_c_uc, n_c_all, (n_c_uc/n_c_all)*100)
 s_letter_freq__lcase = "\t" + LETTER_FREQ_TEMPLATE.format("LOWER-CASE", n_c_lc, n_c_all, (n_c_lc/n_c_all)*100)
return TEXT_FILE_SUMMARY_TEMPLATE.format(
                      fname,
                    n_words,
                     s_letter_freq__all,
                     s_letter_freq__ucase,
                      s_letter_freq__lcase
 ), d_w_index, d_c_index
```

Helper Functions

```
def tokens_list_to_inverted_index(l_tokens, fn_process_token_to_word=process_token_to_word, debug=False):
    """
    This function converts a list of tokens into two dictionaries:
        1. the first dictionary is keyed by each unquie word and the corresponding value is the count of that word
        2. the second dictionary is keyed by each unquie character and the corresponding value is the count of that character
```

```
The above happens AFTER the token is cleaned by the function specified by the fn_process_token_to_word argument

"""

d_w_index = {}

d_c_index = {}

for tkn in l_tokens:

w = fn_process_token_to_word(tkn)

if w is not None:

w_lower = w.lower()

d_w_index[w_lower] = d_w_index.get(w_lower, 0) + 1

for c in w:

d_c_index[c] = d_c_index.get(c, 0) + 1

return d_w_index, d_c_index
```

```
def process_token_to_word(tkn):
    """
    This function's sole purpose is to "clean" a token and return a word (or None if the token is not actually a word).
    For instance, we want to strip preceding and trailing whitespace if any exists.

We also want to remove punctuation characters.
    """

if tkn is None:
    return None

tkn = tkn.strip()

# strip punctuation
tkn = re.sub(r"[^\w\s]", "", tkn)

if len(tkn) == 0:
    return None

return tkn
```

The logic of the <code>summarize_text_file()</code> function is very straightforward. It first loads the text file specified via the <code>words_file_to_list()</code> function. Then it calls <code>tokens_list_to_inverted_index()</code> to create the required dictionaries: the first dictionary is keyed by each unique word and the corresponding value is the count of that word, and the second dictionary is keyed by each unique character

and the corresponding value is the count of that character. But when <code>tokens_list_to_inverted_index()</code> creates these dictionaries, it first calls <code>process_token_to_word()</code> in order to "clean" a token and return a word (or <code>None</code> if the token is not actually a word). Note that this function strips punctuation characters by using <code>regular expressions</code>.

Converting lower to upper-case (and vice versa) characters occurring within "mobysmall.txt" is very straightforward. The words_file_to_toggle_case() function that accomplishes this is as follows:

```
def words_file_to_toggle_case(fname_in, fname_out):
    try:
        with open(fname_in, 'r') as f_words_in:
            with open(fname_out, 'w') as f_words_out:
                for words_line_in in f_words_in:
                    words_line_out = ""
                    for c_in in words_line_in:
                        if c in.isalpha():
                            if c_in.isupper():
                                words_line_out += c_in.lower()
                                words_line_out += c_in.upper()
                            words_line_out += c_in
                    f_words_out.write(words_line_out)
                f_words_out.close()
                print(f"{fname_out} file written")
            f words in.close()
    except Exception as e:
        print(f"words_file_to_toggle_case: ***RUNTIME ERROR caught***: {e}")
```

This function simply opens "mobysmall.txt" for reading, and "mobysmall-case-toggled.txt" for writing. It then reads "mobysmall.txt" line-by-line, followed up by parsing each line read into each corresponding character. If the character is alphabetic, it then simply toggles the case of that character – from upper to lower and vice versa. It then writes that modified character (in the same order) to "mobysmall-case-toggled.txt". That is all. Very straightforward.

As for problem "H", we chose to "absorb" it into this problem (G) since problem "H" only involves writing the results of summarize_text_file() to a new text file (which we named "mobysmall-summary.txt"). The code that accomplishes the creation of this file uses the results output

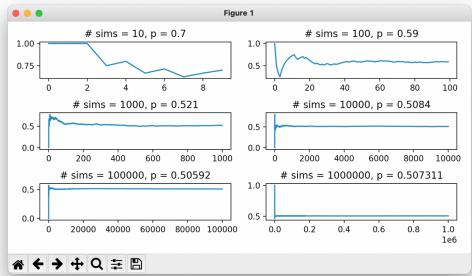
from the prior call to <code>summarize_text_file()</code>, and is toward the end of the "main" driver code and is all but self-explanatory:

