5 $D^4(x_j) \simeq \dfrac{f(x_j+2) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_j-1) + f(x_j-2)}{h^4}$

Usamos Series de Taylor

$f(x_j+2) = f(x_j) + 2hf'(x_j) + \frac{4h^2}{2!}f''(x_j) + \frac{(2h)^3}{3!}f'''(x_j) + \frac{(2h)^4}{4!}f''''(x_j) + \ldots + O(h^2)$

$-4f(x_j+1) = -4f(x_j) - 4hf'(x_j) + \frac{4h^2}{2!}f''(x_j) + \frac{(4h)^3 f'''(x_{j+1})}{3!} + \frac{(4h)^4}{4!}f''''(x_{j+1}) + \ldots + O(h^2)$

$6f(x_j) = 6f(x_j)$

$-4f(x_j-1) = -4f(x_j) + 4hf'(x_{j-1}) + \frac{4h^2}{2}f''(x_{j-1}) + \frac{(4h)^3}{3!}f'''(x_{j-1}) + \frac{(4h)^4}{4!}f''''(x_{j-1}) + \ldots + O(h)$

$f(x_j-2) = f(x_{j-2}) - 2hf'(x_{j-2}) + \frac{(2h)^2 f''(x_{j-2})}{2!} + \frac{(2h)^3}{3!}f'''(x_j-2) + \frac{(\cancel{4}2h)^4}{4!}f''''(x_j-2) + \ldots + O(h^2)$

cancelamos terminos y obtenemos

$f(x_j+2) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_j-2) = h^4 f''''(x_j) + O(h^2)$

entonces obtenemos la definicion de

$$\boxed{D^4 f(x_j) = \frac{f(x_j+2) - 4f(x_j+1) + 6f(x_j) - 4f(x_j-1) + f(x_j-2)}{h^4} \quad \text{con un error de } O(h^2)}$$