Punto 2

Encontrar el error para la regla de trapecio simple

$\Rightarrow \boxed{h = b-a}$, $E = \int_a^b \epsilon(x)\,dx = \frac{-h^3}{12} f''(\xi)$

entonces para $f(x)$ continua y derivable clase C^2 en $[a,b]$

$$f(x) = P_1(x) + \epsilon(x)$$

& $\Rightarrow \epsilon(x) = \frac{f''(\xi)}{2}(x-a)(x-b)$, $a \leq \xi \leq b$

ahora

$$E = \int_a^b \epsilon(x)\,dx = \int_a^b \frac{f''(\xi)}{2}(x-a)(x-b)\,dx$$

$$\rightarrow \frac{f''(\xi)}{2}\int_a^b (x-a)(x-b)\,dx$$

$$\Rightarrow \frac{f''(\xi)}{2}\int_a^b x^2 - bx - ax + ab\,dx$$

$$\Rightarrow \frac{f''(\xi)}{2}\left(\frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + \frac{abx}{2}\right)\Big|_a^b$$

$$\Rightarrow \frac{f''(\xi)}{2}\left(\frac{b^3}{3} - \frac{a^3}{3} - \frac{b^3}{2} + \frac{ba^2}{2} - \frac{ab^2}{2} + \frac{a^3}{2} + ab^2 - a^2b\right)$$

$$\Rightarrow E = \frac{f''(\xi)}{2}\left(\frac{-b^3+a^3-3ba^2+3ab^2}{6}\right)$$

$$\Rightarrow \frac{f''(\xi)}{2}\left(\frac{-(b-a)^3}{6}\right)$$

$$\Rightarrow \frac{f''(\xi)}{12}\left[-(b-a)^3\right]$$

$$\boxed{b-a = h}$$

$$\Rightarrow \boxed{\frac{-h^3}{12} f''(\xi)}$$