# Lab Assignment 7

Submit it to iCollege dropbox by **10/21 11:00PM**. Late assignment submissions will receive 50% of the original score.

**Question 1.** Show the sum of the first *n* positive even integers is $n^2 + n$.

1. Rephrase the statement as a formal math expression.

$$\sum_{i=1}^{n} 2i = n^2 + n$$

2. Prove it using mathematical induction.

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|---|---|
| $2(1) = (1)^2 + (1) \rightarrow 2 = 2$ | Base Case |
| $\sum_{i=1}^{n+1} 2i = (n+1)^2 + (n+1)$ | Inductive Hypothesis |
| $(n^2 + n) + 2(n+1) = (n+1)^2 + (n+1)$ | Inductive Step |
| $n^2 + n + 2n + 2 = (n+1)^2 + (n+1)$ | |
| $n^2 + 3n + 2 = (n+1)^2 + (n+1)$ | LHS |
| $n^2 + 3n + 2 = n^2 + 2n + 1 + n + 1$ | |
| $n^2 + 3n + 2 = n^2 + 3n + 2$ | RHS |

**Question 2.** Show that $3^n \geq 2n + 5$ for all $n > 1$ with mathematical induction.

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|---|---|
| $n = 2 \quad 3^{(2)} \geq 2(2) + 5 \rightarrow 9 = 9$ | Base Case |
| $3^n \geq 2n + 5 \qquad n > 1$ | Inductive Hypothesis |
| $3^{n+1} \geq 2(n+1) + 5 \rightarrow 2n + 7$ | Inductive Step |
| $3^n \cdot 3 \rightarrow 3(2n+5) \rightarrow 6n + 15$ | |
| $6n + 15 \geq 2n + 7$ | Place 2n+7 to the left side. |
| $4n + 8 \geq 0$ | |
| $n + 2 \geq 0$ | |
| Inductive step holds | |