Punto 3

$$I = \int_a^b f(x)\,dx$$ Metodo Simpson Simple 1/3

El este metodo se usa un polinomio interpolador de grado 2

$$f(x) = P_2(x) = \frac{(x-b)(x-X_m)}{(a-b)(a-X_m)} f(a) + \frac{(x-a)(x-b)}{(X_m-a)(X_m-b)} f(X_m) + \frac{(x-a)(x-X_m)}{(b-a)(b-X_m)} f(b)$$

dondo $\boxed{X_m = \frac{a+b}{2}}$, $x \in [a,b]$, $\boxed{h = \frac{b-a}{2}}$

ahora

$$I = \int_a^b f(x)\,dx = \int_a^b P_2(x)\,dx$$

$$\Rightarrow \int_a^b P_2(x)\,dx \Rightarrow \int_a^b \frac{(x-b)(x-X_m)}{(a-b)(a-X_m)} f(a) + \int_a^b \frac{(x-a)(x-b)}{(X_m-a)(X_m-b)} f(X_m) + \int_a^b \frac{(x-a)(x-X_m)}{(b-a)(b-X_m)} f(b)$$

$$\Downarrow$$

$$I \Rightarrow I_1 + I_2 + I_3$$

entonces

$$I_1 = \int_a^b \frac{(x-b)(x-X_m)}{(a-b)(a-X_m)} f(a)\,dx \Rightarrow \frac{f(a)}{(a-b)(a-X_m)} \int_a^b (x-b)(x-X_m)\,dx$$

entonces $\int_a^b (x-b)(x-x_m)\,dx \Rightarrow x-x_m \frac{(x-b)^2}{2}\Big|_a^b - \frac{1}{2}\int_a^b (x-b)^2\,dx$

$= -(a-x_m)\frac{(a-b)^2}{2} - \frac{(x-b)^3}{6}\Big|_a^b$

$\Rightarrow -(a-x_m)\frac{(a-b)^2}{2} + \frac{(a-b)^3}{6} \Rightarrow -(-h)\frac{(-2h)^2}{2} + \frac{-2h^3}{6}$

$\Rightarrow 2h^3 - \frac{8h^3}{6} \Rightarrow \boxed{\frac{2h^3}{3}}$

ahora

$I_1 = \frac{f(a)}{(a-b)(a-x_m)} \cdot \frac{2h^3}{3} = \frac{f(a)}{2h^2} \cdot \frac{2h^3}{3} \Rightarrow \boxed{\frac{h}{3} f(a)}$

ahora

$I_2 = \frac{f(x_m)}{(x_m-a)(x_m-b)} \int_a^b (x-a)(x-b)\,dx$

$\Rightarrow \int_a^b (x-a)(x-b)\,dx \Rightarrow (x-a)\frac{(x-b)^2}{2}\Big|_a^b - \frac{1}{2}\int_a^b (x-b)^2\,dx$

$\Rightarrow -\frac{(x-b)^3}{6}\Big|_a^b \Rightarrow \frac{(a-b)^3}{6} \Rightarrow \frac{(2h)^3}{6} \Rightarrow \frac{-8h^3}{6} \Rightarrow \boxed{-\frac{4}{3}h^3}$

$I_2 = \frac{f(x_m)}{(x_m-a)(x_m-b)} \cdot \frac{-4}{3}h^3 \Rightarrow \boxed{\frac{4}{3}h\, f(x_m)}$

$$I_3 = \frac{f(b)}{(b-a)(b-X_m)} \int_a^b (x-a)(x-X_m)\,dx$$

$$\Rightarrow \int_a^b (x-a)(x-X_m)\,dx \Rightarrow \frac{(x-X_m)(x-a)^2}{2}\Bigg|_a^b - \frac{1}{2}\int_a^b (x-a)^2\,dx$$

$$\Rightarrow (b-X_m)\frac{(b-a)^2}{2} - \frac{(x-a)^3}{6}\Bigg|_a^b \Rightarrow$$

$$\Rightarrow b-X_m \frac{(b-a)^2}{2} - \frac{(b-a)^3}{6} \Rightarrow h\cdot\frac{(2h)^2}{2} - \frac{(2h)^3}{6} \Rightarrow \boxed{\frac{2h^3}{3}}$$

entonces

$$\Rightarrow I_3 = \frac{f(b)}{(b-a)(b-X_m)} \cdot \frac{2h^3}{3} = \boxed{\frac{h}{3} f(b)}$$

por lo tanto

$$I = I_1 + I_2 + I_3 = \boxed{\frac{h}{3}\left(f(a) + 4f(X_m) + f(b)\right)}$$