```
try:
    with open("mobysmall-summary.txt", "w") as f_summary_out:
        f_summary_out.write(s_text_file_summary)
        f_summary_out.close()
        print(f"mobysmall-summary.txt file written")
    except Exception as e:
    print(f"write textfile summary: ***RUNTIME ERROR caught***: {e}")
```

Finally, we conclude with the creation of the "histogram" – it's actually a "bar" plot – of the letter frequencies of the characters within the "mobysmall.txt" file. Again, we leverage the results output from the prior call to $summarize_text_file()$ – the d_c_index dictionary, specifically, which is keyed by each unique character and the corresponding value is the count of that character. The matplotlib library is used to generate the bar plot, with the (alphabetically) sorted sequence of characters as the "x" axis and the counts of each character (provided by the d_c_index dictionary) as the "y" axis. The code that accomplishes this is as follows.

```
# histogram (bar chart) creation
fig = plt.figure(figsize=(8,4))
x = d_c_index.keys()
y = [d_c_index[c] for c in x]
plt.bar(x, y)
plt.title(f"Letter frequency for {fname}")
plt.show()
```

Screenshots of Test Case Executions



```
Testing remove_duplicates()...
    TEST remove_duplicates(l=steven): ['e', 'n', 's', 't', 'v']
    TEST remove_duplicates(l=[1, 2, 3, 4, 2, 1]): [1, 2, 3, 4]
    TEST remove_duplicates(l=['s', 't', 'e', 'v', 'e', 'n']): ['e', 'n', 's', 't', 'v']

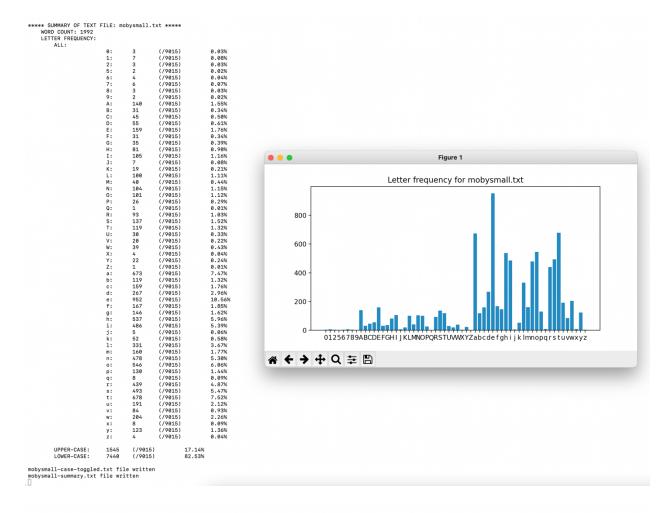
Running 100000 words_file_to_list() iterations...
    DONE: Out of 100000 iterations, List.append() method was more efficient 30.0% of the time, while t += [x] idiom was more efficient 70.0% of the time.

Testing bisect()...
    loading word list from mobysmall.txt...
    DONE

1 is not sorted! sorting...

DONE

bisecting 1 for target value -->a<-- ...
    midpoint (index) of 1 between index 0 and 2001 is: 1000 and 1[1000]==if
    midpoint (index) of 1 between index 0 and 1000 is: 550 and 1[550]==againe
    midpoint (index) of 1 between index 0 and 500 is: 250 and 1[250]==IN
    midpoint (index) of 1 between index 250 and 500 is: 375 and 1[375]==TEN
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 430 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 430 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 430 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between index 437 and 500 is: 437 and 1[437]==WhALE.
    midpoint (index) of 1 between i
```



THAT'S ALL FOLKS! Thanks for playing. Bye bye.

Video

You can view our video at https://youtu.be/3X-L12RSRwA.

References

Birthday problem - Wikipedia. (2021). Retrieved from https://en.wikipedia.org/wiki/Birthday_problem

Cormen, T., Balkcom, D., & Khan Academy. (2021). Binary search. Retrieved from https://www.khanacademy.org/computing/computer-science/algorithms/binary-search/a/binary-search

Python Program for Binary Search (Recursive and Iterative) - GeeksforGeeks. (2021). Retrieved from https://www.geeksforgeeks.org/python-program-for-binary-search